# Online Appendix of the article "Expectations, Stagnation and Fiscal Policy: a Nonlinear Analysis"

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Note: the equation and footnote numbers continue from those of the main article.

## A Derivations of model equations, section 2

**Consumption decisions:** The consumption Euler equation is

$$
\begin{aligned}
(c_{t,i} + \xi g_t)^{-1} &= \beta R_t \hat{E}_{t,i} \left( \pi_{t+1}^{-1} (c_{t+1,i} + \xi g_{t+1})^{-1} \right) \\
&= \beta \hat{E}_{t,i} \left( r_{t+1} (c_{t+1,i} + \xi g_{t+1})^{-1} \right),
\end{aligned} \tag{16}
$$

provided $c_{t,i} > 0$. The household's consumption decision rule is obtained by combining iterations of this with the household intertemporal budget constraint and its perceived intertemporal budget constraint for the government.

Ricardian households are assumed to internalize the intertemporal budget constraint (IBC) of the government. The flow budget constraint of the government is

$$
b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + r_t b_{t-1}, \tag{17}
$$

where we now write $r_t = R_{t-1}\pi_t^{-1}$. Setting $\Delta_t = g_t - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}$ we have

$$b_t = \Delta_t + r_t b_{t-1.}$$

Note that $\Upsilon_t + m_t - m_{t-1}\pi_t^{-1}$ is total tax revenue, equal to the sum of lump-sum taxes and seigniorage.

Substituting in recursively we obtain

$$0 = r_t b_{t-1} + \sum\nolimits_{j=1}^{s} D_{t,t+j}^{-1}\Delta_{t+j} + \Delta_t - D_{t,t+s}^{-1} b_{t+s} \text{ where } D_{t,t+j} = \prod\nolimits_{i=1}^{j} r_{t+i}.$$

Imposing $\lim_{s\to\infty} D_{t,t+s}^{-1} b_{t+s} = 0$ gives the IBC of the government,

$$0 = r_t b_{t-1} + \sum\nolimits_{j=0}^{\infty} D_{t,t+j}^{-1}\Delta_{t+j},$$

where for convenience we set $D_{t,t} = 1$.

For households the flow budget constraint

$$c_{t,i} + m_{t,i} + b_{t,i} + \Upsilon_{t,i} = m_{t-1,i}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,i} + (P_{t,i}/P_t) y_{t,i}$$

can be written as

$$b_{t,i} = \Lambda_{t,i} + r_t b_{t-1,i}, \text{ where } \Lambda_{t,i} = \frac{P_{t,i}}{P_t} y_{t,i} - \Upsilon_{t,i} - c_{t,i} - m_{t,i} + m_{t-1,i}\pi_t^{-1}.$$

Hence $0 = r_t b_{t-1,i} + \sum_{j=0}^{s} D_{t,t+j}^{-1}\Lambda_{t+j,i} - D_{t,t+s}^{-1} b_{t+s,i}$ and imposing $\lim_{s\to\infty} D_{t,t+s}^{-1} b_{t+s,i} =$

0 gives the household IBC

$$0 = r_t b_{t-1,i} + \sum\nolimits_{j=0}^{\infty} D_{t,t+j}^{-1} \Lambda_{t+j,i}. \tag{18}$$

We have representative agents and assume they believe future lump-sum taxes and seigniorage revenue provided to the government will be identical across agents, so that $\Upsilon_{t,i} - m_{t,i} + m_{t-1,i}\pi_t^{-1} = \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}$ and $\Lambda_{t,i} = \frac{P_{t,i}}{P_t} y_{t,i} - \Upsilon_t - c_{t,i} - m_t + m_{t-1}\pi_t^{-1}$. It follows that

$$\Lambda_{t+j,i} = \frac{P_{t+j,i}}{P_{t+j}} y_{t+j,i} - c_{t+j,i} - g_{t+j} + \Delta_{t+j}. \tag{19}$$

Incorporating the government IBC into the household IBC yields the Ricardian household IBC, which we assume holds in expectation, and with point expectations becomes

$$0 = \sum\nolimits_{j=0}^{\infty} D_{t,t+j,i}^{e^{-1}} \left( \frac{P_{t+j,i}^e}{P_{t+j}^e} y_{t+j,i}^e - c_{t+j,i}^e - g_{t+j}^e \right).$$

Finally, to obtain the household consumption function we make use of their consumption Euler equation

$$(c_{t,i} + \xi g_t)^{-1} = \beta \hat{E}_{t,i} \left( r_{t+1}(c_{t+1,i} + \xi g_{t+1})^{-1} \right).$$

Iterating and assuming point expectations gives

$$c_{t+j,i}^e = -\xi g_{t+j,i}^e + \beta^s \left( D_{t,t+j,i}^e \right) (c_{t,i} + \xi g_t).$$

Substituting for $c^e_{t+j,i}$ in the household IBC and solving for $c_t$ gives the consumption function

$$\begin{aligned} c_{t,i} &= (1-\beta)\left[\frac{P_{t,i}}{P_t}y_{t,i} - g_t\left(1+\frac{\xi\beta}{1-\beta}\right)\right] + \\ &\quad (1-\beta)\sum_{s=1}^{\infty}\left(D^e_{t,t+s,i}\right)^{-1}\left[\left(\frac{P_{t+s,i}}{P_{t+s}}\right)^e y^e_{t+s,i} - g^e_{t+s,i}(1-\xi)\right]. \end{aligned}$$

Imposing the non-negativity constraint $c_{t,i} \geq 0$ gives (3) in the main text.

Impose now the representative agent assumption, $c_{t,i} = c_t$, $y_{t,i} = y_t$, $\Xi_{t,i} = P_{t,i}/P_t = 1$, $D^e_{t,t+s,i} = D^e_{t,t+s}$, $y^e_{t+s,i} = y^e_{t+s}$ and $g^e_{t+s,i} = g^e_{t+s}$. Assuming also agents have learned that $\Xi_{t,i} = 1$ we have $\Xi^e_{t+s,i} = (P_{t+s,i}/P_{t+s})^e = 1$. The market clearing equation $y_t = c_t + g_t$ then yields

$$y_t = \max\left\{g_t, \beta(1-\xi)g_t + (1-\beta)\left[y_t + \sum_{s=1}^{\infty}\left(D^e_{t,t+s}\right)^{-1}\left(y^e_{t+s} - (1-\xi)g^e_{t+s}\right)\right]\right\}.$$

Solving for $y_t$ gives the temporary equilibrium output equation (5) in the main text.

*Remark:* In the Ricardian case, with monetary policy specified as an interest-rate rule, it is unnecessary to track money supply and demand. However, it is straightforward to show that with our utility function real money demand satisfies $m_{t,i} = \chi\beta\left(1-R_t^{-1}\right)^{-1} c_{t,i}$. The cashless limit corresponds to $\chi \to 0$.

**Production decisions:** The adjustment cost function $\Phi(\frac{P_{t,j}}{P_{t-1,j}})$ is the Linex function (see Kim and Ruge-Murcia (2009)) , centered on $\pi^*$, given by

$$(20)\quad \Phi\left(P_{t,i}/P_{t-1,i}\right) \equiv \left(\phi/\psi^2\right)\left[\exp(-\psi(P_{t,i}/P_{t-1,i} - \pi^*)) + \psi(P_{t,i}/P_{t-1,i} - \pi^*) - 1\right],$$

where $\phi > 0$, and we assume the case $\psi > 0$, consistent with asymmetric adjustment costs. The function $\Phi'(\pi)\pi = (\phi/\psi)\pi(-\exp(-\psi(\pi - \pi^*)) + 1)$ is monotonically increasing above a critical value $\breve{\pi}$, given by the condition $\frac{d}{d\pi}\Phi'(\pi)\pi = 0$. We restrict attention to regions for which $\pi > \breve{\pi}$. We compute the derivative

$$\frac{d}{dP_{t,i}}\left[\Phi\left(\frac{P_{t,i}}{P_{t-1,i}}\right)\right] = \frac{\phi}{\psi}P_{t-1,i}^{-1}\left[-\exp(-\psi(P_{t,i}/P_{t-1,i} - \pi^*)) + 1\right].$$

Note that $\Phi'(\pi_i) = \frac{\phi}{\psi}(-\exp(-\psi(\pi_i - \pi^*)) + 1)$, so

$$\frac{d}{dP_{t,i}}\left[\Phi\left(\frac{P_{t,i}}{P_{t-1,i}}\right)\right] = P_{t-1,i}^{-1}\Phi'\left(\frac{P_{t,i}}{P_{t-1,i}}\right).$$

The agent's period utility is

$$U_{t,i} = \log(c_{t,i} + \xi g_t) + \varkappa \log\left(\frac{M_{t-1,i}}{P_t}\right) - (1+\varepsilon)^{-1}h_{t,i}^{1+\varepsilon} - \Phi\left(\frac{P_{t,i}}{P_{t-1,i}}\right),$$

and the first-order condition for optimal price setting is

$$\begin{aligned}(21)\quad 0 \;=\; & \frac{\partial U_{t,i}}{\partial P_{t,i}} + \beta E_{t,i}\frac{\partial U_{t+1,i}}{\partial P_{t,i}} = \frac{\nu_t}{\alpha}h_{t,i}^{\varepsilon+1}\frac{1}{P_{t,i}} - \Phi'(\pi_{t,i})\frac{1}{P_{t-1,i}} \\ & +(c_{t,i}+\xi g_t)^{-1}(1-\nu_t)\,y_t\left(\frac{P_{t,i}}{P_t}\right)^{-\nu_t}\frac{1}{P_t} + \beta\Phi'(\pi_{t+1,i}^e)\left(\frac{P_{t+1,i}}{P_{t,i}^2}\right)^e,\end{aligned}$$

where again we have used point expectations and here $\pi_{t,i} = P_{t,i}/P_{t-1,i}$. Multiplying the right-hand side by $P_{t,i}$ we can write this equation as

$$(22)\quad \Phi'(\pi_{t,i})\pi_{t,i} = \frac{\nu_t}{\alpha}h_{t,i}^{\varepsilon+1} + (c_{t,i}+\xi g_t)^{-1}(1-\nu_t)\,y_t\left(\frac{P_{t,i}}{P_t}\right)^{1-\nu_t} + \beta\Phi'(\pi_{t+1,i}^e)\pi_{t+1,i}^e.$$

We now discuss the properties of

$$\Phi'(\pi)\pi = \frac{\phi}{\psi}\pi(-\exp(-\psi(\pi - \pi^*)) + 1).$$

The function $\Phi'(\pi)\pi$ is monotonically increasing above a critical value $\breve{\pi}$ which is given by the condition

$$\frac{d}{d\pi}\Phi'(\pi)\pi = 0. \tag{23}$$

We compute the derivative

$$\frac{d}{d\pi}\Phi'(\pi)\pi = \frac{\phi}{\psi}(1 - (1 - \phi\pi)\exp(-\psi(\pi - \pi^*)),$$

so the condition giving $\breve{\pi}$ can be written as

$$1 = (1 - \phi\pi)\exp(-\psi(\pi - \pi^*)).$$

This equation has a unique solution $\breve{\pi} < \phi^{-1}$. It is easily seen that (i) $\breve{\pi}$ is increasing in $\psi$ with $\lim_{\psi\to\infty}\breve{\pi} = 1/\phi^{-1}$ and (ii) $\breve{\pi}$ is decreasing in $\phi$ with $\lim_{\phi\to\infty}\breve{\pi} = 0$ *ceteris paribus*. Throughout the paper we restrict attention to regions for which $\pi > \breve{\pi}$. In the calibrated model we will compute $\breve{\pi}$ to check and impose the inequality $\pi > \breve{\pi}$ when solving for the temporary equilibrium.

Using the production function, $\zeta_{t,i}$ in the main text is

$$\begin{aligned}\zeta_{t,i} &= \frac{\nu_t}{\alpha}h_{t,i}^{\varepsilon+1} - (\nu_t - 1)(c_{t,i} + \xi g_t)^{-1} y_t \left(\frac{P_{t,i}}{P_t}\right)^{1-\nu_t} \\ &= \frac{\nu_t}{\alpha}\left(\frac{y_{t,i}}{A_t}\right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1)(c_{t,i} + \xi g_t)^{-1} y_t \left(\frac{P_{t,i}}{P_t}\right)^{1-\nu_t}.\end{aligned}$$

Here

$$\frac{y_{t,i}}{A_t} = \frac{\int_0^1 c_{t,j}(i)dj + g_t(i)}{A_t} = \frac{c_t(i) + g_t(i)}{A_t}$$

is the total demand for variety $i$.

Note that the term $y_t \left(\frac{P_{t,i}}{P_t}\right)^{1-\nu_t}$ combines $y_t$, which is exogenous to the firm, with the relative price $\frac{P_{t,i}}{P_t}$, in which the aggregate price level is exogenous while $P_{t,i}$ is a decision variable of the firm. Iterating forward we get the expression (4)

$$\Phi'(\pi_{t,i})\pi_{t,i} = \zeta_{t,i} + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s,i}^e,$$

which is our infinite-horizon pricing decision rule. Here $\zeta_{t+s,i}^e$ is the point expectation of

$$\zeta_{t+s,i} = \frac{\nu_{t+s}}{\alpha}\left(\frac{y_{t+s,i}}{A_{t+s}}\right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s} - 1)\, y_{t+s} \left(\frac{P_{t+s,i}}{P_{t+s}}\right)^{1-\nu_t} \times (c_{t+s,i} + \xi g_{t+s})^{-1},$$

where

$$y_{t+s,i} = c_{t+s}(i) + g_{t+s}(i)$$

is the future market demand for variety $i$.

# B Implementation of stochastic model

From Section 2 we have the representative agent NK PC temporary equilibrium (TE) equation

$$(24) \qquad Q(\pi_t) = \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e, \text{ where } Q(\pi_t) = \Phi'(\pi_t)\pi_t,$$

with $Q(\pi_t) > 0$ for $\pi_t > \check{\pi}$, and where

$$\zeta_t = \frac{\nu_t}{\alpha} \left( \frac{y_t}{A_t} \right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1)\, y_t \times (y_t - (1-\xi) g_t)^{-1}$$

and

$$(25) \qquad \zeta_{t+s}^e = \frac{\nu_t}{\alpha} \left( \frac{y_{t+s}^e}{A_{t+s}^e} \right)^{(1+\varepsilon)/\alpha} - \left( \nu_{t+s}^e - 1 \right) y_{t+s}^e \times (y_{t+s}^e - (1-\xi) g_{t+s})^{-1},$$

for $s = 1, 2, 3, \ldots, T$. As we discuss below, we will set $\zeta_{t+s}^e$ at its perceived mean value after $T+1$ periods, for some (suitably large) period $T$. It is assumed that $\Xi_t^e \equiv 1$ as discussed in Section 2. Note that $\nu_t$ is stochastic and we assume that, faced with stochastic shocks in a nonlinear setting, agents use point expectations. We also assume that the future path of government spending is credibly announced and implemented; hence $g_{t+s}^e = g_{t+s}$.

Also from Section 2, the aggregate demand temporary equilibrium equation (5) is obtained by combining the IH consumption function, market clearing, i.e. $y_t = c_t + g_t$, and $\Xi_t^e \equiv 1$. This yields the TE equation for output

$$(26) \qquad y_t = (1-\xi)\, g_t + \left(\beta^{-1} - 1\right) \sum_{s=1}^{\infty} \left(D_{t,t+s}^e\right)^{-1} \left(y_{t+s}^e - (1-\xi)\, g_{t+s}^e\right),$$

where

$$D_{t,t+s}^e = \prod_{j=1}^{s} r_{t+j}^e \text{ and } r_{t+j}^e = \frac{R_{t+j-1}^e}{\pi_{t+j}^e}.$$

Here for $j = 1$ we have $R_t^e = R_t$.[47] The interest-rate rule is assumed known and is given by a forward-looking rule (8). As before, point expectations for forecasting all unknown future values is assumed.

We turn next to the data-generating process for the stochastic shocks. As assumed in Section 4, $\ln A_t$ and $\ln \nu_t$ are independent stationary exogenous AR(1) processes

$$\ln\left(A_{t+1}/\bar{A}\right) = \rho_A \ln\left(A_t/\bar{A}\right) + \ln \varepsilon_{A,t+1}$$

where $0 \leq \rho_A < 1$, $\ln \varepsilon_{A,t} \overset{iid}{\sim} N\left(0, \sigma_A^2\right)$, and

$$\ln\left(\nu_{t+1}/\bar{\nu}\right) = \rho_\nu \ln\left(\nu_t/\bar{\nu}\right) + \ln \varepsilon_{\nu,t+1}$$

where $0 \leq \rho_\nu < 1$, $\ln \varepsilon_{\nu,t} \overset{iid}{\sim} N\left(0, \sigma_\nu^2\right)$.

It follows that

$$\nu_{t+1}/\bar{\nu} = \left(\nu_t/\bar{\nu}\right)^{\rho_\nu} \varepsilon_{\nu,t+1} \text{ and } A_{t+1}/\bar{A} = \left(A_t/\bar{A}\right)^{\rho_A} \varepsilon_{A,t+1},$$

[47] As we explain below we need to replace $r_{t+j}^e = R_{t+j-1}^e/\pi_{t+j}^e$ by $r_{t+j}^e = \beta^{-1}$ for $j \geq T_1$ for some positive $T_1$.

and that

$$\nu_{t+s}/\bar{\nu} = (\nu_t/\bar{\nu})^{\rho_\nu^s} \prod_{j=0}^{s-1} \varepsilon_{\nu,t+j}^{\rho_\nu^j}.$$

Under point expectations $\ln \varepsilon_{\nu,t+j}^e = 0$ and $\varepsilon_{\nu,t+j}^e = 1$ so that

$$\nu_{t+s}^e = \bar{\nu} \left(\nu_t/\bar{\nu}\right)^{\rho_\nu^s},$$

and analogously we have

$$A_{t+s}^e = \bar{A} \left(A_t/\bar{A}\right)^{\rho_A^s}.$$

In Section 4, the PLMs for output and inflation use a linear forecasting rule based of the observed exogenous variables. To first-order these correspond to a stochastic REE at a steady state. Thus the perceived laws of motion are

$$\begin{aligned} \ln(y_t) &= f_y + d_{yA} \ln\left(A_t/\bar{A}\right) + d_{y\nu} \ln\left(\nu_t/\bar{\nu}\right) + \eta_{yt} \\ \ln(\pi_t) &= f_\pi + d_{\pi A} \ln\left(A_t/\bar{A}\right) + d_{\pi\nu} \ln\left(\nu_t/\bar{\nu}\right) + \eta_{\pi t}, \end{aligned}$$

where $\eta_{yt}, \eta_{\pi t}$ are perceived white noise shocks.

Under recursive least squares (RLS) learning the coefficient vectors, $\phi_y, \phi_\pi$ where $\phi_y' = (f_y, d_{yA}, d_{y\nu})$ and $\phi_\pi' = (f_\pi, d_{\pi A}, d_{\pi\nu})$, are time-varying and updated over time using recursive least squares regressions of $(\ln(y_t), \ln(\pi_t))$ on $x_t' = (1, \tilde{A}_t, \tilde{\nu}_t)$. The recursive updating equations, which are standard,[48] are

$$(27)\qquad \begin{aligned} \phi_{yt} &= \phi_{y,t-1} + \omega_t \mathcal{R}_t^{-1} x_{t-1} \left( y_{t-1} - \phi'_{y,t-1} x_{t-1} \right) \\ \phi_{\pi t} &= \phi_{\pi,t-1} + \omega_t \mathcal{R}_t^{-1} x_{t-1} \left( \pi_{t-1} - \phi'_{\pi,t-1} x_{t-1} \right) \\ \mathcal{R}_t &= \mathcal{R}_{t-1} + \omega_t ( x_{t-1} x'_{t-1} - \mathcal{R}_{t-1} ). \end{aligned}$$

Note that $\phi_{yt}, \phi_{\pi t}$ are updated based on their most recent forecast errors. Here $\mathcal{R}_t$ is an estimate of the second-moment matrix of regressors. RLS updating equations allow for a time-varying gain $\omega_t$. We focus on the constant gain case $\omega_t = \omega$ for $0 < \omega < 1$. In the decreasing gain case, $\omega$ is replaced by $0 < \omega_t < 1$ with $\omega_t \to 0$ at an appropriate rate, for instance at rate $t^{-1}$.

Under constant gain RLS learning the coefficients $\phi = (f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu})$ are time-varying and updated over time using recursive least squares regressions of $(\ln(y_t), \ln(\pi_t))$ on $(1, \tilde{A}_t, \tilde{\nu}_t)$.

Letting $f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu}$ now denote the time $t$ values of their estimates, expectations of output $s$ steps ahead are given by

$$y^e_{t+s} = e^{f_y} \tilde{A}_t^{\rho_A^s d_{yA}} \tilde{\nu}_t^{\rho_\nu^s d_{y\nu}} \text{ for } s = 1, \ldots, T,$$

where as usual point expectations are assumed. Here $T$ denotes the period after which agents believe that all relevant processes will have reverted to their mean

[48] See, for example, Chapter 2 of Evans and Honkapohja (2001) or Evans and Honkapohja (2009).

steady-state values. Thus

$$y^e_{t+s} = e^{f_y}, \nu^e_{t+s} = \bar{\nu}, A^e_{t+s} = \bar{A} \text{ and } g_{t+s} = \bar{g} \text{ for } s \geq T+1.$$

Here $\bar{g}$ is the original level of $g$ to which $g_t$ reverts after the fiscal stimulus, and we assume $t+T+1 > T_p$, where $T_p$ is the length of the fiscal stimulus. Similarly for $\zeta^e_{t+s}$ with $s \geq T+1$ we replace (25) with

$$\zeta^e_{t+s} = \bar{\zeta} \equiv \frac{\bar{\nu}}{\alpha}\left(\frac{e^{f_y}}{\bar{A}}\right)^{(1+\varepsilon)/\alpha} - (\bar{\nu}-1)\, e^{f_y} \times (e^{f_y} - (1-\xi)\bar{g})^{-1} \text{ for } s \geq T.$$

Using these expectations $\pi_t$ is determined by the temporary equilibrium equation

$$Q(\pi_t) = \zeta_t + \sum_{s=1}^{T} \beta^s \zeta^e_{t+s} + \frac{\beta^{T+1}}{1-\beta}\bar{\zeta}. \tag{28}$$

We now turn to the aggregate demand temporary equilibrium equation (5). Expectations $y^e_{t+s}$ and $\pi^e_{t+j}$ are given as above. For the discount factors we have

$$D^e_{t,t+s} = \prod_{j=1}^{s} \frac{R^e_{t+j-1}}{\pi^e_{t+j}}.$$

where $R^e_{t+j-1}$ is given by the forward-looking $R$-rule (8). Here $R_t = R(\pi^e_{t+1}, y^e_{t+1})$ with $R^e_{t+j-1} = R(\pi^e_{t+j}, y^e_{t+j})$, so that

$$D^e_{t,t+s} = \prod_{j=1}^{s} \frac{R(\pi^e_{t+j}, y^e_{t+j})}{\pi^e_{t+j}}, \text{ for } s \leq T_1 - 1.$$

The restriction $s \leq T_1 - 1$ is included because in order to ensure that consumption and output is positive and finite we need discount factors $D^e_{t,t+s}$ to be bounded above 1. This can be an issue because the interest rate $R(\pi^e, y^e)$ can be less than $\pi^e$ for some $\pi^e$ between $\pi_L$ and $\pi^*$ and for a range of $y^e$ if the rule also depends on $y^e$. This difficulty is avoided by assuming that after $T_1$ periods the expected real interest rate factor is the steady-state value $\beta^{-1}$. Thus we assume $r^e_{t+j} = \beta^{-1}$ for $j \geq T_1$ implying

$$D^e_{t,t+s} = \prod_{j=1}^{T_1-1} \frac{R(\pi^e_{t+j}, y^e_{t+j})}{\pi^e_{t+j}} \beta^{-(s-(T_1-1))}, \text{ for } s \geq T_1.$$

We assume that the "truncation" parameter $T > T_1, T_p$.

Incorporating the assumption that expectations $y^e, \pi^e$ return to their perceived steady-state values after $T$ periods, we arrive at the the aggregate demand temporary equilibrium equation

$$\text{(29)} \qquad y_t = (1-\xi)\, g_t + \left(\beta^{-1} - 1\right) \sum_{s=1}^{T} \left(D^e_{t,t+s}\right)^{-1} \left(y^e_{t+s} - (1-\xi)\, g^e_{t+s}\right) + \left(\beta^{-1} - 1\right) \left(e^{f_y} - \bar{g}\,(1-\xi)\right) \sum_{s=T+1}^{\infty} \left(D^e_{t,t+s}\right)^{-1}, \text{ where}$$

$$\sum_{s=T+1}^{\infty} \left(D^e_{t,t+s}\right)^{-1} = (D^e_{t,t+T_1-1})^{-1} (1-\beta)^{-1} \beta^{T-T_1+2}.$$

The latter equation is obtained from

$$\sum_{s=T+1}^{\infty} \left(D^e_{t,t+s}\right)^{-1} = (D^e_{t,t+T_1-1})^{-1} \sum_{s=T+1}^{\infty} \beta^{s-T_1+1}.$$

The forward-looking $R$-rule has the advantage that $\pi_t, y_t$ can be solved explicitly using the above equations (28) and (29). As already noted, the results for the contemporaneous rule are similar.

Next, we discuss the interpretation of the numerical magnitudes pointed out at the end of Section 4. To understand the magnitude of the expectation shocks given in Table 1, it is helpful to consider a reinterpretation of the role of $y^e$ in the temporary equilibrium model. For the consumption function (3), assuming the representative agent case with $\Xi_t^e \equiv 1$, it can be seen that consumption, and hence temporary equilibrium output $y_t$, depend to first-order on $\{y_{t+s}^e\}_{s=1}^{\infty}$ through its present value

$$\mathrm{PV}\left(\{y_{t+s}^e\}_{s=1}^{\infty}\right) = \sum_{s=1}^{\infty} \left(D_{t,t+s}^e\right)^{-1} y_{t+s}^e \approx \sum_{s=1}^{\infty} \beta^s y_{t+s}^e.$$

We have interpreted steady state learning as agents acting as if $y_{t+s}^e = y_t^e$ for all horizons $s = 1, 2, 3 \ldots$. However this is behaviorally equivalent to assuming that agents have an expected output profile with the same present value as $\mathrm{PV}\left(\{y_{t+s}^e = y_t^e\}_{s=1}^{\infty}\right)$.

In particular suppose that agents believe $y_{t+s}^e = \hat{y} < y^*$ for $s = 1, \ldots, L$ periods, interpreted as quarters, followed by $y_{t+s}^e = y^*$ for $s > L$, i.e. a recession of $L$ periods followed by a return to targeted steady state. Then the PV of the $L$-period recession output expectation sequence equals the PV of a constant sequence $\tilde{y}^e < y^*$ when

$$\sum_{s=1}^{\infty} \beta^s \tilde{y}^e = \sum_{s=1}^{L} \beta^s \hat{y} + \sum_{s=L+1}^{\infty} \beta^s y^* \text{ or } \tilde{y}^e = \hat{y} + \beta^L (y^* - \hat{y}). \tag{30}$$

As an example assume $\beta = 0.99$ and consider an expectations shock $\hat{y}$ lasting two years i.e. $L = 8$. Then compute $\hat{y}$ that is equivalent to permanent shock at $\tilde{y}^e =$

0.99745 which in Table 1 is slightly above the boundary of the stochastic domain of attraction at $\pi^e = \pi^*$ for the targeted steady state.[49] Then (30) yields $\hat{y} =$ 0.96663 which corresponds to an expected recession approximately equal to over 3.34% reduction of expected GDP relative to target $y^*$ during two years, followed by a return to normal value.

# C Calibration details

The parameter values are $\pi^* = 1.005$, $\beta = 0.99$, $\alpha = 0.7$, $\xi = 0.4$, $\bar{A} = 1.113$, $\nu = 13.5$, $\phi = 75$, $\psi = 20$, $\varepsilon = 1$, $\bar{g} = 0.2$, $B = 1.5/R^*$, $\phi_y = 8.25$. We set $\phi = 75$, $\psi = 20$ based on comparing Linex-type functions to a quadratic adjustment cost function at the most common range for $\pi$. Here are some comments about these values.

The parameter values in the main text are chosen as follows. $\alpha = 0.7$, $\beta = 0.99$ and $\varepsilon = 1$ are standard. There are various suggestions for $\xi$ and we set $\xi = 0.4$. The frequency of price change is that $1/3$ $(= 1-\eta)$ of firms change prices per quarter. This is consistent with Nakamura and Steinsson (2008) and Kehoe and Midrigan (2015). Various estimates of $\nu$ or of the markup $\nu/(\nu-1)$ have been used with estimates of $\nu$ ranging from 21 to 3.5. Keen and Wang (2007) give the relation between these

[49] The computation assumes unchanged fiscal policy and unchanged monetary policy given by the specified Taylor-rule.

parameters and the Rotemberg quadratic adjustment cost parameter

$$\gamma = \frac{(\nu - 1)\,\eta}{(1-\eta)\,(1-\beta\eta)}.$$

We choose $\nu = 13.5$, a markup of about 8%, and $\eta = 0.67$ which gives $\gamma = 75$.

For Linex adjustment cost functions the parameter estimates for $\phi, \psi$ vary widely. Note that $\phi \rightarrow \gamma$ as $\psi \longrightarrow 0$. In most papers adjustment costs are assumed to be proportional to output or profit, whereas we use a non-proportional setup to avoid multiplicities. However as we normalize steady-state target output to $y = 1$ then the parameters are comparable. Also near the steady state marginal utilities drop out to first order. We choose $\phi = 75$ and $\psi = 20$, which gives a fairly close approximation to the quadratic in the range $\pi = 1.00$ to $\pi = 1.01$, i.e. 0 to 4% annual inflation.

For technology we set $\bar{A} = 1.113$. A high steady state is $\bar{y} \approx 1.00003 \approx 1$ with $\bar{g} = 0.2$. For productivity and mark-up shock calibrations we set first-order autocorrelation parameters to $\rho_A = \rho_v = 0.5$ and standard deviations for the log innovations, in decimal form, to $\sigma_A = 0.0015$ and $\sigma_\nu = 0.0001$. Both the serial correlation and autocorrelation parameters are smaller than those found by Smets and Wouters (2007), but their estimates are for models under RE and with additional frictions. Adaptive learning dynamics add additional volatility relative to RE, particularly in purely forward-looking models.

# D Calibrating the Taylor rule

To calibrate the interest rate rule

$$R_t = R(\pi^e_{t+1}, y^e_{t+1}) = 1 + (R^* - 1)\left(\frac{\pi^e_{t+1}}{\pi^*}\right)^{BR^*/(R^*-1)}\left(\frac{y^e_{t+1}}{y^*}\right)^{\phi_y}, \tag{31}$$

where $y^*$ is output level at the target steady state, we relate (31) to the usual Taylor rule. Rearranging and taking logs we get

$$\begin{aligned} & \log(R_t - 1) - \log(R^* - 1) \\ = & \frac{BR^*}{R^* - 1}(\log \pi^e_{t+1} - \log \pi^*) + \phi_y(\log y^e_{t+1} - \log y^*). \end{aligned}$$

Multiplying by $(R^* - 1)$ and approximating log differences by percentage changes we get

$$R_t - R^* = BR^*\left(\frac{\pi^e_{t+1} - \pi^*}{\pi^*}\right) + (R^* - 1)\phi_y\left(\frac{y^e_{t+1} - y^*}{y^*}\right).$$

Thus $BR^*$ is the inflation coefficient and $(R^* - 1)\phi_y$ is the output coefficient in the usual linear Taylor rule. Assuming a quarterly calibration one should have

$$\begin{aligned} BR^* &= 1.5 \\ (R^* - 1)\phi_y &= \frac{0.5}{4}. \end{aligned}$$

At the target steady state $R^* = \beta^{-1}\pi^*$ we get

$$\phi_y = \frac{0.5}{4}/(0.01515) \approx 8.25$$

when $\beta = 0.99$ and $\pi^* = 1.005$.

# E Proof of Proposition 1

We start by computing the partial derivatives of the right-hand sides of differential equations (12)-(13):

$$\frac{\partial F_\pi}{\partial \pi^e} = D_y G_1 D_{\pi^e} G_2 - 1, \; \frac{\partial F_\pi}{\partial y^e} = D_{y^e} G_1 + D_y G_1 D_{y^e} G_2$$

and

$$\frac{\partial F_y}{\partial \pi^e} = D_{\pi^e} G_2, \; \frac{\partial F_y}{\partial y^e} = D_{y^e} G_2 - 1.$$

The E-stability differential equations in vector form are

$$\begin{pmatrix} \frac{\partial \pi^e}{\partial \tau} \\ \frac{\partial y^e}{\partial \tau} \end{pmatrix} = \begin{pmatrix} F_\pi(\pi^e, y^e) \\ F_y(\pi^e, y^e) \end{pmatrix},$$

where $F_y(.)$ and $F_\pi(.)$ are given in (12) and (13). We get the Jacobian

$$DFI = \begin{pmatrix} D_y G_1 D_{\pi^e} G_2 - 1 & D_{y^e} G_1 + D_y G_1 D_{y^e} G_2 \\ D_{\pi^e} G_2 & D_{y^e} G_2 - 1 \end{pmatrix}.$$

**Proof of Proposition 1:** (a) (i) Consider the case $\phi_y = 0$. Calculating the derivatives of the Jacobian at the target steady state we get

$$
\begin{aligned}
(Q^{-1})' &= (\Phi''\pi + \Phi')^{-1} = (\phi\pi^*)^{-1} > 0 \text{ so} \\
D_y G_1 &= (Q^{-1})' \left( \frac{\nu(1+\varepsilon)}{\alpha^2}(y^*/A)^{(1+\varepsilon)/\alpha - 1} + \frac{(\nu-1)(1-\xi)g}{(y^* - (1-\xi)g)^2} \right) > 0.
\end{aligned}
$$

$$
D_{\pi^e} G_2 = (\beta^{-1} - 1)(y^* - g(1-\xi)) \left( \frac{R(\pi_t^e, y_t^e) - \pi_t^e D_\pi R(\pi_t^e, y_t^e)}{(R(\pi_t^e, y_t^e) - \pi_t^e)^2} \right) < 0.
$$

As

$$
D_\pi R(\pi, y) = \frac{BR^*}{\pi^*} \left( \frac{\pi}{\pi^*} \right)^{(R^*(B-1)+1)/(R^*-1)} \left( \frac{y}{y^*} \right)^{\phi_y}
$$

we have $D_\pi R(\pi, y) = \frac{BR^*}{\pi^*}$ and so $R(\pi^*, y^*) - \pi^* D_\pi R(\pi^*, y^*) = \beta^{-1}\pi^*(1-B) < 0$ at the target steady state. Also

$$
\begin{aligned}
D_{y^e} G_1 &= (Q^{-1})' \frac{\beta}{1-\beta} \left[ \frac{\nu(1+\varepsilon)}{\alpha^2}(y^*/A)^{(1+\varepsilon-\alpha)/\alpha} + \frac{(\nu-1)(1-\xi)g}{(y^* - (1-\xi)g)^2} \right] > 0 \\
D_{y^e} G_2 &= 1
\end{aligned}
$$

if $\phi_y = 0$. So in this case we get

$$
DFI = \begin{pmatrix} - & + \\ - & 0 \end{pmatrix}
$$

which has negative trace and positive determinant and is thus a stable matrix. The result follows by continuity of eigenvalues.

If $\phi_y$ is not zero, we have

$$D_{y^e}G_2 = 1 + (\beta^{-1} - 1)\left[(1 - g(1-\xi))\left(\frac{\pi}{D_yR(\pi, y) - \pi}\right)\right].$$

For the interest rate rule we get

$$D_yR(\pi, y) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{(R^*B/(R^*-1)}\frac{\phi_y}{y^*}\left(\frac{y}{y^*}\right)^{\phi_y - 1},$$

so

$$D_yR(\pi^*, y^*) = (R^* - 1)\phi_y/y^*$$

at the target steady state and thus

$$D_{y^e}G_2 = 1 + (\beta^{-1} - 1)\left[(1 - g(1-\xi))\left(\frac{\pi^*}{(R^* - 1)\phi_y/y^* - \pi^*}\right)\right].$$

Now $R^* - 1 \gtrapprox 0$ is small, something like 0.02, while $\phi_y \approx 8$ and $y^* \approx 1$, whereas $\pi^* \gtrapprox 1$. Then $D_{y^e}G_2 < 1$ and the targeted steady state is E-stable.

(ii) Doing calculations similar to above, set first $\phi_y = 0$ and we get

$$\begin{aligned}
(Q^{-1})' &= (\frac{\phi(1 - (1 - \psi\pi_L)\exp(\pi_L - \pi^*))}{\psi})^{-1} > 0 \text{ normally so} \\
D_yG_1 &= (Q^{-1})'\left(\frac{\nu(1+\varepsilon)}{\alpha^2}(y_L/A)^{1+\varepsilon-\alpha)/\alpha} + \frac{(\nu - 1)(1-\xi)g}{(y_L - (1-\xi)g)^2}\right) > 0.
\end{aligned}$$

$$D_{\pi^e}G_2 = (\beta^{-1} - 1)(y_L - g(1-\xi))\left(\frac{R(\pi_L, y_L) - \pi_L D_\pi R(\pi_L, y_L)}{(R(\pi_L, y_L) - \pi_L)^2}\right) < 0,$$

as

$$1+(R^*-1)\left(\frac{\pi_L}{\pi^*}\right)^{BR^*/(R^*-1)}\left(\frac{y_L}{y^*}\right)^{\phi_y} < B\beta^{-1}\pi^*\left(\frac{\pi_L}{\pi^*}\right)^{BR^*/(R^*-1)}\left(\frac{y_L}{y^*}\right)^{\phi_y} \text{ normally.}$$

Also

$$\begin{aligned} D_{y^e}G_1 &= (Q^{-1})'\frac{\beta}{1-\beta}\left[\frac{\nu(1+\varepsilon)}{\alpha^2}(y_L/A)^{(1+\varepsilon-\alpha)/\alpha}+\frac{(\nu-1)(1-\xi)g}{(y_L-(1-\xi)g)^2}\right]>0 \\ D_{y^e}G_2 &= 1-\frac{\phi_y(y_L/y^*)(\beta^{-1}\pi_L-1)}{(\beta^{-1}-1)\pi_L}. \end{aligned}$$

Normally, $D_{y^e}G_1 + D_yG_1D_{y^e}G_2 > 0$ and considering the case $\phi_y = 0$, we get

$$DFI = \begin{pmatrix} ? & + \\ + & 0 \end{pmatrix}.$$

It is seen that the determinant of $DFI$ is negative, so the $(\pi_L, y_L)$ is not E-stable.

The case $\phi_y > 0$ but not too large also leads to instability depending on the parameter values. This is true in numerical analyses where $\phi_y = 8.25$.

(iii) At the stagnation steady state $y_t = G_2(\pi_t^e, y_t^e)$ has to be locally constant, so $D_{\pi^e}G_2 = D_{y^e}G_2 = 0$. Then the Jacobian matrix becomes

$$DFI = \begin{pmatrix} -1 & D_{y^e}G_1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & + \\ 0 & -1 \end{pmatrix}$$

as

$$D_{y^e}G_1 = (Q^{-1})'\frac{\beta}{1-\beta}\left[\frac{\nu(1+\varepsilon)}{\alpha^2}(g)^{1+\varepsilon-\alpha)/\alpha}+\frac{(\nu-1)(1-\xi)g}{(\xi g)^2}\right]>0.$$

Given the sign pattern for $DFI$, local stability is evident.

(b). In non-stochastic models, with constant-gain, steady-state learning is locally stable for sufficiently small gains if and only if E-stability holds. We provide a sketch of the proof. Consider a linear(-ized) model $y_t = Ty_t^e$, assuming zero intercept without loss of generality, and the constant-gain rule $a_t = a_{t-1} + \gamma(y_t - a_{t-1})$, where $0 < \gamma < 1$. The PLM is $y_t^e = a_{t-1}$. Then $a_t = a_{t-1} + \gamma(Ta_{t-1} - a_{t-1})$ and the system is convergent if matrix $\gamma T + (1-\gamma)I$ has eigenvalues $t_i$ inside the unit circle; equivalently $T + \gamma^{-1}(1-\gamma)I$ has eigenvalues inside the circle with radius $\gamma^{-1}$. Eigenvalues of the latter matrix are equal to $t_i + \gamma^{-1}(1-\gamma)$, so the $t_i$ values must lie inside unit circle with origin at $(1-\gamma^{-1}, 0)$ and radius $\gamma^{-1}$. Letting $\gamma \to 0$ yields the E-stability condition that real parts of $t_i$ must be less than 1. ■

# F Figure 1 and Figure 7 Details

Figure 1 data details:

Figure 1, left panel: The interest-rate rule curve takes the form $\mathcal{I} = A * \exp(B\Pi)$, where $\Pi$ denotes net inflation and $\mathcal{I}$ denotes the net interest rate. Japan switched the policy target in 2013 to monetary base.

Figure 1, right panel: Macrobond data base which in turn utilizes standard data sources. GDP data is volume data with 2010 as reference year and in local currency. GDP data is annualized. This was specifically done for the Euro area by multiplying quarterly data by 4. Population data is total population and it is interpolated for quarters.

Figure 7 simulation details:

In the simulations used for Figure 7, the pricing friction $\Phi$ is modified so that $Q(\pi) \equiv \Phi'(\pi)\pi$ is replaced by $QQ(\pi)$. Over $\pi_{Lc1} < 1 < \pi_{Rc1}$ $Q$ is unchanged, i.e. $QQ(\pi) = Q(\pi)$. For $\pi_{Lc2} < \pi_{Lc1}$ and for $\pi_{Rc1} < \pi_{Rc2}$ the function $QQ(\pi)$ modifies $Q(\pi)$ using logit-type asymptotes that give elastic effective bounds on inflation and deflation. Because $\pi_{Lc1} < \pi_L < \pi^* < \pi_{Rc1}$, the targeted and unintended steady states $(\pi^*, y^*)$ and $(\pi_L, y_L)$ are unchanged, as are their local dynamics under adaptive learning. The stagnation steady state remains locally stable under learning but has a deflation rate corresponding to $\pi_{Lc2}$.

The Figure 7 scatterplots combine simulated data from three stochastic simulations, each of 80 periods length. These correspond to three expectations starting points: (i) $(\pi^e, y^e)$ close to $(\pi_L, y_L)$ but with $\pi^e < \pi_L$ and $y^e < y_L$; (ii) initial $(\pi^e, y^e)$ close to $(\pi^*, y^*)$, but with initial $y^e$ slightly below $y^*$, (iii) initial $(\pi^e, y^e)$ close to $(\pi^*, y^*)$, but with initial $\pi^e$ slightly below $\pi^*$. In Figure 7 simulations we set $T_1 = 8$, which also acts to moderate the response of output and inflation to expectations.

For simulations (ii) and (iii) normal policy is followed. For simulation (i) stimulative monetary and fiscal policies are followed after delays. Specifically, in simulation (i) normal policies are initially followed for 12 quarters. During this period inflation and output, and their expectations, gradually fall, with the negative output gap reaching 3%. After this delay monetary policy drops the net interest rate to an "effective lower bound" of 0.8% per year and, using forward guidance, holds it there for 14 quarters. (If a credit friction were included a similar outcome would arise with a policy net interest rate at zero). This policy is not enough to begin a sustained recovery, and in period 26 policy adds a large fiscal stimulus (increasing $g$ from 0.20 to 0.34 for 18 quarters). These measures together increase output and inflation substantially, eventually returning the economy to the targeted steady state.

# G Further Numerical Results, Section 5

A systematic analysis of the case $y_0^e = 0.997 \times y^*$ of Section 5 is now conducted. The magnitude and length of fiscal policy are varied and the estimated probability of the economy going back to target steady state is computed. The expectation shock is $y_0^e = 0.997 \times y^*$, $\pi_0^e = \pi^*$ and Table A.1 gives the estimated probabilities of convergence to the targeted steady state (vs. eventual convergence to the stagnation steady state) for alternative values of the length $T_p$ and the magnitude $\bar{g}'$ of fiscal policy.

| $T_p \backslash \bar{g}'$ | 0.2 | 0.225 | 0.25 | 0.275 | 0.3 | 0.325 | 0.35 | 0.375 | 0.4 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 94 | 100 | 100 | 100 | 100 |
| 2 | 0 | 0 | 97 | 100 | 100 | 100 | 100 | 100 | 100 |
| 3 | 0 | 0 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 4 | 0 | 95 | 100 | 100 | 100 | 100 | 100 | 67 | 1 |
| 5 | 0 | 99 | 100 | 100 | 100 | 100 | 5 | 0 | 0 |
| 6 | 0 | 100 | 100 | 100 | 100 | 2 | 0 | 0 | 0 |
| 7 | 0 | 100 | 100 | 100 | 4 | 0 | 0 | 0 | 0 |
| 8 | 0 | 100 | 100 | 44 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 100 | 100 | 3 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 100 | 96 | 1 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 100 | 30 | 1 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 100 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Table A.1: Percentage of simulations in which fiscal policy successfully results in convergence to the targeted steady state starting from pessimistic output expectations $y_0^e = 0.997 \times y^*$. Based on 100 replications with length 500.

Table A.1 shows that the sequence of serially correlated random productivity and mark-up shocks can matter: for a fiscal policy that is usually successful, a particularly unfavorable sequence of shocks can adversely affect expectations enough to prevent the policy from working. However, for a substantial range of policies, with $\bar{g}'$ between 0.25 and 0.35 with $T_p$ between 2 and 4 quarters, a fiscal stimulus is successful approximately 100% of the time.

In these cases the cumulative fiscal spending multipliers would of course be huge, reflecting the fact that a temporary fiscal stimulus prevents the economy from descending into stagnation and pushes it back toward convergence to the targeted steady state.

It can also be seen that in many cases a fiscal stimulus that is too long can be counterproductive. For example, for $g = 0.30$ the effectiveness of the stimulus decreases greatly if $T_p$ is increased to $T_p = 7$ quarters or longer. This is a reflection of the negative effect on consumption of the tax burden associated with higher government spending, which we assume is correctly foreseen by households. In particular, the impact on aggregate output is largest in the first period when a fiscal policy of a given magnitude $\Delta g$ for $T_p$ periods is initiated. In this case the present value of the tax burden is simply $\Delta g$ and the direct impact of this on consumption is $-(1-(1-\xi)\beta)\Delta g$, which is small compared to the increase in aggregate demand for output from government spending $\Delta g$. For larger $T_p$ the present value of the tax burden is larger; consequently the reduction in consumption in the initial period is greater, leading to a smaller initial increase in aggregate output and inflation. Against this, of course, a larger $T_p$ means that the increase in demand continues for a longer period of time.

# H Convergence probabilities for $y_0^e = 0.985$

Each table gives the probabilities for the specified value of $T_m$ and ranges of values for $\bar{g}'$ and $T_p$ based on 100 replications.

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 57 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 36 | 60 | 32 | 4 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 54 | 35 | 2 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 57 | 5 | 1 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 55 | 5 | 1 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$, gain $= 0.01, T_m = 1$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 51 | 61 | 62 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 62 | 66 | 17 | 4 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 56 | 27 | 1 | 0 |
| 7 | 0 | 0 | 0 | 0 | 50 | 12 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 46 | 15 | 1 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain=0.01$, T_m = 2$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 27 | 55 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 35 | 54 | 66 | 44 | 10 |
| 5 | 0 | 0 | 0 | 0 | 0 | 59 | 62 | 13 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 3 | 55 | 21 | 2 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 57 | 19 | 1 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 21 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain=0.01, $T_m = 3$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 46 | 55 | 64 | 71 |
| 4 | 0 | 0 | 0 | 0 | 6 | 57 | 60 | 66 | 28 | 6 | 0 |
| 5 | 0 | 0 | 0 | 32 | 61 | 58 | 10 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 3 | 56 | 25 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 55 | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 38 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain=0.01, $T_m = 4$

| $T_p\backslash\bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 43 | 58 | 62 |
| 3 | 0 | 0 | 0 | 0 | 15 | 55 | 61 | 69 | 52 | 24 | 8 |
| 4 | 0 | 0 | 0 | 41 | 65 | 73 | 27 | 6 | 0 | 0 | 0 |
| 5 | 0 | 0 | 34 | 65 | 43 | 8 | 1 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 58 | 34 | 3 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 38 | 48 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain$= 0.01, T_m = 5$

| $T_p\backslash\bar{g}_1$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 37 | 64 | 60 | 69 | 67 | 41 |
| 3 | 0 | 0 | 0 | 45 | 62 | 73 | 36 | 12 | 3 | 1 | 0 |
| 4 | 0 | 0 | 60 | 69 | 37 | 6 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 32 | 65 | 30 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 64 | 48 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 57 | 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 31 | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain=0.01$, T_m = 6$

| $T_p \backslash \bar{g}_1$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 39 | 49 | 58 |
| 2 | 0 | 0 | 0 | 42 | 61 | 65 | 67 | 34 | 14 | 8 | 2 |
| 3 | 0 | 12 | 61 | 62 | 33 | 9 | 2 | 0 | 0 | 0 | 0 |
| 4 | 0 | 63 | 65 | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 25 | 65 | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 49 | 46 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 50 | 21 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 59 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$,gain $= 0.01, T_m = 7$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 32 | 45 | 53 | 63 | 60 | 61 | 50 |
| 2 | 0 | 32 | 55 | 58 | 41 | 19 | 8 | 1 | 1 | 1 | 0 |
| 3 | 21 | 64 | 46 | 17 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 4 | 58 | 53 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 66 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 53 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.985$, gain $= 0.01, T_m = 8$

# I Convergence probabilities for $y_0^e = 0.98$

Each table gives the probabilities for the specified value of $T_m$ and ranges of values for $\bar{g}'$ and $T_p$ with 100 replications.

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 38 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 29 | 19 | 2 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 31 | 4 | 0 | 0 |

$y_0^e = 0.98$, gain=0.01, $T_m = 1$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 31 | 22 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22 | 20 | 3 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 8 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain=0.01, $T_m = 2$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 37 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 43 | 23 | 3 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 24 | 3 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 19 | 3 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain=0.01, $T_m = 3$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 40 | 35 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 42 | 2 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 31 | 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 23 | 2 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain=0.01, $T_m = 4$

| $T_p \backslash \bar{g}_1$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 30 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 40 | 38 | 12 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 29 | 28 | 6 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 41 | 18 | 1 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 24 | 15 | 1 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain=$0.01, T_m = 5$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 41 | 45 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 40 | 35 | 6 | 1 |
| 6 | 0 | 0 | 0 | 0 | 2 | 1 | 37 | 12 | 4 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 12 | 21 | 2 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 19 | 2 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 6$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 17 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 21 | 40 | 29 | 14 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 45 | 25 | 6 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 41 | 15 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 37 | 11 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 22 | 11 | 1 | 1 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 7$

| $T_p \backslash \bar{g}'$ | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 28 | 39 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 42 | 21 | 6 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 40 | 24 | 6 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 42 | 16 | 2 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 19 | 15 | 2 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 8$

# J Additional results with higher values for $\bar{g}'$

The next three tables show the additional rows and columns (with at least one non-zero value) to the corresponding table above with results continued to be based on 100 replications. Note: column for $\bar{g}' = 0.8$ is included to see continuity to the earlier table.

| $T_p \backslash \bar{g}'$ | 0.8 | 0.85 | 0.9 | 0.95 | 1.0 | 1.05 | 1.1 | 1.15 | 1.2 | 1.25 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 1 | 21 | 34 | 44 | 44 | 49 | 24 |
| 4 | 30 | 41 | 52 | 18 | 5 | 3 | 0 | 0 | 0 | 0 |
| 5 | 12 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 5$

| $T_p \backslash \bar{g}'$ | 0.8 | 0.85 | 0.9 | 0.95 | 1.0 | 1.05 | 1.1 | 1.15 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 0 | 1 | 24 | 39 | 44 | 47 | 25 | 17 | 10 |
| 4 | 45 | 28 | 4 | 3 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 6$

| $T_p \backslash \bar{g}'$ | 0.8 | 0.85 | 0.9 | 0.95 | 1.0 | 1.05 | 1.1 | 1.15 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 17 | 35 | 42 | 36 | 20 | 8 | 2 | 0 | 1 |
| 4 | 14 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

$y_0^e = 0.98$, gain $= 0.01, T_m = 7$

# K Numerical Computation of Size of the Domain of Attraction

In this model it is possible to compute an approximation of the area of the domain of attraction (DOA). The figure below which is like Figure 3 shows the DOA in the basic NK model as the area which is inside the global stable manifold (blue curve). The global stable manifold (GSM) can be numerically computed by solving two boundary value problems for the E-stability differential equation as the unstable steady state is a saddle point. The unstable steady state is the end point for the two curves that form the GSM. The numerical solutions of these curves can be obtained by solving the end-point problems where the trajectory approaches the unstable steady state from South-East or North-West direction as $\tau \to \infty$.

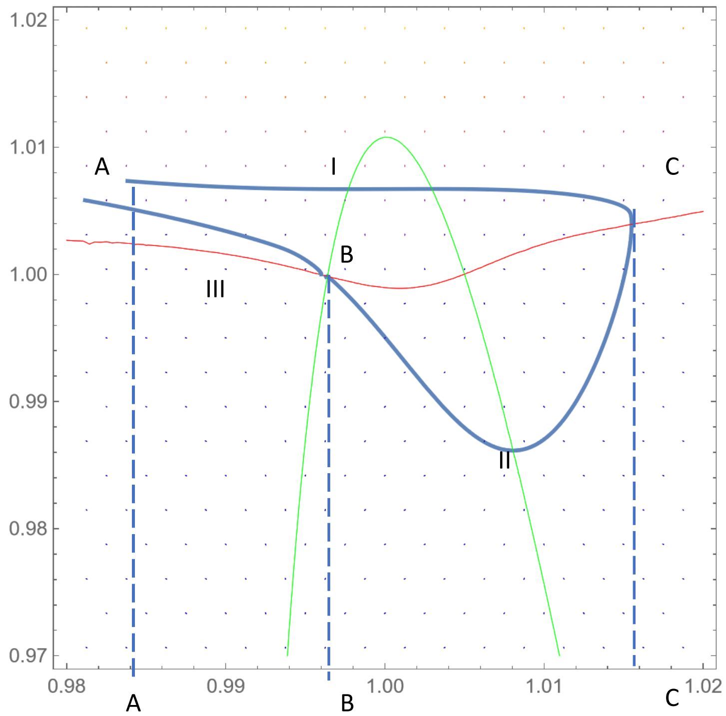


Figure A.1: Numerical computation of the area of domain of attraction.

To compute the area of the DOA, the GSM is divided into three segments which are determined using the intersections of the vertical straight lines with GSM as follows. Segment I is the "top" curve between the intersections of GSM with AA and CC. Segment II is the "bottom" curve between the intersections of GSM with BB and CC. Segment III is the "bottom" curve between the intersections of GSM with AA and BB.

Using standard formula for determining the area below a curve in parametric form, one computes the line integral of each curve

$$\int_{\tau_1}^{\tau_2} y^e(\tau) F_\pi(\pi^e(\tau), y^e(\tau)) d\tau$$

when bounds for the independent variable $\tau$ are implicitly obtained from the relevant intersection points. Denote these integrals by $Num(i)$, where $i = I$, $II$ and $III$. It is seen from the Figure that an approximation for the area of the domain of attraction is then given by

$$Num(I) - Num(II) - Num(III).$$

When applying the formula, the numerical integration is made difficult by the fact that each curve is given implicitly by the solution to a differential equation and the solution is obtained from solving end-point problems for the E-stability differential equation. Some of the values for $\tau_1$ and $\tau_2$ in the integral must be obtained by trial and error method.

**References, Appendices**.

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