Expectations, Learning and Macroeconomic Policy

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Lecture 2

Optimal Monetary Policy and Learning

Stability of Optimal Monetary Policy in NK models

Evans&Honkapohja (REStud 2003, ScanJE 2006)

We start from:

– the standard NK "new Phillips curve/IS curve" model with optimal monetary policy under RE

and we look at two potential problems

– indeterminacy (multiple equilibria), and instability under private agent learning.

We find:

– a well chosen "expectations based" i_t rule is superior to purely "fundamentals based" rules.

MACRO MODEL

The structural model is:

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t \qquad \text{(IS)}$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \qquad \text{(PC)}$$

$x_t =$ "output gap" and $\pi_t =$ inflation rate,

g_t, u_t are observable with

$$g_t = \mu g_{t-1} + \tilde{g}_t \text{ and } u_t = \rho u_{t-1} + \tilde{u}_t.$$

See e.g. Woodford (various) and "The Science of Monetary Policy," Clarida, Gali & Gertler (JEL, 1999)

OPTIMAL MONETARY POLICY WITH COMMITMENT UNDER RE

To complete the model we add a policy rule for i_t.

The policy maker aims to minimize

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\alpha x_{t+s}^2 + \pi_{t+s}^2 \right).$$

Note: x target of 0 (no inflation bias), π target of 0 (for simplicity).

We focus on the <u>full commitment</u> case.

OPTIMAL POLICY WITH COMMITMENT

From the FOCs we obtain

$$\begin{aligned} \lambda\pi_t &= -\alpha x_t \\ \lambda\pi_{t+s} &= -\alpha(x_{t+s} - x_{t+s-1}), \text{ for } s = 1, 2, \ldots \end{aligned}$$

- Optimal discretionary policy is $\lambda\pi_t = -\alpha x_t$, all t

- Optimal policy with commitment is time inconsistent

- We adopt the timeless perspective optimal policy (see Woodford and McCallum/Nelson),

  $$\lambda\pi_t = -\alpha(x_t - x_{t-1}), \text{ all } t, \tag{OPT}$$

  i.e. follow same rule in first period too.

OPTIMAL SOLUTION UNDER RE

Combining PC and OPT $\longrightarrow$ optimal REE

$$\begin{aligned} x_t &= \bar{b}_x x_{t-1} + \bar{c}_x u_t, \\ \pi_t &= \bar{b}_\pi x_{t-1} + \bar{c}_\pi u_t. \end{aligned}$$

where $\bar{b}_x$ is the root $0 < \bar{b}_x < 1$ of

$$\beta \bar{b}_x^2 - \gamma \bar{b}_x + 1 = 0,$$

and $\gamma = 1 + \beta + \lambda^2/\alpha$.

We still need an <u>interest rate reaction function</u> that implements the optimal REE.

FUNDAMENTALS FORM OF THE OPTIMAL POLICY REACTION FUNCTION

- Compute $E_t \pi_{t+1}$ and $E_t x_{t+1}$ for the optimal REE.

- Insert into IS curve to get the "fundamentals based reaction function"
$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t,$$
where ψ_i depend on $\lambda, \alpha, \rho, \varphi, \beta$.

This i_t rule is consistent with the optimal REE. But

- Will it lead to "determinacy"?

- Will it lead to stability under learning?

DETERMINACY RESULTS: FUNDAMENTALS BASED REACTION FUNCTION

Proposition 1: *Under the fundamentals based reaction function there are parameter regions in which the model is determinate and other parameter regions in which it is indeterminate.*

Calibrations

W: $\beta = 0.99$, $\varphi = (0.157)^{-1}$, $\lambda = 0.024$.
CGG: $\beta = 0.99$, $\varphi = 1$, $\lambda = 0.3$
MN: $\beta = 0.99$, $\varphi = 0.164$, $\lambda = 0.3$.

Indeterminate for $\alpha < \hat{\alpha}$, where
$\hat{\alpha} = 0.16$ (W), 7.5 (CGG), 277 (MN)
Hence in some cases this i_t rule is also consistent with inefficient REE.

LEARNING

The NK model with the fundamentals based interest-rate rule can be put into our standard first-order form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = M \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + N \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + P \begin{pmatrix} g_t \\ u_t \end{pmatrix}.$$

$$y_t = M E_t^* y_{t+1} + N y_{t-1} + P v_t.$$

Recall that the optimal REE takes the form

$$y_t = \bar{a} + \bar{b} y_{t-1} + \bar{c} v_t.$$

Under learning agents use LS to estimate and update (a, b, c) over time. Does

$$(a_t, b_t, c_t) \to (\bar{a}, \bar{b}, \bar{c})?$$

The answer is obtained by computing the E-stability conditions as described earlier.

INSTABILITY RESULT

Proposition 2: *The fundamentals based reaction function leads to instability under learning for all structural parameter values.*

Partial Intuition: Fix all PLM parameters except a_π. Then

$$\Delta T_{a_\pi}(a_\pi) = (\beta + \lambda\varphi)\Delta a_\pi$$

via IS,PC. This tends to destabilize if $\beta + \lambda\varphi > 1$.

Conclusion: The fundamentals based reaction function can lead to indeterminacy and it always leads to instability under learning of the optimal REE.

Question: Is there an alternative interest rate setting rule that guarantees determinacy and stability?

AN EXPECTATIONS BASED OPTIMAL RULE

- The instability problem can be overcome if expectations of private agents are observable and policy is conditioned on them.

- To get an optimal rule of this form solve for i_t from structural equations (IS), (PC) and the optimality condition (OPT), <u>without</u> imposing RE.

- That is, solve

$$x_t = -\varphi(i_t - E_t^*\pi_{t+1}) + E_t^* x_{t+1} + g_t \quad \text{(IS)}$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \quad \text{(PC)}$$

$$\lambda \pi_t = -\alpha(x_t - x_{t-1}), \text{ all } t, \quad \text{(OPT)}$$

for i_t in terms of $x_{t-1}, E_t^* x_{t+1}, E_t^* \pi_{t+1}, g_t, u_t$.

We obtain

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t,$$

where

$$\begin{aligned}
\delta_L &= \frac{-\alpha}{\varphi(\alpha + \lambda^2)}, \\
\delta_\pi &= 1 + \frac{\lambda\beta}{\varphi(\alpha + \lambda^2)}, \\
\delta_x &= \delta_g = \varphi^{-1}, \\
\delta_u &= \frac{\lambda}{\varphi(\alpha + \lambda^2)}.
\end{aligned}$$

We call this the *expectations based reaction function*, or the expectations-based optimal i_t - rule.

This derivation made no specific assumption about expectation formation.

DETERMINACY AND STABILITY

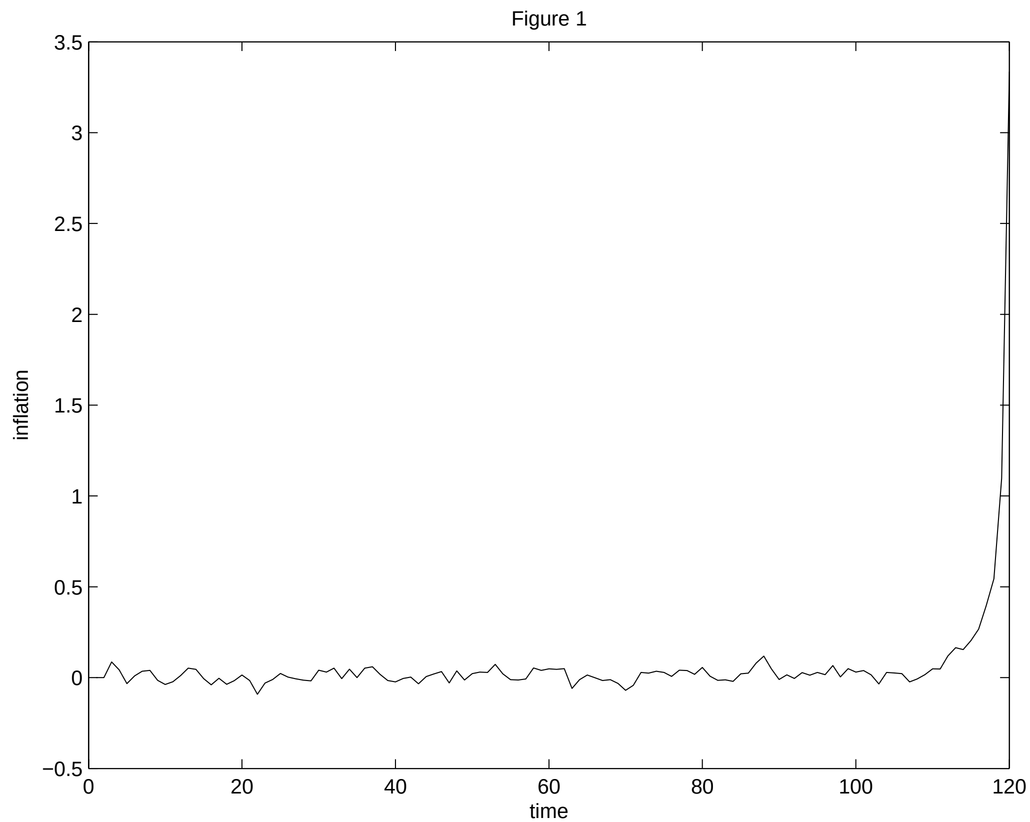
Proposition 2: *Under the expectations-based i_t - rule, the REE is determinate for all structural parameter values.*

Proposition 3: *Under the expectations-based i_t - rule, the optimal REE is stable under learning for all structural parameter values.*

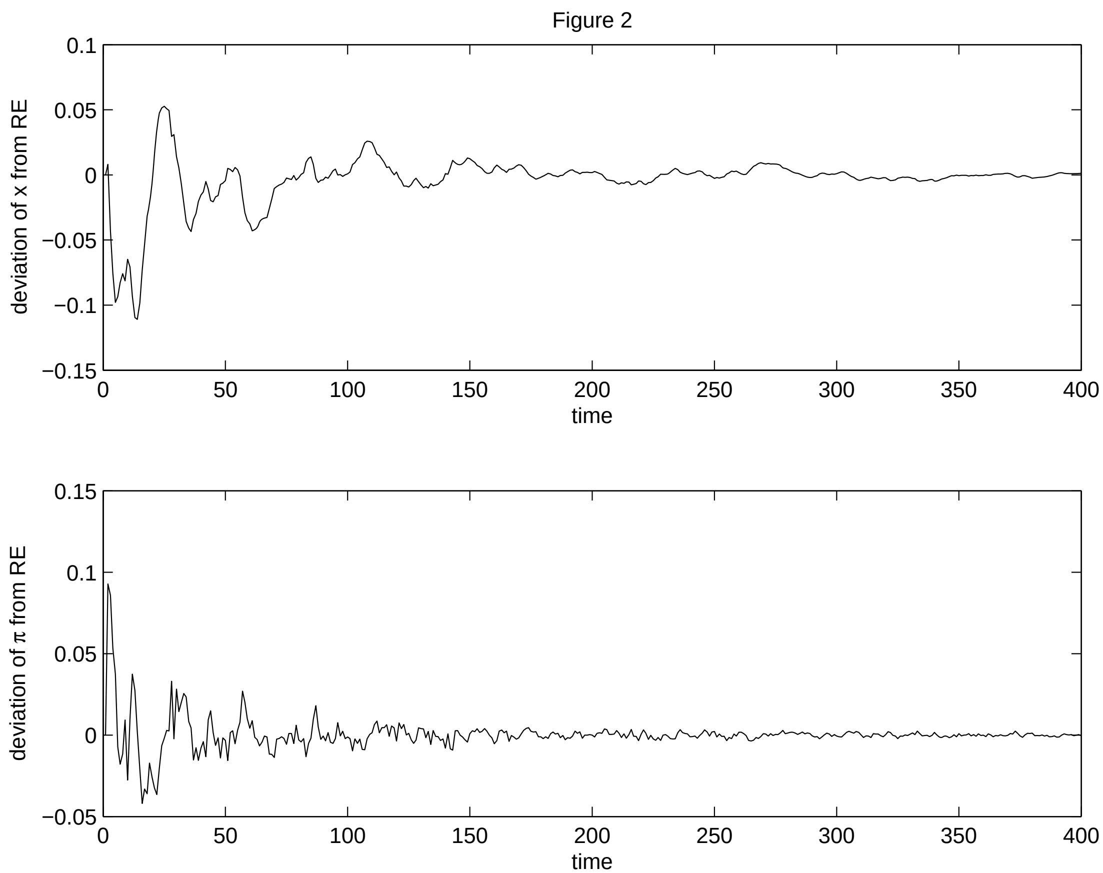
Partial intuition: $\uparrow E_t^* \pi_{t+1} \longrightarrow \uparrow\uparrow i_t \longrightarrow \downarrow x_t, \pi_t$.

Conclusion: if expectations are observable then the optimal policy can be achieved using the expectations-based i_t rule.

See numerical illustrations of instability and stability



Instability under fundamnetals-based rule



Stability under expectations-based rule

- Remarks:

- Stability of expectations-based rule would also hold for some variations of LS learning and even some misspecified learning schemes such as adaptive expectations.

- Determinacy and stability of EB rule holds if (a) expectations observed with white noise error, or (b) VAR proxies for expectations are used by Central Bank.

- Two important **extensions**:
  (i) structural parameter learning by Central Bank can be combined with learning by private agents, EH (JMCB, 2003).
  (ii) robustness to structural parameter uncertainty is important – see Evans & McGough (JMCB, 2007).

CONCLUSIONS

– Optimal monetary policy design should not simply assume RE.

– The economy will diverge under private agent learning if the fundamentals based i_t rule is followed. Indeterminacy may also arise.

– Under our expectations-based i_t rule the optimal REE is always stable under learning, and indeterminacies are avoided.

– If there is a high degree of uncertainty about structural parameters, the CB should follow "optimal constrained" rules, designed to be optimal subject to always delivering determinacy and stability under learning.

– General point: Monetary policy must treat expectations as subject to shocks and be designed to be stable under learning.

Monetary Policy under Perpetual Learning.

Orphanides and Williams (2005a), "Imperfect knowledge, inflation expectations and monetary policy"

– The learning literature looks not just at issues of stability under learning but also at the possibility of new learning dynamics.

– A particularly simple approach that is widely used is known as "constant gain LS learning" or "discounted LS learning." Agents are assumed to discount past data.

– As a result there is not full convergence to RE. It turns out that this can make a big difference.

– O&W investigate the implications for optimal monetary policy on a New Classical model.

– Lucas-type aggregate supply curve for inflation π_t:

$$\pi_{t+1} = \phi \pi^e_{t+1} + (1 - \phi)\pi_t + \alpha y_{t+1} + e_{t+1},$$

– Output gap y_{t+1} is set by monetary policy up to white noise control error

$$y_{t+1} = x_t + u_{t+1}.$$

– Policy objective function $\mathcal{L} = (1 - \omega)Var(y) + \omega Var(\pi - \pi^*)$ gives rule

$$x_t = -\theta(\pi_t - \pi^*).$$

where under RE $\theta = \theta^P(\omega, \phi, \alpha)$.

Learning: Under RE inflation satisfies

$$\pi_t = \bar{c}_0 + \bar{c}_1 \pi_{t-1} + v_t.$$

Under learning agents estimate this AR(1) model and forecast

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t.$$

Letting $c' = (c_0, c_1)$, the recursive LS scheme is

$$\begin{aligned} c_t &= c_{t-1} + \kappa_t R_t^{-1} X_t (\pi_t - X_t' c_{t-1}) \\ R_t &= R_{t-1} + \kappa_t (X_t X_t' - R_{t-1}), \\ \text{where } c_t &= (c_{0,t}, c_{1,t})' \text{ and } X_t = (1, \pi_{t-1})'. \end{aligned}$$

– The "gain" under LS is $\kappa_t = 1/t$ ("decreasing gain")

– Under "constant gain" learning we have $\kappa_t = \kappa$ for some $0 < \kappa < 1$.

Under learning private agents estimate coefficients by **constant gain** (or **discounted**) least squares. Older data dated discounted at rate $(1-\kappa)$. κ is called the "gain."

- Discounting of data natural if agents are concerned to track structural shifts.

- There is empirical support for constant gain ("perpetual") learning

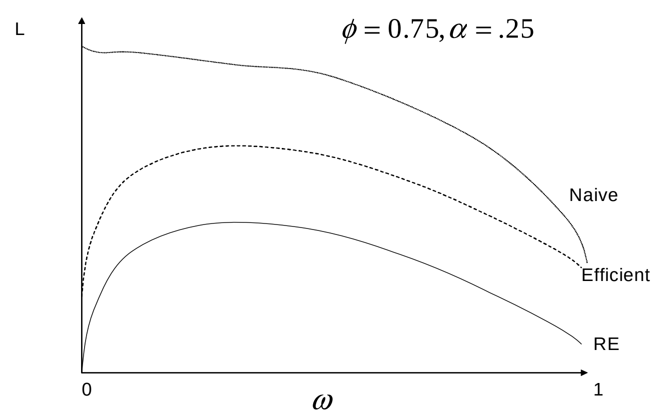
- With constant gain, LS estimates fluctuate randomly around $(\bar{c}_0, \bar{c}_1)$: there is "perpetual learning" and

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t.$$

Results:

- Perpetual learning increases inflation persistence.

- Naive application of RE policy leads to inefficient policy. Incorporating learning into policy response can lead to major improvement.



Policymaker's loss

- Efficient policy is more hawkish, i.e. under learning policy should increase θ to reduce persistence. This helps guide expectations.

- Following a sequence of unanticipated inflation shocks, inflation doves (i.e. policy-makers with low θ) can do very poorly, as expectations become detached from RE.

- If agents know π^* and only estimate the AR(1) parameter the policy trade-off is more favorable.

Postscript

Constant gain learning has been used in a numerous other applications in macroeconomics and finance.