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EXPECTATIONS, STAGNATION, AND FISCAL POLICY: A NONLINEAR ANALYSIS*

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Stagnation and fiscal policy are examined in a nonlinear stochastic New-Keynesian model with adaptive learning. There are three steady states. The steady state targeted by policy is locally but not globally stable under learning. A severe pessimistic expectations shock can trap the economy in a stagnation regime, underpinned by a low-level steady state, with falling inflation and output. A large fiscal stimulus may be needed to avoid or emerge from stagnation, and the impacts of forward guidance, credit frictions, central bank credibility, and policy delay are studied. Our model encompasses a wide range of outcomes arising from pessimistic expectations shocks.

1. INTRODUCTION

The sluggish macroeconomic performance of advanced market economies in the years following the Great Recession has raised interest in the possibility of the economy becoming stuck for long periods in a distinct stagnation regime associated with the zero lower bound (ZLB) for the policy interest rate.[1] The global COVID-19 pandemic has also raised longer-term concerns about stagnation. One possible explanation for a stagnation regime is that it is caused by a wide-spread lack of confidence on the part of economic agents. Specifically, the economy can become confined to a region with low output, deflation or below-target inflation, and interest rates constrained by the ZLB.

The recent pattern within many economies of extended periods of below target inflation rates, negative output gap, and near-zero policy interest rates, can be seen in the two panels of Figure 1, showing quarterly data from 2002Q1 to 2021Q2, of the United States, Japan, and the Euro area. The left panel gives a scatterplot of (core) inflation versus the policy interest rate, for each country/area, as originally done in Bullard (2010) for Japan and U.S. data and extended by Honkapohja (2016) to include Euro area data, in the context of the Fisher equation and an interest-rate policy rule. These equations identify the two steady states emphasized by rational expectations (RE): the targeted steady state corresponding to a 2%

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[1] For alternative explanations of long-lasting stagnation, see, for example, Summers (2013), Teulings and Baldwin (2014), Eggertsson et al. (2019), and Benigno and Fornaro (2018).

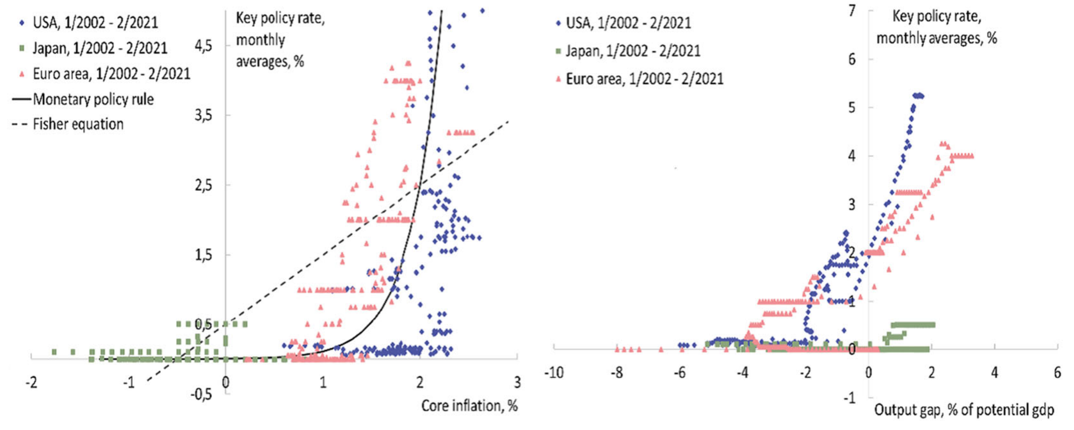

FIGURE 1

LEFT PANEL: INFLATION VERSUS POLICY INTEREST RATE; RIGHT PANEL: OUTPUT GAP VERSUS POLICY INTEREST RATE
NOTES: SEE ONLINE APPENDIX F FOR DETAILS.

inflation target and an unintended steady state with mild deflation at near-zero net interest rates.

The second panel plot, for each country/area gives the data on the output gap versus the policy interest rate. This panel makes it evident that policy interest rates near or at the ZLB are frequently associated with negative output gaps. Since the unintended RE deflation steady state has a negligible output gap in standard new Keynesian (NK) models, it is clearly challenging to interpret coordination on this steady state as the main focus for explaining macroeconomic outcomes at the ZLB.

We develop an extension of a standard NK model in which there exists a stagnation regime—a region of pessimistic expectations anchored by a stagnation steady state. Our analysis goes beyond RE by assuming that economic agents make forecasts using adaptive learning (AL). We show that in the stagnation region expectations become trapped, with the stagnation steady state acting as an attractor, preventing a return to the targeted steady state. Existence of this stagnation regime is consistent with the observation above that, under the ZLB constraint, real economic performance of the United States, Japanese and the Euro area economies appears to be clearly worse than in the earlier period before the ZLB became binding.

Our approach centers squarely on the role of expectations. In line with the AL literature, our agents are assumed to be boundedly rational: expectations are formed using statistical models that have the potential to converge to RE, but which can also sometimes follow trajectories away from the targeted steady state. Much of the RE literature focuses on managing an economy subject to large finite-duration, exogenous discount rate, or financial shocks, the stochastic properties of which are known, whereas our story stresses the role of pessimistic expectational overhang, continuing after the cessation of fundamental shocks, which can prevent the economy from returning to the targeted steady state.[2]

As in the RE literature, the ZLB plays a key role in our model, but our focus under AL is on local and global stability, not on indeterminacy or on self-fulfilling rational "sunspot" equilibria. In Section 2, we employ the basic Rotemberg adjustment-cost version of the NK model with AL, extended to include partial substitutability between private and public consumption. Section 3 develops the central result that this version of the benchmark NK model has three steady states, two of which are locally stable under AL: the targeted steady state and a subsistence-level "stagnation" steady state. The third ("unintended") indeterminate steady

[2] See Section 7 for more detailed discussion of the literature.

state remains of interest because it lies on the edge of the domain of attraction (DOA) of the targeted steady state: for a range of pessimistic expectations, the economy is drawn toward it before veering either to the targeted steady state or into the stagnation regime.

After establishing these central features of the economy under AL, we consider fiscal policy in the face of an adverse expectation shock. The analysis is carried out in a stochastic nonlinear economy in Section 4. At each point in time, aggregate output, consumption, and inflation arise as the temporary equilibrium implied by exogenous shocks and agents' decision rules. The latter in turn depend on point expectations of future variables obtained from forecast rules based on observed shocks, with coefficients updated over time using recursive least-squares (RLS) learning.

The starting point for our approach is that low output and inflation during a period of adverse exogenous shocks, may have made agents pessimistic about the future. These pessimistic expectations may continue for a time after the shocks have ceased, and the subsequent dynamics can depend sensitively on the position of these expectations relative to the DOA of the targeted steady state.[3] If expectations are too pessimistic, the economy can become trapped in the stagnation regime under normal policy. A key policy question in this case is whether fiscal policy can prevent stagnation and return the economy to the targeted steady state.

Section 5 turns to policy, focusing on situations in which output expectations are sufficiently pessimistic that with high likelihood they would lead, under unchanged policy, to the economy becoming trapped in the stagnation regime. We provide numerical results for the success of a fiscal stimulus, of stated magnitude and duration, in moving the economy to a path converging to the targeted steady state. The impact of fiscal policy is highly nonlinear: for a given duration, a small stimulus may be unsuccessful, whereas a larger temporary stimulus can be effective in returning the economy to the targeted steady state. Success is stochastic since convergence to the targeted steady state depends in part on the sequence of stochastic shocks. The probability of success, that is, avoiding stagnation, depends on the magnitude and length of fiscal stimulus.

Section 6 considers the implications of important extensions: (i) Combining expansionary fiscal policy with forward guidance in monetary policy can be beneficial; (ii) Policy delays reduce the efficacy of fiscal policy; (iii) With financial frictions the stagnation regime can include points with normal output expectations and positive but low inflation expectations; (iv) A higher inflation target enlarges the DOA of the targeted steady state; and (v) The likelihood of stagnation is reduced if the inflation target has substantial credibility. This section also illustrates the potential of our model to fit observed data using scatterplots from simulations.

A discussion of related literature is set out in Section 7. Section 8 concludes. The Online Appendices contain numerous technical details and further results.

2. THE MODEL

Our model is a generalization of Benhabib et al. (2014). There is a continuum of identical household-producers $i \in [0, 1]$. Agent i maximizes utility subject to flow budget and production function constraints:

$$E_{0,i} \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_{t,i} + \xi g_t) + \varkappa \log\left(\frac{M_{t-1,i}}{P_t}\right) - (1+\varepsilon)^{-1} h_{t,i}^{1+\varepsilon} - \Phi\left(\frac{P_{t,i}}{P_{t-1,i}}\right) \right\} \tag{1}$$

$$\text{s.t. } c_{t,i} + m_{t,i} + b_{t,i} + \Upsilon_{t,i} = m_{t-1,i}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,i} + \frac{P_{t,i}}{P_t} y_{t,i} \quad \text{and} \quad y_{t,i} = A_t h_{t,i}^{\alpha}.$$

[3] A similar point arises in connection with the large negative productivity, labor supply, and sectoral shocks since 2020 due to the coronavirus pandemic. The course of the economy will depend heavily on the course of the "intrinsic" virus shocks. However, even after these shocks have receded, there may well be a pessimistic overhang of the type considered in this article.

Here, $0 < \alpha, \beta < 1$. $c_{t,i}$ is the consumption aggregator consumed by i, $M_{t,i}$, and $m_{t,i} = M_{t,i}/P_t$ denote nominal and real money balances, $h_{t,i}$ is the labor input into production of good variety i, and $b_{t,i}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent i at the end of period t. g_t is exogenous government spending per capita, $\Upsilon_{t,i}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest-rate factor between $t-1$ and t, $P_{t,i}$ is the price of consumption good i, $y_{t,i}$ is output of good i, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. $\Phi \geq 0$ captures a convex pricing friction, with $\Phi(\pi^*) = 0$, where π^* is the inflation rate targeted by policymakers. A_t is a productivity shock to all firms with mean $\bar{A} > 0$. The household is subject to the usual "no Ponzi game" condition.

Utility of consumption includes both private consumption $c_{t,i}$ and public consumption $g_t > 0$ of the goods aggregator, with relative weight parameter $0 < \xi \leq 1$ capturing the degree of substitution between private and public consumption as in Christiano and Eichenbaum (1992). Under standard policy, $g_t = \bar{g} > 0$. Note that $\Phi(.)$ gives the (utility) cost of adjusting prices, which arises if agent i changes prices at a different rate from the central bank inflation target. We use the utility Rotemberg formulation, with household-producers, instead of either an output cost version or the Calvo model of price stickiness, because this enables us to study global dynamics in the nonlinear system. The parametric form of Φ is discussed below.

The consumption aggregator takes the usual constant elasticity of substitution (CES) form, with elasticity of substitution between two goods $\nu_t > 1$, where ν_t is a stationary AR(1) process. Output is differentiated and firms operate under monopolistic competition. Each household-firm faces a downward-sloping demand curve,

$$P_{t,i} = \left(\frac{y_{t,i}}{y_t}\right)^{-1/\nu_t} P_t, \quad \text{where } P_t = \left[\int_0^1 P_{t,i}^{1-\nu_t}\, di\right]^{1/(1-\nu_t)}. \tag{2}$$

Finally, the government faces the usual flow budget constraint—see Online Appendix A—and households are assumed Ricardian in the sense that they expect the government's intertemporal budget constraint to be satisfied. In particular, the tax implications for an increase in government spending would be fully anticipated by the households.

In line with the AL literature, our approach consists of three key pieces:

- Specification of agent decision rules, for consumption and price setting, conditional on current and expected future variables.
- Temporary equilibrium equations for a representative agent (RA) economy, based on aggregation and market clearing, given monetary and fiscal policy.
- Updating of agent forecast rule parameters using statistical learning.

The equilibrium path is then determined recursively. This general setup essentially implements the temporary equilibrium concept, introduced by Hicks (1939) and the Stockholm school of economic thought, within a dynamic setting in which expectations are updated over time in accordance with the AL approach. In the context of infinite-horizon agents solving dynamic optimization problems, our approach can be viewed as a version of the "anticipated utility" approach formulated by Kreps (1998), discussed in Sargent (1999) and Cogley and Sargent (2008).

We now turn to the formal description of the model. Online Appendix A gives the details. The decision rule of agent i for consumption $c_{t,i}$ is obtained by combining their iterated consumption Euler equations, under subjective expectations, with the household's Ricardian perceived intertemporal budget constraint.[4] An additional bounded-rationality assumption is imposed concerning expressions with conditional expectations of nonlinear functions of future random variables. Even if agents knew the required joint probability distributions this would

[4] Non-Ricardian households were considered in Benhabib et al. (2014).

be a difficult calculation, and since the distributions are unknown, they would need to be estimated. We make the assumption, which we view as realistic, that agents instead use *point expectations*, treating the conditional expectation of a nonlinear function of random variables as equal to the nonlinear function of the conditional expectations. Put differently, they act as if all conditional probability density of each random variable were concentrated at its expected value. This assumption is natural because it can plausibly be implemented by agents to approximate optimal decision making.

Using superscript e to denote subjective expectations, and letting $\Xi_{t,i} = P_{t,i}/P_t$, it is shown in Online Appendix A that consumption is given by

$$
\begin{aligned}
(3) \qquad c_{t,i} = {} & (1-\beta)[\Xi_{t,i} y_{t,i} - g_t(1+\xi\beta/(1-\beta))] \\
& + (1-\beta)\sum_{s=1}^{\infty} \left(D^e_{t,t+s,i}\right)^{-1}\left[\Xi^e_{t+s,i} y^e_{t+s,i} - g^e_{t+s,i}(1-\xi)\right],
\end{aligned}
$$

if this is nonnegative, else $c_{t,i} = 0$. Here, $D^e_{t,t+s,i} = \prod_{j=1}^{s} r^e_{t+j,i}$, for $r_{t+j} \equiv R_{t+j-1}/\pi_{t+j}$, are the perceived discount factors. This decision rule depends on forecasts of future incomes $\Xi^e_{t+s,i} y^e_{t+s,i}$, government consumption g^e_{t+s}, and discount factors $D^e_{t,t+s}$.

The agent's production and pricing decisions are governed by the pricing Euler equation. For the adjustment cost function $\Phi(P_{t,j}/P_{t-1,j})$, we use the Linex function given in Online Appendix A. This form makes deflation more costly, which is often regarded as more plausible, and provides a flexible way to capture downward price rigidity. Online Appendix A shows that iterating the Euler equation for price setting and assuming point expectations, yields the long-horizon pricing equation

$$
(4) \quad \Phi'(\pi_{t,i})\pi_{t,i} = \zeta_{t,i} + \sum_{s=1}^{\infty} \beta^s \zeta^e_{t+s,i}, \quad \text{where}
$$

$$
\zeta_{t,i} = \nu_t \alpha^{-1}(y_{t,i}/A_t)^{(1+\varepsilon)/\alpha} - (\nu_t - 1)(c_{t,i} + \xi g_t)^{-1} y_t \Xi_{t,i}^{1-\nu_t} \quad \text{and} \quad \Xi_{t,i} = P_{t,i}/P_t.
$$

Decision rules (3) and (4) require agents to make forecasts of various future variables, and to proceed further we make an additional bounded-rationality assumption below. To forecast $\zeta^e_{t+s,i}$, agent i needs to forecast future exogenous variables ν_{t+s}, A_{t+s}, g_{t+s}, aggregate output y_{t+s}, the discount factor $D_{t,t+s}$, but also the agent's relative price $\Xi_{t+s,i} = P_{t+s,i}/P_{t+s}$, market demand $y_{t+s,i}$, and marginal utility $(c_{t+s,i} + \xi g_{t+s})^{-1}$. Thus formally (3) and (4) are conditional decision rules.

Our approach is to assume that agents use these conditional decision rules supplemented by forecasts of future variables, including some that they themselves will be setting. Thus we share with Eusepi and Preston (2010) the assumption that agents are infinite-horizon anticipated-utility optimizers, but in contrast to them our agents do not assume that their future pricing decisions $P_{t+s,i}$, for example, will be consistent with what would be their optimal choices under current expectations of the variables exogenous to their decision making, including future aggregate inflation and aggregate output. Instead we assume agents use AL based on observed data to forecast $\Xi_{t+s,i}$, $y_{t+s,i}$, and $(c_{t+s,i} + \xi g_{t+s})^{-1}$, an assumption we view as plausible and which also simplifies our model and makes possible a nonlinear global analysis.[5]

In line with the anticipated utility approach agents update forecasts over time but do not explicitly take into account that their forecasting model parameters will change over time. This is a boundedly rational decision-making approach widely used in the AL literature; see Cogley and Sargent (2008) and Sargent (2008).

[5] The assumption that agents forecast some future variables that are under their control has also been used by Eusepi and Preston (2012) and Woodford (2013).

Because of our RA framework, in which all agents behave identically, in temporary equilibrium $\Xi_{t,i} = 1$ for all agents i at all times t. Under AL, agents will therefore learn over time that $\Xi^e_{t+s,i} \to 1$ with probability one. Similarly, $y_{t,i} = y_t$ and $c_{t,i} = c_t$, so that under AL we would have $y^e_{t+s,i} \to y^e_{t+s}$ and $c^e_{t+s,i} \to c^e_{t+s}$. Although we could allow for initial out-of-equilibrium expectations for these variables, this would add little to our analysis. Thus, we now assume agents have learned that $\Xi^e_{t+s,i} = 1$, $y^e_{t+s,i} = y^e_{t+s}$, and $c^e_{t+s,i} = c^e_{t+s}$. Furthermore, from market clearing $y_t = c_t + g_t$, and $c_t + \xi g_t = y_t - (1-\xi)g_t$, so we can assume $c^e_{t+s,i} + \xi g^e_{t+s} = y^e_{t+s} - (1-\xi)g^e_{t+s}$.

We can now list the RA temporary equilibrium equations:

$$y_t = \max\left\{g_t, (1-\xi)g_t + (\beta^{-1}-1)\left[\sum_{s=1}^{\infty}\left(D^e_{t,t+s}\right)^{-1}\left(y^e_{t+s} - (1-\xi)g^e_{t+s}\right)\right]\right\} \tag{5}$$

$$\Phi'(\pi_t)\pi_t = \zeta_t + \sum_{s=1}^{\infty}\beta^s \zeta^e_{t+s}, \quad \text{where} \tag{6}$$

$$\zeta^e_{t+s} = \alpha^{-1}\nu^e_{t+s}\left(y^e_{t+s}/A^e_{t+s}\right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s}-1)y^e_{t+s}\left(y^e_{t+s} - (1-\xi)g^e_{t+s}\right)^{-1} \quad \text{for } s \geq 0. \tag{7}$$

There remains only to specify the policy variables and to discuss $D^e_{t,t+s}$. In normal times $g_t = \bar{g}$ is fixed. When active fiscal policy is used, it will follow an announced exogenous path. Monetary policy follows the forward-looking interest-rate rule[6]

$$R_t = R\left(\pi^e_{t+1}, y^e_{t+1}\right) = 1 + (R^* - 1)\left(\pi^e_{t+1}/\pi^*\right)^{BR^*/(R^*-1)}\left(y^e_{t+1}/y^*\right)^{\phi_y}, \tag{8}$$

where $B > 1$ and $\phi_y \geq 0$. Here, $R^* = \beta^{-1}\pi^*$ and y^* is the target level of output, assumed equal to the output level at the nonstochastic targeted steady state. Note $R_t \geq 1$, that is, the R_t satisfies the ZLB for net interest rates. Agents are assumed to know the interest-rate rule. Hence,

$$r^e_{t+j} \equiv R\left(\pi^e_{t+j}, y^e_{t+j}\right)/\pi^e_{t+j}, \quad \text{and} \quad D^e_{t,t+s} = \prod_{j=1}^{s} r^e_{t+j}. \tag{9}$$

Given the exogenous variables A_t, ν_t, g_t and expectations $\{y^e_{t+s}\}, \{\pi^e_{t+s}\}, \{A^e_{t+s}\}, \{\nu^e_{t+s}\}, \{g^e_{t+s}\}$, Equations (5)–(9) determine the temporary equilibrium.

3. STEADY STATES AND LEARNING DYNAMICS

To examine our model under AL, we begin with a nonstochastic setting. The model has three interior perfect-foresight steady states. Under AL expectations are revised over time and the learning dynamics can be studied. AL rules are particularly simple in the nonstochastic case; thus, formal results for local stability can be obtained and the global dynamics characterized. We then extend AL to the stochastic model and study the learning dynamics numerically.

[6] We have also considered contemporaneous rules $R_t = R(\pi_t, y_t)$ with a similar functional form, and the main results appear unchanged. The rule (8) is formally and computationally simpler to implement. One interpretation of (8) is that the policy rate reacts to private-sector expectations.

3.1. *Steady States and Learning in Nonstochastic Case.* The nonstochastic model sets $\nu_t = \nu > 1$, $A_t = A$, and $g_t = \bar{g}$. AL in nonstochastic models usually study agents attempting to learn a steady state using expectations based on long-run averages. Introducing the notation $y^e_{t+s} = y^e_t$, and $\pi^e_{t+s} = \pi^e_t$ for time t expectations of future values for all horizons $s > 0$, AL takes the simple form

$$y^e_t = y^e_{t-1} + \omega\big(y_{t-1} - y^e_{t-1}\big) \quad \text{and} \quad \pi^e_t = \pi^e_{t-1} + \omega\big(\pi_{t-1} - \pi^e_{t-1}\big), \tag{10}$$

where $0 < \omega < 1$ is the learning "gain" parameter.[7] The focus is usually on ω small and thus examines local stability of steady states for sufficiently small $\omega > 0$. Rules of this form are often called "steady-state" learning, because they are simple adaptive rules that can converge to perfect-foresight steady states.

For this setting, the temporary equilibrium equations simplify to [8]

$$\begin{aligned}
y_t &= \max\left\{\bar{g}, \bar{g}(1-\xi) + (\beta^{-1} - 1)[(y^e_t - \bar{g}(1-\xi))\pi^e_t/(R(\pi^e_t, y^e_t) - \pi^e_t)]\right\}, \\
\pi_t &= Q^{-1}\Big[(\nu/\alpha)(y_t/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)y_t(y_t - (1-\xi)\bar{g})^{-1} \\
&\quad + \beta(1-\beta)^{-1}\Big[(\nu/\alpha)(y^e_t/A)^{(1+\varepsilon)/\alpha} - (\nu-1)y^e_t(y^e_t - (1-\xi)\bar{g})^{-1}\Big]\Big], \quad \text{or} \\
y_t &= G_2(\pi^e_t, y^e_t) \text{ and } \pi_t = G_1(y_t, y^e_t)
\end{aligned}$$

in general notation. Here, $Q(\pi) \equiv \Phi'(\pi)\pi$.

In a perfect-foresight steady state, with $y_t = y^e_t = y$ and $\pi_t = \pi^e_t = \pi$, the temporary equilibrium equation for π_t implies

$$(1-\beta)\Phi'(\pi)\pi = (\nu/\alpha)(y/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)y \times (y - (1-\xi)\bar{g})^{-1}. \tag{11}$$

The consumption Euler equation implies $\beta^{-1} = r = R/\pi$, provided $c > 0$. The steady state targeted by monetary policy is at $\pi = \pi^*$ with a corresponding output level $y^* > \bar{g}$ given by (11). This is the value y^* used in (8), together with $R^* = \pi^*/\beta$.

Smooth interest-rate rules that obey the Taylor principle, $(d/d\pi^e)R(\pi^*, y^*) > \beta^{-1}$, imply a second steady state (π_L, y_L) with $\pi_L < \pi^*$. In this "unintended" steady state $R = \pi_L/\beta$ and $y_L > \bar{g}$ is determined by (11). If the ZLB were strictly binding at $\pi = \pi_L$, so that $R = 1$, then we would have $\pi_L = \beta$, that is, there would be a net deflation rate of $1 - \beta$. Under our calibration of (8), $1 > \pi_L > \beta$ with $\pi_L \approx \beta$.

In our model, there is also generally a third "stagnation" steady state, at $y = \bar{g}$, $c = 0$ and deflation. The corresponding inflation rate $\pi_S < \pi_L$ is determined from (11) by

$$(1-\beta)\Phi'(\pi_S)\pi_S = (\nu/\alpha)(\bar{g}/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)\xi^{-1}.$$

The condition for existence of the stagnation steady state is $\bar{g}/A < (\alpha(\nu-1)/\nu\xi)^{\alpha/(1+\varepsilon)}$ as $\Phi'(\pi)\pi < 0$ if and only if $\pi < \pi^*$. For the calibration below, the condition $\bar{g} < 1.338$, approximately, is required.

In the stagnation steady state, the consumption Euler equation, and the Fisher equation, are not satisfied with equality, since households are at the corner solution to $c \geq 0$. The nominal interest rate $R \geq 1$ is very close to the ZLB, that is, $R \approx 1$, so the real interest rate is high, $r \approx 1/\pi_S$. However, households cannot increase their saving because their income net of taxes

[7] In the stochastic case, "decreasing gains" ω_t are sometimes used, in which ω_t is proportional to t^{-1}. See Online Appendix B for the RLS equations.

[8] We remark that for a range of values of π^e_t, y^e_t, it is possible that $R(\pi^e_t, y^e_t) < \pi^e_t$. This issue is discussed further in Section 4 but it does not arise for local stability under steady-state learning.

is zero. They are required to pay their taxes, levied by the government to finance the production of public consumption goods $\bar{g}$. Households use their labor to produce and sell sufficient goods to cover these taxes. They could increase their income further by increasing their labor and production, but the needed reduction in prices to sell the output would, due to the Rotemberg pricing friction, cause greater disutility. Household-producers are at a corner solution with private consumption zero but with positive public consumption and marginal utility bounded above zero.

We emphasize that the stagnation steady state is extreme, and there is no suggestion that the economy has been or is likely to be in this steady state. Its importance and role is that it is a well-defined steady state that, as we will see, acts as an attractor outside the DOA of the targeted steady state.

We now turn to stability of the three steady states π^*, π_L, and π_S under AL. E-stability, defined in terms of the ordinary differential equation (ODE) given below, is known to be the condition for local convergence of steady state learning to a (steady-state) fixed point. In general, for a vector of learning parameters θ, the E-stability ODE is $d\theta/d\tau = T(\theta) - \theta$, where $T(\theta)$ gives the corresponding actual temporary equilibrium outcome parameters corresponding to given perceived law of motion (PLM) parameters θ. Here, τ denotes "notional" time, which can, however, be linked to real time t. From the above temporary equilibrium equations, we obtain the E-stability differential equations:

$$d\pi^e/d\tau = F_\pi(\pi^e, y^e) \equiv G_1(G_2(\pi^e, y^e), y^e) - \pi^e, \quad \text{and} \tag{12}$$

$$dy^e/d\tau = F_y(\pi^e, y^e) \equiv G_2(\pi^e, y^e) - y^e, \tag{13}$$

so in our model $\theta = (\pi^e, y^e)^T$ and $T(\theta) = [F_\pi(\pi^e, y^e), F_y(\pi^e, y^e)]^T$. We have the following results, which are proved in Online Appendix E:

PROPOSITION 1. *(a) (i) The targeted steady state at (π^*, y^*) is E-stable provided ϕ_y is not too large. (ii) The steady state (π_L, y_L) is not E-stable if ϕ_y is not too large. (iii) The steady state (π_S, y_S) is E-stable. (b) Hence, provided ϕ_y is not too large and for all $\omega > 0$ sufficiently small, under the learning rule (10), the steady state (π_L, y_L) is not locally stable and the steady states (π^*, y^*) and (π_S, y_S) are locally stable.*

The condition ϕ_y not too large is standard and known to be necessary, with forward-looking interest-rate rules, to avoid indeterminacy of the targeted steady state.

With temporary equilibrium of the nonlinear system fully specified, we next extend our analysis numerically to look at the global system under learning.

3.2. *Global Analysis of E-Stability Dynamics.* For the nonstochastic system, the dynamics of the differential equations (12)–(13) give the global learning dynamics under (10) corresponding to small learning gain $\omega > 0$. In the current section, we assume government spending is constant, that is, $g = \bar{g}$. The numerical values of the parameters are typical and correspond to a quarterly calibration given in Online Appendices C and D.

Figure 2 provides a sketch of the global E-stability dynamics that includes all three steady states: the targeted steady state (π^*, y^*), the unintended liquidity trap steady state (π_L, y_L), and the boundary stagnation steady state $(\pi_S, y_S = \bar{g})$.[9] Steady states (π^*, y^*) and (π_L, y_L) have been widely discussed in the RE literature.

Figure 2 illustrates that the steady state at π^* is locally stable under learning dynamics, whereas the one at π_L is locally unstable under learning; these observations are well known,

[9] Under our calibration, $\pi^* = 1.005$, $y^* = 1.00003$, and $R^* = \beta^{-1}\pi^* \approx 1.01515$. At the unintended steady state, $\pi_L = 0.996393$, $y_L = 0.999862$, and $R_L = \beta^{-1}\pi_L \approx 1.00646$. At the stagnation steady state, $\pi_S = 0.647161$, $y_S = 0.2$, and $R_S \approx 1$.

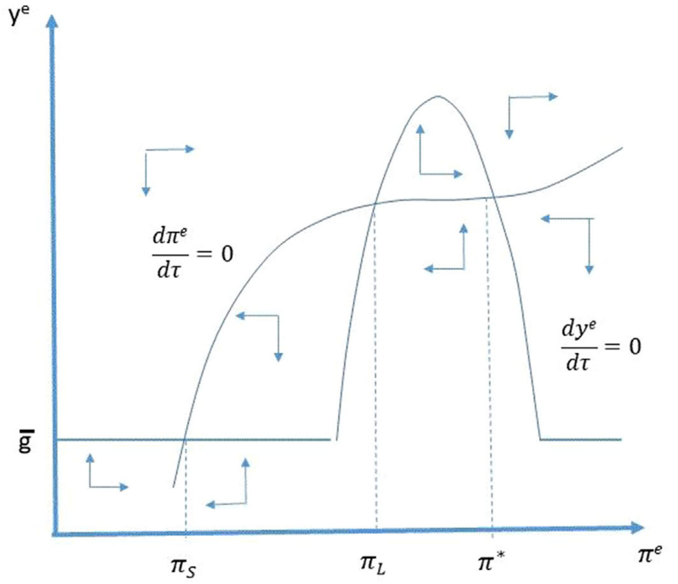

FIGURE 2

GLOBAL E-STABILITY DYNAMICS

see, for example, Benhabib et al. (2014).[10] At the third, stagnation, steady state, output $y = \bar{g}$ is at the minimal level, with households receiving only $\bar{g}$ as subsistence consumption (private consumption is zero), and there is rapid deflation and a high real interest rate. $(\pi_S, \bar{g})$ is locally stable, and more specifically is a sink with dynamics nearby that are not oscillatory. Interestingly, above the targeted steady state the bound $y_t \geq \bar{g}$ is also binding for sufficiently high values of π_t^e. This is because the real interest rate $R(\pi_t^e, y_t^e)/\pi_t^e$ then becomes very high (due to the Taylor rule) reducing c_t to zero.[11]

Noting the saddle-point nature of the unstable middle steady state (π_L, y_L) in Figure 2, it is possible to construct the DOA for the locally stable targeted steady state under the E-stability dynamics.[12] It will be convenient henceforth to refer to the targeted steady-state domain of attraction as the DOA. In Figure 3 (left panel), the DOA is the "liver-shaped" region bounded by the thick solid (blue) curve with a narrow tail toward the northwest and is shaded (yellow) in the figure.[13] The targeted steady state is at $\pi^* = 1.005$ and $y^* = 1.00003$ and is shown by the star in Figure 3 (left panel). For any expectations (π^e, y^e) inside the DOA, in the nonstochastic case under consideration, the economy will converge under learning to the targeted steady state, whereas it will diverge to the stagnation steady state from all points outside this domain.

In other words, under imperfect knowledge, there is a real possibility that after significant shocks, leading to an adverse shift in expectations (π^e, y^e), the economy can move into, and become trapped in, a region leading to stagnation under unchanged monetary and fiscal

[10] Quantitatively, y_L is only slightly smaller than y^*. This result is not sensitive to ξ, the degree of substitutability between private and public consumption. Ceteris paribus, increases in ξ lead, via the labor-leisure choice, to approximately equal decreases in y^* and y_L.

[11] In Figure 2, this phenomenon would appear in the curve $d\pi^e/d\tau = 0$ which gradually turns near-horizonal for large $\pi^e > \pi^*$.

[12] We note one issue for global E-stability dynamics, which is that $R(\pi^e, y^e) < \pi^e$ for some configurations (π^e, y^e). This issue does not arise in Figure 3, but would arise if $y^e < 0.9$ and $\pi^e \approx 1$. This point is addressed in Section 4 in the context of global numerical simulations.

[13] The DOA extends beyond the range shown in the figure but becomes increasingly narrow.

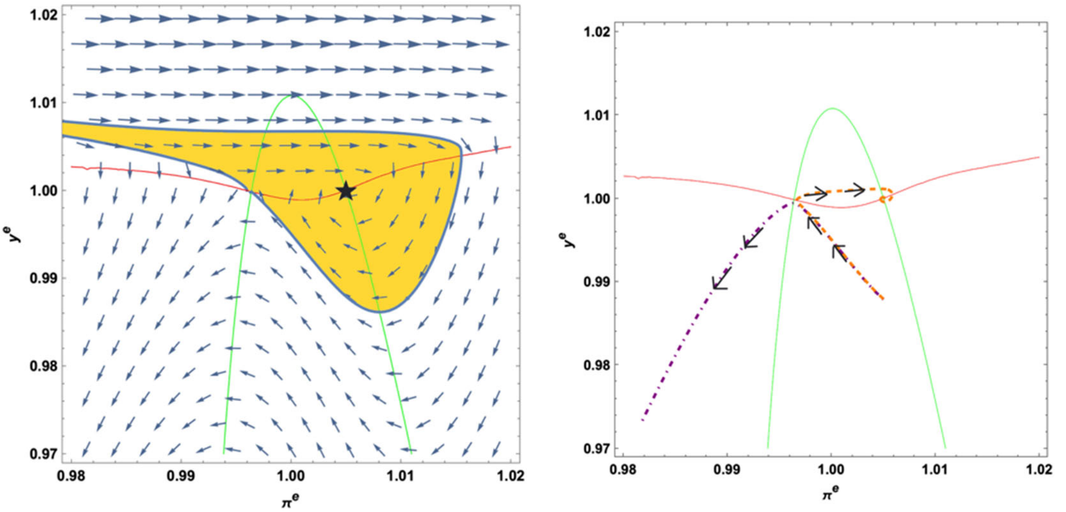

FIGURE 3

THE LEFT PANEL SHOWS THE DOMAIN OF ATTRACTION (DOA) OF THE TARGETED STEADY STATE (π^*, y^*)
NOTES: THE DOMAIN OF ATTRACTION IS SHADED (YELLOW) IN THE FIGURE AND THE TARGET STEADY STATE IS INDICATED BY THE STAR. THE RIGHT PANEL SHOWS THE SENSITIVE DEPENDENCE OF DYNAMICS ON INITIAL CONDITIONS. THE ARROWS SHOW THE DIRECTION OF MOVEMENT OF THE VARIABLES.

policy. [14] It is convenient to refer to the part of the DOA of the stagnation steady state in which $\pi^e < \pi_L$ and $d\pi^e/d\tau < 0$ as the stagnation "regime" or "region" or as the "stagnation trap." (The related term "deflation trap" is also used in the literature.) The underlying forces are, first, that the interest rate is at or near the ZLB and, second, that with output low, inflation and expected inflation are falling. Consequently, expected real interest rates are high and increasing. This in turn leads to lower demand and output leading to self-reinforcing stagnation dynamics. For future reference, when $\pi^e = \pi^*$ the lower boundary of the DOA is approximately $y^e = 0.98792$.

Figure 3 (right panel) shows that the model has sensitive dependence on initial conditions in a relevant area of the state space. Consider time paths of the economy from a starting point at $\pi_0^e = \pi^*$ and y_0^e slightly below or slightly above the boundary of DOA $y^e = 0.98792$. The dotted-dashed (purple) curve shows the time path from an initial value y_0^e slightly below 0.98792 whereas the dashed (orange) time path corresponds to y_0^e slightly above 0.98792. The two time paths are very close to each other until they get near the middle steady state (π_L, y_L). They then evolve in very different ways: one path moving deep into the stagnation region, and the other path eventually converging to the targeted steady state in dampening oscillations.

This sensitivity to initial conditions is local to the boundary of the DOA, but it occurs in a critical area and complicates decision making for policymakers. Looking at the illustration in Figure 3 (right panel), it can be difficult to know, for some time, whether or not aggressive policies need to be, or retrospectively should have been, followed. For both paths shown, over an extended stretch of time, y^e is low but improving and π^e is below target and falling, with interest rates (not shown) near the ZLB, as the unstable middle steady state (π_L, y_L) is approached. Only then, after a possibly extended period near (π_L, y_L), does it become evident whether the economy will recover or will instead deteriorate and move deep into the stagnation region.

[14] Parameter values of the monetary policy rule matter for the size of the DOA, for example, for $\phi_y = 0$ the DOA is smaller than in our base case. The preference parameter ξ affects steady-state outputs y^* and y_L but the impact on the size of the DOA is small.

The preceding discussion suggests that there will be some challenges in the design of fiscal and monetary policy. As we will see in Section 5, if the economy is outside the DOA and fiscal policy is used to try to direct the economy to the targeted steady state it will be important to choose the magnitude and length of the fiscal stimulus carefully. Finally, as also discussed in detail in Section 5 , an aggressive policy change is required if expectations are quite pessimistic.

4. EXTENSION TO THE STOCHASTIC ECONOMY

We now turn to the model under AL when the economy is subject to stochastic shocks. We use the RLS learning approach to expectation formation as developed in Bray and Savin (1986), Marcet and Sargent (1989), and Evans and Honkapohja (2001). Under this approach, agents forecast like econometricians, regressing variables to be forecasted on observed explanatory variables, updating the forecast rule coefficients as new data become available.

The productivity shocks A_t and mark-up shocks ν_t are assumed to be independent of each other and to take the form

$$\ln(A_t/\bar{A}) = \rho_A \ln(A_{t-1}/\bar{A}) + \ln(\varepsilon_{A,t}) \quad \text{and} \quad \ln(\nu_t/\bar{\nu}) = \rho_\nu \ln(\nu_{t-1}/\bar{\nu}) + \ln(\varepsilon_{\nu,t}),$$

where $0 \leq \rho_A, \rho_\nu < 1$, and where $\ln(\varepsilon_{A,t}) \sim IIN(0, \sigma_A^2)$ and $\ln(\varepsilon_{\nu,t}) \sim IIN(0, \sigma_\nu^2)$. We assume A_t, ν_t are observable, and, for convenience, the parameters $\bar{A}, \bar{\nu}, \rho_A, \rho_\nu$ are assumed to be known to agents. (If the parameters were unknown it would be straightforward for agents to use consistent estimates of them). We assume the forecast rules include linear dependence on the observable A_t and ν_t. Alternative assumptions could be entertained at the cost of further analytical complexity.

Specifically, agents have a PLM taking the form

$$\begin{aligned}
\ln(y_t) &= f_y + d_{yA} \ln(\tilde{A}_t) + d_{y\nu} \ln(\tilde{\nu}_t) + \eta_{yt} \\
\ln(\pi_t) &= f_\pi + d_{\pi A} \ln(\tilde{A}_t) + d_{\pi\nu} \ln(\tilde{\nu}_t) + \eta_{\pi t},
\end{aligned}$$

where $\eta_{yt}, \eta_{\pi t}$ are perceived white noise shocks, where $\tilde{A}_t \equiv A_t/\bar{A}$ and $\tilde{\nu}_t = \nu_t/\bar{\nu}$. To form forecasts y_{t+s}^e and π_{t+s}^e at time t, agents estimate the parameters of the PLM using data up to period $t-1$ and iterate the estimated PLM forward to period $t+s$.

The PLMs are estimated using constant-gain RLS, see Online Appendix B for formal details. Letting $f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu}$ now denote the time t values of the parameter estimates, expectations of output and inflation s steps ahead, based on the observed exogenous shocks $\tilde{A}_t$ and $\tilde{\nu}_t$, are given by

$$y_{t+s}^e = e^{f_y} \tilde{A}_t^{\rho_A^s d_{yA}} \tilde{\nu}_t^{\rho_\nu^s d_{y\nu}} \quad \text{and} \quad \pi_{t+s}^e = e^{f_\pi} \tilde{A}_t^{\rho_\pi^s d_{\pi A}} \tilde{\nu}_t^{\rho_\nu^s d_{\pi\nu}}.$$

With these expectations, the temporary equilibrium at time t is given by (5)–(9), subject to the modification described in the beginning of Subsection 4.1.

The dynamic path under AL is then specified recursively. At the beginning of time $t+1$ estimates of $\phi_y' = (f_y, d_{yA}, d_{y\nu})$ and $\phi_\pi' = (f_\pi, d_{\pi A}, d_{\pi\nu})$ are updated to include the time t data point using the RLS equations. Then, after the time $t+1$ exogenous random variables are drawn, the temporary equilibrium equations determine $y_{t+1}, c_{t+1}, \pi_{t+1}$, and R_{t+1}. Given initial conditions and continuing in this way generates a time path of temporary equilibria $\{y_t, c_t, \pi_t, R_t\}_{t=0}^\infty$ for the economy under AL. For further details, see Online Appendix B.

4.1. *Simulation Results.* For our numerical analysis, we conduct stochastic simulations over long periods of time and one must allow for trajectories that can go very far from steady states. Consequently, two modifications in our simulations are made. First, it is assumed that after T periods the transitory stochastic component of output and inflation forecasts can be ignored by agents, that is, we set $y^e_{t+s} = e^{f_{yt}}$ and $\pi^e_{t+s} = e^{f_\pi}_t$ for $s \geq T$. This is a convenient way of speeding up computations. In the simulations, we set $T = 28$. Second, agents are assumed to believe that after T_1 periods the real interest rate reverts to its steady-state value β^{-1}, that is, $r^e_{t+s} = \beta^{-1}$ for $s \geq T_1$. Some assumption like this is needed for examining global dynamics since there are some regions of the expectational parameter space for which the expected real interest-rate factor would be less than one, implying undefined consumption.

In our benchmark simulations, we set $T_1 = 20$, that is, at each time t agents believe real interest rates will return to their steady-state value after five years.[15] Thus, T_1 (and T) are rolling windows. Although an assumption like this is needed for technical reasons, it can also be viewed as making a substantive assumption about expectations: agents believe that periods of persistently high or low real interest rates will end after five years.[16] One could, of course, use higher values for T and T_1.

Before turning to numerical results we discuss the role of the gain sequence ω_t. Consider first the decreasing gain case in which $\omega_t \to 0$, as $t \to \infty$. As with the nonstochastic case, if the variances of stochastic shocks are not too large, and with additional plausible assumptions, we can expect there to be fixed forecast parameters $\bar{\phi}_y, \bar{\phi}_\pi$ that correspond to an equilibrium near the targeted steady state. The resulting equilibrium, usually called a "*restricted perceptions equilibrium*" (RPE), is a generalization of a rational expectations equilibrium (REE): the forecast coefficients $\bar{\phi}$ are minimum mean squared error within the restricted class of linear forecast models used by agents, though in principle better nonlinear forecast rules may exist.[17] The RPE also differs from the REE due to our boundedly optimal agents' use of point expectations in their forecasting. However, the RPE can be viewed as an approximation to the REE centered at the targeted steady state.[18]

The *E-stability principle* states that, in the decreasing gain case, with suitable additional assumptions, this RPE will be locally stable under RLS learning, so that for initial expectations near the RPE parameters $\bar{\phi}_y, \bar{\phi}_\pi$ we will have $\phi_{yt} \to \bar{\phi}_y$ and $\phi_{\pi t} \to \bar{\phi}_\pi$. Similarly, we can expect an RPE at the stagnation steady state to be locally stable but for the middle steady state (π_L, y_L) to be locally unstable under RLS learning. Thus in the stochastic model, local stability of the equilibrium paths under RLS learning is inherited from E-stability of the steady states.

In practice, in applied macromodels, a constant gain $\omega_t = \omega$ with $0 < \omega < 1$, is almost invariably assumed. This allows agents to track structural change and changes in policy, but also results in "perpetual learning dynamics" around an REE or RPE. An advantage of this in empirical applications is that the learning dynamics are part of a stationary system.[19] In our numerical simulations, constant-gain learning is employed. Some theoretical stochastic approximation results are available for constant-gain learning in the stochastic model in the limiting case $\omega > 0$ sufficiently small,[20] based on an ODE approximation to the RLS system.

In the current setting, the E-stability principle confirms local stability of both the targeted and the stagnation steady state, and local instability of the middle steady state.[21] However,

[15] This is consistent with expected long real rates varying over horizons longer than five years.

[16] Of course, monetary policy can in principle commit to a path of future *nominal* interest rates over a much longer period. In Section 6, we explore the impact of credible forward guidance by the Central Bank about future nominal rates.

[17] For discussion of RPE, see, for example, Evans and Honkapohja (2001), Ch. 13, and Branch (2006). For applications in nonlinear models, see Evans and McGough (2020a, 2020b).

[18] For A_t, v_t with finite support, REE and RPE coincide as $\sigma_A, \sigma_v \to 0$ and $\rho_A, \rho_v \to 0$.

[19] Discussion and applications of constant-gain learning in economics include Sargent (1999), Evans and Honkapohja (1993), Cho et al. (2002), McGough (2006), Milani (2007), Orphanides and Williams (2007), Branch and Evans (2011), and Eusepi and Preston (2011).

[20] See, for example, Subsection 7.4 and Chapter 14 of Evans and Honkapohja (2001), Cho et al. (2002), Evans and Honkapohja (2009), and Williams (2019).

[21] The intercepts of the expectations functions govern the evolving means of y_t and π_t in the sequence of temporary equilibria, so that the preceding E-stability analysis remains central to the model's dynamics.

TABLE 1
PERCENTAGE CONVERGENCE TO TARGET OR STAGNATION UNDER UNCHANGED POLICY, 100 REPLICATIONS, GAIN = 0.01

Initial Expectation	$P(target)$	$P(stagn.)$
$y_0^e/y^* = 0.9980$	100	0
$y_0^e/y^* = 0.9975$	69	31
$y_0^e/y^* = 0.99745$	15	85
$y_0^e/y^* = 0.99742$	1	99
$y_0^e/y^* = 0.9974$	0	100

as was already noted, with constant-gain learning perpetual fluctuations remain, for example, near the targeted steady-state RPE. The forecast rule parameters $\phi_{yt}, \phi_{\pi t}$ have means near their RPE values and variances approximately proportional to the gain ω. Stochastic approximation results based on the ODE approximation to the updating equations (see Online Appendix B) can be used to compute the "mean dynamics" globally, but in practice it is convenient to study the dynamics directly using stochastic simulations.

We are particularly interested in how the size of an initial pessimistic expectations shock affects whether the economy returns to the targeted steady-state RPE or whether it is pushed into the stagnation regime along a path toward the stagnation steady state. To study this using simulations of our calibrated model, we consider the impact over time of an unmodeled adverse shock to output expectations y_0^e, such as might have occurred following the 2007–8 financial crisis, lowering agents' estimates of future output and incomes.

Assume the economy is initially in the targeted steady state (with $y^e = y^* = 1.00003 \approx 1$) when a shock to expectations occurs. Because our model is now stochastic, we anticipate that, at least for a range of initial y_0^e, whether the economy returns to the targeted steady state will itself be a stochastic event. Under AL dynamics, the gain parameter must be specified and in our numerical simulations we set this to $\omega = 0.01$. For long-horizon models, because of the high sensitivity of temporary equilibrium output and inflation to long-run expectations, the gain is typically set somewhat lower.[22] However, in the presence of a large shock to the economy, and with a possible change in policy, a higher gain is warranted to track the evolving data.

Consider first a small negative shock to y^e which is inside the DOA in Figure 3. A shock of 0.2% to steady-state output expectations (or its present value equivalent), with $\pi^e = \pi^*$ unchanged, shifts output expectations to $y_0^e = \lambda y^*$, where $\lambda = 0.998$. In this case, the economy will converge back with very high probability to the targeted steady state. This is as expected since this shock places expectations substantially inside the DOA: the lower boundary of the DOA is approximately $y^e = 0.98792$ when $\pi^e = \pi^*$.

Larger adverse shocks to y_0^e lead to an increasing likelihood of failure to return to the targeted steady state under unchanged policy. The key results are shown in Table 1. For $\lambda = 0.9975$, the probability of convergence to the targeted steady state is 69% and for $\lambda = 0.99745$ this probability is only 15%. Thus, for a range of expectation shocks the dynamics of the economy can depend sensitively on the sequence of exogenous random shocks affecting output and inflation.

The numerical results seen in Table 1 show that failure to converge to the targeted steady state arises even for pessimistic output expectations well inside the theoretical E-stability DOA shown in Figure 3. The discrepancy for initial expectations inside the nonstochastic DOA in Figure 3 arises for several reasons. First, our assumption $T_1 = 20$ has a sizable effect. Additional simulations show that at $\pi^e = \pi^*$ the lower boundary y^e to the numerical stochastic DOA of the targeted steady state falls as T_1 increases. The intuition for this is as follows: With $\pi^e = \pi^*$ and $y^e < y^*$, expected nominal and real interest rates are lower, raising

[22] See, for example, Eusepi and Preston (2011).

demand and output. This stabilizing effect of monetary policy, in the face of pessimistic output expectations, is blunted, however, because we impose that expected real interest rates are expected to return to the steady-state value after a finite number of periods T_1. Qualitative results are not affected by the precise choice of T_1.

There are two other factors that arise from our stochastic setup. In the nonstochastic model generating Figure 3 there are only two parameters, corresponding to the intercepts of the RLS system given in Online Appendix B. In our stochastic setup, there are six parameters in ϕ_y, ϕ_π, as well as additional parameters in the estimated second-moment matrix $\mathcal{R}$. The global ODE approximation to the RLS algorithm thus differs from the E-stability dynamics shown in Figure 3. In addition, with constant-gain learning the mean dynamics corresponding to the ODE are only a good approximation for $\omega > 0$ very close to zero and can differ significantly for values even as small as $\omega = 0.01$. The combination of constant $\omega = 0.01$ and stochastic intrinsic shocks leads to sufficient variation in (y_t, π_t) over time so that expectations are more frequently pushed into unstable trajectories.

The key numerical findings are clearly consistent with our general theoretical results. The targeted steady state is locally stable under LS learning, but it is not globally stable. For sufficiently pessimistic initial output expectation shocks, that is, $0 < \lambda < 1$ sufficiently low, the proportion of trajectories that converge to the targeted steady state is near zero. In our stochastic setup, this arises for $\lambda \leq 0.9974$. The numerical results are shown in Table 1.

These results illustrate that with constant fiscal policy in the stochastic model there are situations where the long-run outcome may be either the targeted steady state or stagnation depending on the realization of the exogenous random shocks A_t and ν_t. Related, stochastic simulations can deliver a cloud of points that reflect features of the data shown in Figure 1, see Subsection 6.6 for details. We also see that, on a formal level, the global E-stability analysis of Subsection 3.2, based on nonstochastic one-parameter PLMs, provides key, though approximate, results concerning convergence of real-time constant-gain RLS learning in the stochastic model.

To understand the magnitudes of the expectation shock given in Table 1, it is helpful to consider a reinterpretation of the role of y^e in the temporary equilibrium model. For the consumption function (3), in the RA case with $\Xi_t^e \equiv 1$, temporary equilibrium output y_t, depends to first order on the present value of $\{y_{t+s}^e\}_{s=1}^{\infty}$ of the sequence of output expectations. We have interpreted steady-state learning as agents acting as if $y_{t+s}^e = y_t^e$ for all horizons $s = 1, 2, 3 \ldots$. However, this is behaviorally equivalent to assuming that agents have an expected output profile with the same present value. Further discussion is at the end of Online Appendix B.

In interpreting these results, it is important to bear in mind that we are employing a benchmark NK model without capital and without additional frictions often employed in serious empirical DSGE models, such as indexation, habit persistence, and adjustment costs for capital. Extensions like these, which introduce inertia into the dynamics, would possibly enlarge the DOA, without, however, altering the qualitative features of our model in which there are three steady states, including a locally stable targeted steady state and a stagnation region.

5. FISCAL POLICY

We turn now to fiscal policy. A growing literature has been reconsidering the effects of fiscal policy in light of the relatively large fiscal stimuli adopted in various countries in the aftermath of the Great Recession. For example, Christiano et al. (2011), Corsetti et al. (2010), and Woodford (2011) demonstrate the effectiveness of fiscal policy in models with monetary policy when the ZLB on the interest rate is reached. For a contrary view, see Mertens and Ravn (2014). Most of this literature makes the RE assumption. The AL literature has shown that quite different results can arise both in NK and Real Business Cycle models; see Evans et al. (2008), Benhabib et al. (2014), Mitra et al. (2013), Gasteiger and Zhang (2014), and Mitra et al. (2019).

We examine fiscal policy under AL using the long-horizon anticipated-utility approach advocated by Preston (2005) and Eusepi and Preston (2010), and extended for policy changes in Evans et al. (2009) and Mitra et al. (2013). We consider an economy in which expectations are pessimistic relative to the targeted steady state and in which the path of the economy adjusts through learning. For concreteness, this is modeled as a negative shock to output expectations y^e (other shocks could be studied). We direct our attention to negative expectation shocks sufficiently large so that without policy change the path of the economy would with high probability fail to return to the targeted steady state and would instead be trapped in the stagnation region.[23]

Because Ricardian households are assumed, we examine the impact of changes in the level of government purchases and focus on temporary increases in the level of government spending on goods and services.[24] When there is a change in fiscal policy, agents take account of the tax effects of the announced path of policy. Given the Ricardian assumption, balanced budget increases in spending can be assumed so that the path of taxes matches the path of government spending.

Evidently, fiscal policy needs to be tuned to the size of the exogenous expectations shock. We consider the case where at $t = 1$ the government announces an increase in government spending for T_p periods, that is,

$$g_t = \Upsilon_t = \begin{cases} \bar{g}', & t = 1, \dots, T_p \\ \bar{g}, & t \geq T_p + 1, \end{cases}$$

where $\bar{g}' > \bar{g}$. Thus government spending and taxes are changed in period $t = 1$ and this change is reversed at a later period $T_p + 1$. We assume that the announcement is fully credible and the policy is implemented as announced. These assumptions could, of course, be relaxed at the cost of added complexity in the analysis.

Using stochastic simulations, we study the evolution of the economy, under AL, after a pessimistic shock and examine the potential role for fiscal policy to prevent stagnation or ameliorate bad outcomes.[25] The focus is whether fiscal policy can alter the dynamic path so that there is instead convergence to the targeted steady state. The impact of fiscal policy may depend critically on the size and length of fiscal policy. In addition, the sequence of random shocks A_t and ν_t have an impact on the success of fiscal policy.

As a first illustration, consider the case $y^e = 0.997 \times y^*$ which, based on Table 1, is big enough shock to result in convergence to the stagnation steady state approximately 100% of the time in our calibrated stochastic model.[26] As a specific illustration we set $T_p = 4$, that is, a one-year fiscal package and a range of government spending increases from $\bar{g} = 0.2$ to 0.4. The simulation is replicated 100 times and with length 500 periods. Table 2 gives the results.[27]

Evidently, temporary increases in g are effective in raising output. Small temporary increases in g may lead only to temporary increases in y, but larger temporary increases in g can shift the economy back to a path converging to the targeted steady state. In the latter situation, policy results in a permanent increase in output relative to the paths that would be followed without the fiscal stimulus.

Another observation is that if probability of convergence to target steady state is between 0 and 1, the sequence of serially correlated random productivity and mark-up shocks can

[23] Analogous simulations could of course be done when the y_0^e shock is smaller and there is eventual convergence to targeted steady state without change in policy. The question of interest would then be whether fiscal policy can speed up the recovery back to the targeted steady state.

[24] In further work, it would be of interest to introduce alternative fiscal frameworks with distortionary taxes and/or public debt.

[25] We emphasize that these simulation results are designed to be illustrative, that is, to exhibit the range of possible results that can be obtained in our model. Using the model to fit actual historical episodes is reserved for future research.

[26] Using the present value interpretation of the expectation shock given at the end of the preceding section, the shock $y^e = 0.997 \times y^*$ corresponds to an expected two-year recession of 3.9% of GDP.

[27] Extended version of Table 2 is given as Table A.1 in Online Appendix G.

TABLE 2
PERCENTAGE OF SIMULATIONS IN WHICH FISCAL POLICY SUCCESSFULLY RESULTS IN CONVERGENCE TO THE TARGETED STEADY STATE STARTING FROM $y_0^e = 0.997 \times y^*$

$T_p \backslash \bar{g}'$	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4
4	0	95	100	100	100	100	100	67	1

TABLE 3
PERCENTAGE OF SIMULATIONS IN WHICH FISCAL POLICY SUCCESSFULLY RESULTS IN CONVERGENCE TO THE TARGETED STEADY STATE STARTING FROM VERY PESSIMISTIC OUTPUT EXPECTATIONS $y_0^e = 0.991 \times y^*$

$T_p \backslash \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.575	0.6	0.625	0.65	0.675	0.7
3	0	0	0	0	0	0	0	0	39	75	80	91
4	0	0	0	0	0	83	89	94	90	83	40	11
5	0	0	0	9	92	65	22	3	1	0	0	0
6	0	0	0	86	29	1	0	0	0	0	0	0

NOTES: Based on 100 replications in each cell.

matter: for a fiscal policy that is usually successful, a particularly unfavorable sequence of shocks can adversely affect expectations enough to prevent the policy from working.

It is also seen that a fiscal stimulus that is too large or too long can be counterproductive. In Table 2, an increase from $g = 0.375$ to 0.4 reduces the effectiveness of the stimulus greatly from 67% to 1%.

The case $y^e = 0.997 \times y^*$ is systematically examined in Table A.1 in Online Appendix G which shows the probability of success for a range of both g and T_p. A stimulus for too long can reduce the effectiveness of fiscal policy. This is a reflection of the negative effect on consumption of the tax burden associated with higher government spending, which is assumed correctly foreseen by households.

We now examine the case of a very large expectations shock $y_0^e = 0.991 \times y^*$, which corresponds to an expected two-year recession of 11.7% of GDP. Following this shock, a temporary fiscal stimulus is applied with government spending increased from $\bar{g} = 0.2$ to $\bar{g}' = 0.3, \ldots, 0.7$ for $T_p = 1, \ldots, 6$ quarters. Table 3 shows the probability (in percentages) of cases where the policy is successful. For $T_p = 1, 2$, the probabilities are zero over this range of $\bar{g}'$. The success probabilities are generally lower than those for the case shown Table 2 and Table A.1 of Online Appendix G. Also values of $\bar{g}'$ need to be significantly larger than those in Table A.1 in order to be successful. However, there are still policies with a high degree of success: the highest success rate shown in Table 3 is 94% .

We find from these results that a sufficiently large stimulus of appropriate duration can have a high probability of extracting the economy back to convergence to target even if pessimistic output expectations are deep inside the stagnation region of the stochastic model. However, it should be emphasized that a higher probability of avoiding the stagnation regime can be achieved, with a much smaller stimulus, if the policy is implemented when expectations are less pessimistic. This suggests that following a large adverse shock to expectations, in which there is major risk of the economy descending into the stagnation regime, a fiscal stimulus should be implemented as early as possible.[28] This is discussed in Ssubection 6.2.

An interesting observation in Table 3 and also other tables is that the cases with relatively high success probability lie in a "corridor" taking the form of a "thick diagonal" from Southwest to Northeast. There is a negative trade-off between magnitude and length of stimulus.

The detailed quantitative results also depend on ξ, the degree of substitutability between private and public consumption. It can be seen from Equation (5) that the impact output multiplier $\partial y_t / \partial dg_t = 1 - \xi > 0$ depends negatively on ξ. This is consistent with Ercolani and

[28] This accords with testimony by Lawrence Summers to the Joint Economic Committee hearing on January 16, 2008, that fiscal "stimulus program should be timely, targeted and temporary."

Azevedo (2019). We nonetheless obtain huge output multipliers if an appropriately aggressive fiscal stimulus is used when expectations are pessimistic. This arises because the increases in output and inflation resulting from the fiscal stimulus lead over time, through AL, to upward revisions in expectations sufficient to eventually return the economy to the targeted steady state.

A general implication of our fiscal policy results, which is evident but worth emphasizing, is that the size and impulse response profile of the government spending multiplier depends sensitively on both the current state of expectations, when the policy is initiated, and nonlinearly on the size and duration of the spending increase.

6. EXTENSIONS

We consider several extensions about designing policies to avoid stagnation and discuss features of simulated data of the model.

6.1. *Including Forward Guidance in Monetary Policy.* In the preceding section, it was seen that fiscal policy is not always successful, in the sense of guaranteeing that the economy escapes the stagnation regime, and the probability of success becomes lower with more pessimism. It therefore makes sense to ask whether supplementary unconventional monetary policy can help.

The current framework is well suited to analyze forward guidance, which of course has been one form of unconventional monetary policy that central banks used during and following the Great Recession.[29] We model this as a commitment by the central bank to keep the policy interest rate at the ZLB for the first T_m periods after the expectation shock occurs. With forward guidance the interest-rate rule (8) becomes

$$R_t = \begin{cases} 1, & t = 1, \dots, T_m, \\ R(\pi^e_{t+1}, y^e_{t+1}), & t \geq T_m + 1. \end{cases}$$

We consider output expectation shocks even more pessimistic than used in Table 3. We first set $y^e_0 = 0.985 \times y^*$, which corresponds to an expected two-year recession of 19.5% of GDP in terms of the computations mentioned at the end of Section 4 and discussed at the end of Online Appendix B. Without a change in policy, the economy always, in our simulations, converges toward the stagnation steady state. We explore the effectiveness of various settings of temporary fiscal stimulus $\bar{g}_1$, T_p combined with forward guidance T_m.

If only forward-guidance monetary policy is used, without including fiscal stimulus, the probability of convergence to the targeted steady state is zero for $T_m \leq 10$ or $T_m \geq 15$, and is positive only for $T_m = 11$ (43%), $T_m = 12$ (25%), $T_m = 13$ (11%), and $T_m = 14$ (1%). If instead only fiscal stimulus is employed, the probability of convergence to the targeted steady state is close to less than 10% except for some specific policy settings, and is above 50% in just a few cases: $\bar{g}_1 = 0.75$ with $T_p = 5$ (60%), $\bar{g}_1 = 0.8$ with $T_p = 5$ (53%), and $\bar{g}_1 = 0.9$ with $T_p = 4$ (55%).

Better outcomes can be achieved by combining fiscal policy and forward guidance. Table 4 illustrates the results from detailed analysis of the case with $y^e_0 = 0.985 \times y^*$ in which forward guidance setting $T_m = 6$ is combined with different fiscal stimulus settings. The highest probability of success (convergence to the targeted steady state) is 73% with $\bar{g}_1 = 0.55$ with $T_p = 3$. Similar results are obtained for nearby values of T_m (further results are in the tables in Online Appendix H). Thus, for the case of severely depressed output expectations, there is a significant increase in the probability of escape from stagnation when both policies are actively employed.

[29] Without extensions, the model is not suited to analyzing other forms of unconventional policies, such as large-scale asset purchases.

TABLE 4
PERCENTAGE OF CONVERGENCE TO TARGET FOR DIFFERENT SPECIFICATIONS OF COMBINED POLICY BASED ON 100 REPLICATIONS FOR $y_0^e = 0.985 \times y^*$, $T_m = 6$

$T_p \backslash \bar{g}_1$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
2	0	0	0	0	0	37	64	60	69	67	41
3	0	0	0	45	62	73	36	12	3	1	0
4	0	0	60	69	37	6	0	1	0	0	0
5	0	32	65	30	4	0	0	0	0	0	0
6	0	64	48	4	0	0	0	0	0	0	0

TABLE 5
PERCENTAGE OF CONVERGENCE TO TARGET FOR DIFFERENT SPECIFICATIONS OF COMBINED POLICY FOR $y_0^e = 0.98$, $T_m = 7$, AND 100 REPLICATIONS

$T_p \backslash \bar{g}_1$	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
4	0	0	0	0	21	40	29	14
5	0	0	0	45	25	6	0	0
6	0	0	41	15	0	0	0	0
7	0	37	11	1	0	0	0	0

For even more pessimistic expectations, it can happen that both fiscal policy alone and forward guidance alone are ineffective in moving the economy to the targeted steady state, whereas combined policy can still achieve some success. As an example, consider initial output expectations $y_0^e = 0.98 \times y^*$. This corresponds to an expected two-year recession of 25.9% of GDP. In this case, forward guidance alone is totally ineffective (for $T_m = 1, \ldots, 20$). Using fiscal policy alone is also largely ineffective: in the range $\bar{g}_1 = 0.25, \ldots, 0.90$ there are only a few cases with positive probability for convergence to target, and the highest probability is 28% for $\bar{g}_1 = 0.85$ with $T_p = 6$. However, combined policy improves the chances of converging to the targeted steady state. In Table 5, the highest probability of convergence is 45% when $\bar{g}_1 = 0.6$, $T_p = 5$, and $T_m = 7$. (More results are given in Online Appendices I and J.)

The results of this section show that, for very pessimistic output expectations, adding forward guidance to fiscal policy can substantially improve the chances of converging to the targeted steady state, at least for the wide range of fiscal policies we considered. A different approach might be to use an even larger fiscal stimulus for which there is some improvement but the results are not very encouraging and would require implausibly large increases in $\bar{g}$.

Our framework also has implications that contrast with the literature. Under RE forward guidance of future low interest rates is very effective—so effective that these implications have been called the "forward guidance puzzle." A large literature has shown that for recessions RE overstates the extent to which forward guidance—of near zero interest rates for an extended period—will stimulate GDP, relative to what is found under a range of bounded rationality assumptions. See, for example, Cole (2021), Garcia-Schmidt and Woodford (2019), and Eusepi et al. (2021). These papers have focused on linearized NK models. In our nonlinear framework we find that, following a large negative expectations shock, forward guidance can be unable to return the economy to the targeted steady state unless it is complemented by a fiscal stimulus.

6.2. *Delays in Policy.* In Section 5, we suggested that in the face of a large pessimistic expectations shock it may be important to implement a fiscal stimulus quickly. We here briefly illustrate the effect of policy delays. For a given output expectations shock y_0^e, we consider the effect on the probability of success of a delay by T_s periods. We restrict attention to the case, examined earlier in Table 3, of a large pessimistic initial output expectations shock $y_0^e = 0.991 \times y^*$, and we now assume policy is executed with a delay of four periods (one year). Table 6 reports the relevant part of the table, that is, ranges $T_p = 3, \ldots, 6$ and $\bar{g}' = 0.45, \ldots 0.7$

TABLE 6

PERCENTAGE OF SIMULATIONS IN WHICH FISCAL POLICY $\bar{g}'$ FOR T_p PERIODS AND DELAY OF $T_s = 4$ PERIODS RESULTS IN CONVERGENCE TO TARGET FROM PESSIMISTIC OUTPUT EXPECTATIONS $y_0^e = 0.991 \times y^*$

$T_p\backslash\bar{g}'$	0.45	0.5	0.55	0.575	0.6	0.625	0.65	0.675	0.7
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	4	48	53	24	15
5	0	0	51	32	12	6	0	1	0
6	0	39	7	1	1	0	0	0	0

NOTES: Based on 100 replications with length 500.

based on 100 replications.[30]

It is seen that the percentage of success with delay is generally lower than the corresponding percentage when there is no delay. Most noticeably, if we compare no-delay fiscal policies, with the highest chance of success, with four-period delayed policies with the highest probability of success, the probability of success falls from 94% to 53%. The reason for this is that during the period of delay output expectations deteriorate further and inflation expectations also begin to decline.

6.3. *Credit Frictions and Calibration of the Discount Factor.* In Subsection 3.1, we noted that at the low steady state (π_L, y_L) the (gross) policy interest rate is approximately equal to one whereas the (gross) inflation rate is approximately equal to β. From Figures 2 and 3, it is evident that π_L plays a key role in the expectation dynamics since the unstable steady state (π_L, y_L) is on the edge of the DOA of the targeted steady state and for $\pi^e < \pi_L$ and $y^e < y_L \approx y^*$ the economy lies within the stagnation trap. The appropriate calibration of the discount factor β is thus worth discussing. Our numerical results have used the quarterly calibration of $\beta = 0.99$, that is, a quarterly deflation rate at π_L of 1%.

While $\beta = 0.99$ is fairly standard, there are good reasons to consider alternative, higher, values. The historical average realized net real interest rate on U.S. Treasuries bills is not more than 1% per annum. In an economy without growth, this corresponds to a discount factor of about $\beta = 0.9975$.[31] The critical inflation rate at the edge of the stagnation trap at $y^e < y_L$ is then an annual deflation rate of 1%.

A second factor that can lead to a higher level of the critical inflation rate is the existence of credit frictions. Various models have been proposed that generate a spread between different interest rates on loans. A prominent example within an NK setting is described in Curdia and Woodford (2010) and developed at length in Curdia and Woodford (2015). Their framework posits a heterogeneous agents setup with two types of households, at any given time, experiencing different realizations of taste shocks. This leads to lending from agents who are currently more patient to those who are currently more impatient. Frictions in the financial intermediation sector result in a borrowing rate above the lending rate.

Embedding a heterogeneous agents framework into our model is beyond the scope of this article. However, it is natural to incorporate a shortcut, motivated by Woodford (2011), which is to assume that the market interest rate relevant in household Euler equations for the "intertemporal allocation of expenditure is not the same as the central bank's policy rate" (Woodford, p. 16). Woodford (2011) and Curdia and Woodford (2015) focus on the implications of the time variation in this spread, while for our purposes the key implication is a positive steady-state spread $\varphi = R - i > 0$, where i is the policy rate and R is the interest rate relevant for household decision making. The benchmark calibration in Curdia and Woodford (2015) corresponds to a value $\varphi = 0.0025$, that is, to 1% per annum.

[30] The policy thus starts in period 5 and ends in period $5 + T_p$.

[31] We note Eggertsson (2010) uses a calibration of $\beta = 0.997$ in a model of the U.S. economy during the Great Depression. In its trough deflation reached 10% per year.

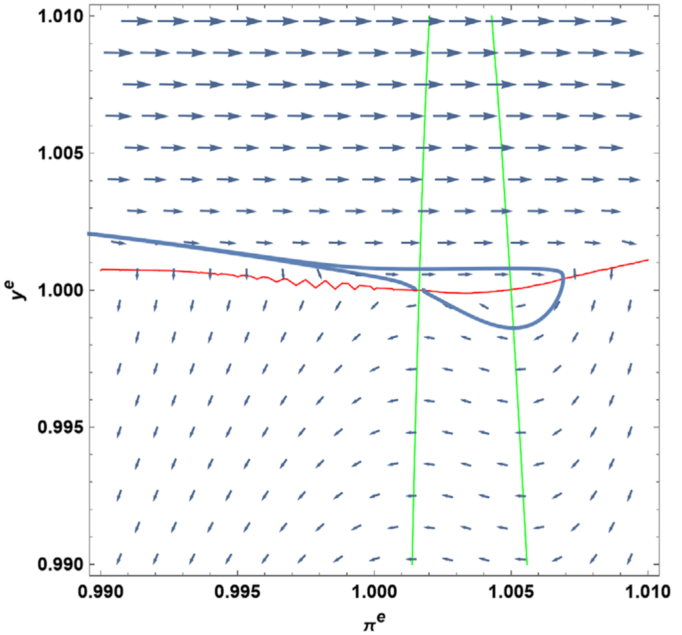

FIGURE 4

DOMAIN OF ATTRACTION WITH CREDIT SPREAD AND $\pi^* = 1.005$
NOTES: $\beta = 0.9975$, $\varphi = 0.0025$, $T_1 = 100$

With credit frictions, we assume a spread $\varphi > 0$ between the market rate R_t and the policy rate $R_t - \varphi$. Since the policy rate obeys the ZLB for net interest rates, the market interest-rate factor relevant for the consumption Euler equation satisfies $R_t \geq 1 + \varphi$. The interest-rate rule (8), with inflation target π^*, is then replaced by

$$R_t = 1 + \varphi + (R^* - (1+\varphi))\left(\pi^e_{t+1}/\pi^*\right)^{BR^*/(R^*-(1+\varphi))}\left(y^e_{t+1}/y^*\right)^{\phi_y}. \tag{14}$$

The positive spread φ increases the low steady-state inflation rate to $\pi_L \approx \beta(1+\varphi)$. This has a number of implications, one of which is particularly relevant for policy: if $\beta(1+\varphi) > 1$ then it is possible to have $1 < \pi_L < \pi^*$, so that the critical inflation rate at (π_L, y_L) is a zero or low positive inflation rate, instead of a deflation rate.

The central consequences of a credit spread can be seen by comparing the domains of attraction of π^* with and without a credit spread. Figure 4 illustrates the DOA of the targeted steady state for the model with a high subjective discount rate and a positive credit spread. The DOA is now significantly smaller than that in the basic model.[32] At $\pi^e = \pi^*$, the value of y^e at the low boundary of the DOA is approximately $y^e = 0.9986$, much higher than the corresponding value in Figure 3. Similarly at $y^e = y^*$, the value of π^e at the low boundary of the DOA is π_L and now corresponds to positive net inflation. Thus, the impact of a higher discount factor and a positive credit spread is to reduce the size of the DOA of (y^*, π^*), making the targeted steady state less robustly stable.

Thus, the qualitative aspects of dynamics shown in Figures 2 and 3 remain unchanged. However, taking into account credit frictions, expected inflation rates significantly below the central bank target, even if positive, increase the possibility of a path toward stagnation and

[32] The truncation of expected real interest rates to a finite horizon in consumer optimization is employed because a wide state space is needed for the analysis. Here, we set T_1 to a fairly high value, $T_1 = 100$, in order to reduce its numerical impact. See Section 4 for discussion of T_1. Using finite T_1 reduces somewhat the DOA.

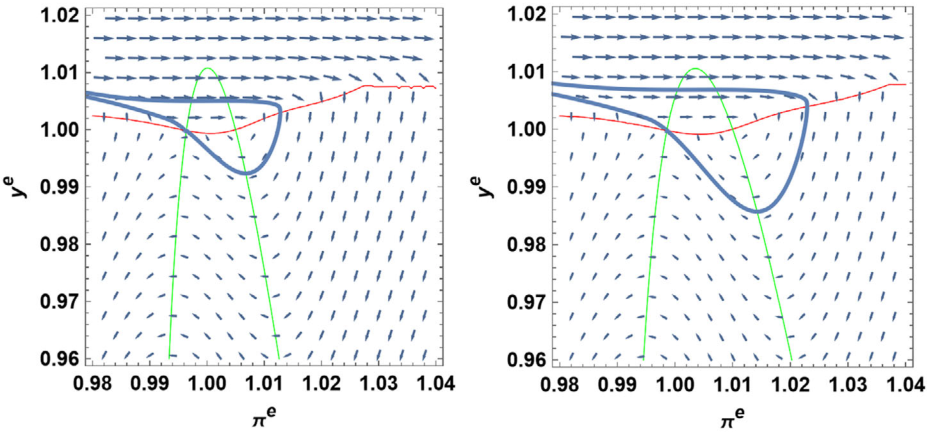

FIGURE 5

DOMAIN OF ATTRACTION FOR DIFFERENT INFLATION TARGETS $\pi^* = 1.005$ (LEFT PANEL) AND 1.01 (RIGHT PANEL) IN QUARTERLY VALUES

the possible need for aggressive policy. We next explore the implications of a higher inflation target which may well be a way to increase the robustness of standard monetary policy.

6.4. *Higher Inflation Target.* Adopting a higher inflation target became a popular though controversial subject in the policy discussion during the Great Recession. See, for example, the influential paper Blanchard et al. (2010). The implications of a higher inflation target in our setup can be examined most readily using the nonstochastic model of Sections 2 and 3. Figure 5 compares the results for an inflation target of $\pi^* = 1.005$, that is, 2% annually (left panel), versus $\pi^* = 1.01$, that is, 4% annually (right panel). The policy with higher inflation target appears clearly effective in the sense that the DOA of the targeted steady state is substantially larger with the higher target. It is possible to compute numerically the area of the DOA—see Online Appendix K for details. Comparing $\pi^* = 1.005$ to 1.01, the DOA increases approximately 2.4-fold.[33] Our finding that the DOA increases with the magnitude of the inflation target holds for alternative model specifications (i) in which the Linex cost of price adjustment $\Phi(P_{t,i}/P_{t-1,i})$ in utility function (1) is centered on 1 instead of π^*, and (ii) the model with credit spread.

These results may appear surprising in light of other results in the literature. Ascari et al. (2017) consider a linearized NK model with Calvo pricing frictions and allowing for trend inflation. They find that with a higher inflation target the set of interest-rate policy parameters giving E-stability is smaller when the inflation target is higher. However, this is a conceptually different exercise from the one examined here. Figure 5 considers an interest-rate rule that is unchanged except for having a higher inflation target and considers the size of the stability region with respect to perturbations in expectations (π^e, y^e) away from the targeted steady state. This important question can only be addressed in a nonlinear setup.

Using a linearized model, Branch and Evans (2017) consider AL rules that allow for autoregressive or VAR dynamics in forecasting inflation and output, and find that an unlucky series of shocks can lead to unstable "escape paths." They emphasize in particular that an *increase* in the inflation target must be done carefully to avoid deanchoring of inflation expectations under AL.

[33] Figure 5 is computed using truncation parameter $T_1 = 100$. An analogous result holds for the untruncated model ($T_1 = \infty$), provided we restrict π^* to values for which convergence issues associated with negative (net) real interest rate do not arise (for the current calibration, this constraint is π^* not greater 3.5% annually).

Although we use a Rotemberg instead of a Calvo pricing friction, we think that the intuition for our finding arises from our nonlinear setup in which the unintended steady state π_L, which does not vary with π^*, is positioned on the edge of the DOA for the targeted steady state. An increase in π^* leads to a greater separation of π^* from π_L and thence to a larger DOA.

6.5. *Blended Expectations.* Inflation targeting has been practiced by a substantial number of Central Banks since it was formally adopted by New Zealand and Canada in 1990 and 1991. In our numerical calibrations, we have used a target of 2%, which, for example, was formally adopted by the Bank of England in 2003. The target of 2% in the United States was formally announced by the Federal Reserve in January 2012, bringing it in line with a number of other countries, but this was preceded by a period in which 2% was believed to be the Fed's informal target. One of the main reasons given for having an explicit inflation target is that this can anchor expectations, so that expected inflation is less sensitive to observed inflation rates or exogenous shocks.

It is certainly possible that having an explicit inflation target helps anchor expectations, for example, see Gurkaynak et al. (2010). Against this, Branch and Evans (2017) have argued that policymakers should take into account that expectations can become deanchored by observed economic data. In this section, we take a balanced approach to this issue by considering "blended expectations," in which inflation expectations are a weighted average of the forecasts arising from our AL rules and the inflation target set by the central bank. Thus, we now set

$$\pi_t^e = \varpi \tilde{\pi}_t^e + (1 - \varpi)\pi^*, \quad \text{for } 0 < \varpi < 1, \tag{15}$$

where ϖ is the weight placed on the AL forecast $\tilde{\pi}_t^e$ and $1 - \varpi$ is the weight on the central bank inflation target.

We now look at global E-stability dynamics with blended expectations in comparison with the benchmark case given in Subsection 3.2. Temporary equilibrium, given expectations (π_t^e, y_t^e), continues to be given by $\pi_t = G_1(y_t, y_t^e)$ and $y_t = G_2(\pi_t^e, y_t^e)$, and the E-stability differential equations are now $dy^e/d\tau = F_y(\pi^e, y^e)$ and $d\tilde{\pi}^e/d\tau = F_\pi(\pi^e, y^e)$. From (15), we have $d\tilde{\pi}^e/d\tau = \varpi^{-1}(d\pi^e/d\tau)$, so that in terms of blended expectations the E-stability equations are

$$dy^e/d\tau = F_y(\pi^e, y^e) \quad \text{and} \quad d\pi^e/d\tau = \varpi F_\pi(\pi^e, y^e).$$

These considerations imply that the earlier analysis is unchanged if the relevant state space is thought to be in terms of blended π^e where the ODE for π^e is the usual ODE for inflation expectations with the right-hand side multiplied by the weight ϖ. (Note that the state space is the usual one when $\varpi = 1$.) Changes in ϖ correspond to changes in the adjustment speed of inflation expectations π^e, so that smaller ϖ means lower value for derivative and slower adjustment. The steady states and their E-stability properties are clearly unchanged, so we have the result:[34]

PROPOSITION 2. *(i) The targeted steady state is E-stable provided ϕ_y is not too large. (ii) The steady state (π_L, y_L) is not E-stable provided ϕ_y is not too large. (iii) The steady state (π_S, y_S) is E-stable.*

Looking at the global picture, the qualitative dynamics for different $\varpi \in (0, 1)$ are unchanged relative to those in Figure 3, which corresponds to $\varpi = 1$. There is, however, a major quantitative change: the DOA becomes larger when the weight $1 - \varpi$ on the fixed

[34] The proof is a straightforward modification of the proof of Proposition 1.

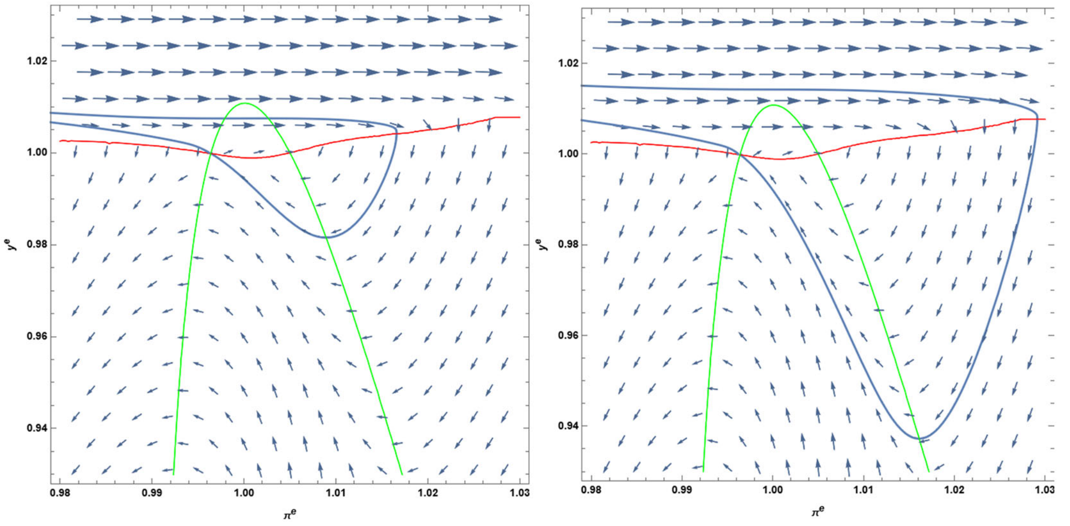

FIGURE 6

THE LEFT PANEL SHOWS THE DOMAIN OF ATTRACTION OF THE TARGET STEADY STATE WITH $\varpi = 0.8$ AND THE RIGHT PANEL SHOWS THE DOMAIN OF ATTRACTION OF THE TARGET STEADY STATE WITH $\varpi = 0.5$

central bank forecast π^* is larger. See the two panels in Figure 6 which should be compared to Figure 3.

One way to see the quantitative significance of the value of ϖ is to consider the value of y^e which is on the lower boundary of the DOA when $\pi^e = \pi^* = 1.005$. In the left panel of Figure 6 with $\varpi = 0.8$, the corresponding value is $y^e \approx 0.985$, whereas in the right panel of Figure 6, with $\varpi = 0.5$, this value falls to $y^e \approx 0.975$. The enlargement of the DOA is also very visible for high values $\pi^e >> \pi^*$. In terms of areas the DOA with $\varpi = 0.5$ is about 2.9 times the magnitude in the case $\varpi = 0.8$. These results suggest that with more anchored inflation expectations fiscal stimulus is needed over a smaller range of pessimistic inflation expectations.

This result has a natural interpretation. $1 - \varpi$ can be viewed as a measure of the Central Bank's credibility in being able to deliver inflation rates in line with its announced target. For $1 - \varpi$ large, π^e will stay near π^* even if econometric forecasts based on recent past data give, say, a much lower forecast. This credibility increases the robustness of the targeted steady state by increasing its DOA under AL.

Again, the qualitative aspects of dynamics shown in Figure 3 remain unchanged in the two panels of Figure 6—the possibility of a stagnation trap remains for y^e, π^e sufficiently pessimistic. A natural extension of the blended expectations approach is reinforcement learning, in which the weight ϖ is made time-varying with ϖ_t evolving based on the relative accuracy of the two forecast rules.[35] Reinforcement learning would limit the degree to which credibility could be maintained if inflation were persistently different from the target. Nonetheless, it is clear that a credible inflation target makes the targeted steady state more robust to expectation shocks under AL.

6.6. *Illustrative Scatterplots.* In Section 1, we noted that in post-2000 data, in addition to a constellation of points centered on the steady state targeted by monetary policy, and another bunching of points with low inflation and interest rates, there has also been an association between very low interest rates and negative output gaps. The combination in our model of a

[35] For an application of this approach, see Honkapohja and Mitra (2020).

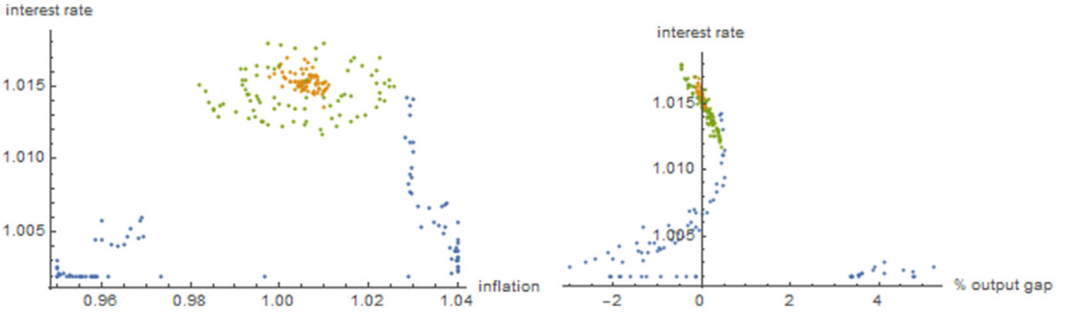

FIGURE 7

SCATTERPLOTS OF DATA FROM THREE SIMULATIONS OF 80 PERIODS, SHOWING INFLATION VERSUS INTEREST RATE (LEFT) AND % OUTPUT GAP VERSUS INTEREST RATE (RIGHT)

locally stable targeted steady state and a stagnation regime has the potential to explain these features of the data, which we illustrate using simulations of a modified version of our model.

As emphasized at the end of Subsection 4.1, to obtain a tractable global stochastic nonlinear setup with AL dynamics, our model has focused on the standard basic NK setup without capital and without other frictions usually introduced in empirical models. Our approach has the advantage of showing that the central qualitative dynamics we identify arise from a standard basic setup; however, an implication is that output and inflation respond immediately and strongly to changes in expectations. To more realistically correspond to historical data, we moderate this sensitivity to expectations by altering the tails of the pricing friction to reduce the range of inflation.[36]

Figure 7 shows scatterplot results combining simulations of 80 periods each for three different starting points for expectations. Two of the simulations start with (y^e, π^e) near (y^*, π^*): in one initial y^e is somewhat below y^* and in the other initial π^e is somewhat below π^*. Both of these initial (y^e, π^e) are within the DOA, and their data clouds, generated under unchanged policy, are centered on the targeted steady state. A third simulation starts near (y_L, π_L), just outside the DOA. For this simulation, expectations gradually become more pessimistic, and both output and inflation decline over time, leading to negative output gaps reaching 3%. After an initial delay, monetary policy reduces the net interest rate to an effective lower bound of 0.8% per year and, with forward guidance, holds it there for 14 quarters. With an additional short delay, there is a large fiscal stimulus for 18 quarters that overlaps with the forward guidance. These combined measures increase output and inflation substantially, eventually returning the economy to the targeted steady state.

Figure 7 exhibits two qualitative features emphasized in the introduction—clouds of points surrounding the targeted steady state and a region of points with low interest rates and negative output gaps, consistent with a stagnation trap that is eventually overcome by active macroeconomic policy. [37]

7. DISCUSSION OF RELATED LITERATURE

Within the context of standard NK models and RE, the implications of the ZLB have been considered from several angles. One natural approach is to examine exogenous shocks to

[36] Recall interest and inflation rates are measured as quarterly factors. The modified pricing friction does not affect (π^*, y^*) and (π_L, y_L), or local dynamics under learning, and easily accommodates the range of the U.S. inflation over the last 100 years. See Online Appendix F for simulation details.

[37] The points with low interest rates and high outputs arise from expansionary policies used to push the economy out of recession and toward the targeted steady state. A more graduated fiscal stimulus, and additional frictions, would smooth this trajectory.

demand that push the economy to the ZLB. Exogenous discount rate or, more plausibly, credit-spread shocks have been emphasized by Eggertsson and Woodford (2003), Christiano et al. (2011), Corsetti et al. (2010), and Woodford (2011). These shocks are often assumed to follow a two-state Markov process in which the credit-spread shock disappears each period with a fixed probability, with aggregate output and inflation recovering as soon as the exogenous shock stops operating.

Although this approach has been fruitful in suggesting suitable policy responses to such shocks, it has several somewhat unattractive features. It relies heavily on the persistence of a shock that evaporates according to an exogenous process, with recession ending as soon as the exogenous negative shocks cease. Furthermore, this approach does not do justice to an independent role for pessimistic expectations.

Another approach, emphasized by Benhabib et al. (2001), focuses squarely on the existence of multiple REE. Under an interest-rate rule that follows the Taylor principle at the targeted steady and which is subject to the ZLB, there is a second, unintended, and indeterminate, steady state at a low inflation, or modest deflation, rate. However, while in this steady state the policy interest rate is at or near the ZLB, the level of aggregate output is only very slightly below that of the targeted steady state. In addition, as we have emphasized and was also stressed in Evans et al. (2008) and Benhabib et al. (2014), this unintended low-inflation steady state is not stable under AL.

In the context of this approach, Benhabib et al. (2002) and chapter 4, pp. 316–17, of Woodford (2003) suggest a strong fiscal expansion that violates the transversality condition of RE equilibrium so that the indeterminate steady state ceases to exist. The economy then coordinates on the remaining nonexplosive RE equilibrium.[38]

A related approach relies on sunspot equilibria based on the targeted and unintended steady states. This can either be a stationary two-state sunspot equilibrium, as in Aruoba et al. (2018) or a two-state sunspot equilibrium with an absorbing state at the targeted steady state, as in Mertens and Ravn (2014). This approach does give full weight to self-fulfilling expectations; furthermore the state corresponding to deflation has lower output due not to a fundamental shock, but to a pure confidence shock. However, in addition to the practical question of exactly which sunspot variable coordinates expectations, there is also the issue of stability under learning.[39] Furthermore, the associated recessions have relatively small magnitude: in the illustrations given in Mertens and Ravn (2014) the impact on output in the sunspot state is -1.6%.

Regime shifts arising from policy shocks that follow a two-state Markov chain also generate REE Markov chains, and these models have some affinity with models of sunspot equilibria. Bianchi and Melosi (2017, 2019) introduce regime switching of monetary-led and fiscally led policy mixes into an NK model under RE.[40] These papers show how some policy regimes can contribute to long-run stability by mitigating occasional recessions subject to the ZLB, and how conflicts between monetary-led and fiscally led policy coordination may lead to adverse outcomes that can be resolved by appropriate sequences of policy regimes.

There is also a substantial related literature focusing on “sentiments” or “confidence.” The term “sentiment” has been used in various ways in both RE and AL setups. Under RE, it can be viewed as similar to an exogenous sunspot variable on which agents coordinate. Angeletos and La’O (2013) shows that if communication frictions between traders are introduced into a standard unique equilibrium model, sunspot-like extrinsic shifts in expectations can be

[38] This approach is potentially problematic as it relies on complete trust of RE asymptotics.

[39] Two-state sunspot equilibria are not locally stable under learning when they are near two steady states, one of which is not locally stable under learning as in this case; for example, see Evans and Honkapohja (2001, Chapter 12).

[40] The regimes correspond to active money/passive fiscal and passive money/active fiscal policy combination in Leeper (1991).

self-fulfilling.[41] For a recent example in a heterogeneous-agents New Keynesian (HANK) model, with labor frictions and two steady states, see Lagerborg et al. (2021).[42]

In the AL literature, sentiment has been viewed as an extension in which serially correlated expectation shocks are added to AL forecasts to capture exogenous waves of optimism or pessimism. With this approach, using an estimated NK model with AL that includes survey data on expectations, Milani (2011) argues that sentiment driven by psychological factors, can explain a significant portion of business cycle fluctuations.[43] Milani (2017) argues that sentiments are particularly important in explaining fluctuations in business investment, and Cole and Milani (2021) provide an extension to heterogeneous expectations. Also closely related to sentiments are the "exuberance" equilibria analyzed by Bullard et al. (2008, 2009).

In contrast to these approaches, this article does not require extrinsic variables to drive expectations. Within a familiar nonlinear global NK setup, expectational dynamics are driven by recent observations of inflation and output. The existence of multiple steady states, with two distinct regions of expectational dynamics, implies a major role for macroeconomic policy in preventing the economy becoming trapped in a stagnation regime.

Schmitt-Grohe and Uribe (2017) and Eggertsson et al. (2019) develop models of secular stagnation with nominal wage stickiness. The Schmitt-Grohe and Uribe (2017) model has representative agents and downward wage rigidity taking the form of a lower bound on nominal wage growth, where the lower bound is negatively related to the unemployment rate. This setup yields two perfect-foresight steady states: the targeted steady state and a steady state with involuntary unemployment and binding downward wage rigidity. From our perspective, a major concern is that their recommended Fisherian monetary policy is premised on RE and is subject to the AL critique discussed in Howitt (1992) and Evans and McGough (2018).

Eggertsson et al. (2019) develop a perfect-foresight/RE analysis with overlapping generations, downward sticky nominal wages and collateral constraints. The basic model assumes households with three-period lifetimes.[44] The model can have a locally determinate stagnation steady state with zero inflation target and negative real interest rate. The latter in turn requires a sufficiently tight collateral constraint that is well below aggregate output at full employment.

In contrast to this literature, our framework uses a benchmark representative agent NK model which is the basis for more elaborate macroeconomic models most frequently used for monetary and fiscal policy analysis. The nonlinear model, relying on a Rotemberg pricing friction, is well known to have two steady states. We have established that when government spending is a partial substitute for private consumption, there is a third stagnation steady state, which is locally stable (under AL), in addition to the targeted and unintended steady states.[45]

8. CONCLUSIONS

Sluggish real economic performance and a long-lasting ZLB has made the possibility of secular stagnation a prominent topic of economic discussion. Japan has mostly been in this situation for over 20 years and the western economies the United States, Euro area, and the

[41] In an RE setup, Benhabib et al. (2015), relying on signal extraction problems, show that sentiment can lead to solutions far away from the usual RE solution.

[42] In the context of an endogenous growth model with multiple steady states, Evans et al. (1998) show that stochastic growth cycles are stable under AL.

[43] Evans and Honkapohja (2003) considered optimal monetary policy under AL when expectations were affected by additional optimistic or pessimistic shocks.

[44] There is also a quantitative 56-period version of the model. Gibbs (2018) shows E-stability of the stagnation equilibrium in the model where agents live for three periods.

[45] In Evans et al. (2008) and Benhabib et al. (2014), AL was introduced into the NK model with two steady states and deflation region arising from the ZLB. However, the destination of the possible deflationary paths was not resolved.

United Kingdom for much of the post-2007 period. Our first objective in this article was to extend a standard NK model in a way that exhibits stagnation as a well-defined regime for the economy, which is present despite the existence of a locally stable regime that includes the steady state targeted by policymakers.

Our model abstracts from exogenous technological progress, as well as population growth, so the stagnation region should be viewed as a "trap" in which economic activity, under normal policy, would tend to remain and fall further below potential output, with declining inflation and inflation expectations. The stagnation region contains a well-defined steady state—a theoretical lower bound on economic activity accompanied by rapid deflation—which acts as an attractor within the stagnation region in the absence of a strong policy response.

A second objective of our article has been to consider the potential for fiscal policy to avoid or extract the economy from the stagnation region in a setting with boundedly rational, AL agents. In this setting a fiscal stimulus, an appropriately chosen increase in government spending, can push the economy out of the stagnation trap. Simulations indicate that effectiveness of a fiscal stimulus (and thus the probability for escaping stagnation) depends not only on the size and length of the policy but also on the realized random sequence of exogenous random productivity and mark-up shocks. Important extensions and alternative policies were also discussed.

The results obtained in the article are all based on the basic standard NK model and simple extensions. Particular crises, such as that due to the ongoing COVID-19 shock, can require substantial extensions of the model to incorporate specific key aspects of the crisis. However, the central features of our model will continue to be relevant. After the exogenous shocks have dissipated, there can be an expectational overhang due to economic experience during the crisis. If output and inflation expectations lie outside the DOA of the targeted steady state, then extraordinary macroeconomic policies may be required.

Supporting Information

Additional online supporting information may be found in the Online Appendix in the Supporting Information section at the end of the article.

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