MATH 243, LECTURE 18

0.1. **Relating hypothesis testing to confidence intervals.** The basic calculations for hypothesis testing and confidence intervals are the same: take a simple random sample, compute its mean, and then calculate its z-statistics using the Central Limit Theorem.

In fact, these two procedures are logically related (seeing how helps us better understand both concepts).

Fact 1. *If some guess for a mean μ_0 is not within a $1-\alpha$ confidence interval about an observed mean $\bar{x}$ then we may reject the null hypothesis $\mu = \mu_0$ at significance level α.*

It helps to sketch a picture of what this is saying and to choose some concrete numbers.

Example 2. *Suppose a variable has a standard deviation of* 5 *and a measured mean of* 172.1 *from a sample size of* 80.

- *Find a confidence interval with $C = 95\%$ for this variable.*
- *Show that the null-hypothesis of a mean equal to* 175 *can be rejected at level* 5%.

1. Caution: garbage in, garbage out

The most common mistakes one sees in applications of statistics are not in the mathematics of means and deviations – with practice one can master these methods, and there are sophisticated computer packages to help out – but in starting with biased data, whose error is comparable to the confidence intervals in question.

Example 3. *What if in our UO height example, two of the sample taken were members of the basketball team over 80 inches tall? If these outliers are thrown out, what is our new confidence level for the hypothesis that UO men are above average height? Recall heights obey $N(69.3, 2.8)$. We took a SRS of 32 undergrad men from the UO and found $\overline{x} = 70.4$.*

One of the topics of Chapter 15 is to elaborate on problems which can plague samples and surveys, in light of their effects on confidence intervals and hypothesis tests. This material mostly applies common sense, not mathematics, as we did when we looked at designing surveys and experiemens.

We will not at all cover the material from Chapter 15 on type I and II errors and power of tests. This development of language is straightforward enough so that you could learn it on your own if you encountered these terms outside of class.

2. Estimating the mean *without* knowing σ

In §13 the method we learned for estimating our population mean μ had the serious drawback that we had to know the standard deviation for our population. We now wish to approximate μ *without* knowing the standard deviation.

Basic Idea: Take a SRS. Calculate the standard deviation of our sample, s. This is called the *standard error* to distinguish it from the unknown standard deviation of our population.

Calculate the mean of our sample $\overline{x}$. Use s to estimate σ and then techniques we've already learned to estimate μ from $\overline{x}$.

TERMINOLOGY: We use

$$\mu = \text{for our (unknown) population mean,}$$

and

$$\overline{x} = \text{for our sample mean.}$$

We now also need to distinguish between two standard deviations.

$$\sigma = \text{for our (unknown) populations standard deviation}$$

and

$$s = \text{for our sample standard deviation.}$$

Recall that if our population has distribution $N(\mu, \sigma)$ and we look at samples of size n, our standardized sample mean

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

has distribution $N(0, 1)$.

If we look instead at the random variable

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

this is called the *t-statistic*.

The t-statistic looks much like the z-statistic we used when we assumed knowledge of the true deviation. We give the t-statistic a distinct name because unlike the z-statistic it is *not* normally distributed. If we applied the 68-95-99.7 rule, we would get incorrect answers!

Instead, we must understand where the t-statistic fits on what is called the *t-distribution with* $n-1$ *degrees of freedom*. This distribution is abbreviated $t(n-1)$, and looks like a normal distribution with "fat tails", which arise because of the uncertainty of not knowing the true standard deviation. If k is large, $t(k) \sim N(0, 1)$.

The critical values for t-distributions are in Table C. We can check by comparing with the last row that with a t-statistic you need to be more deviations away from the mean to have the same level of confidence as a z-statistic.

Example 4. *Use t-statistics to estimate population mean with confidence 95% if we have an SRS of size* $n = 11$ *with* $\overline{x} = 27$ *and* $s = 2$.

Example 5. *With data as above, say whether or not the null-hypothesis of* $\mu = 30$ *can be rejected.*

Example 6. *If you sample* 200 *bacterial lifespans and find an average of* 10.41 *days and a deviation of* 2.1*, does this finding support the hypothesis that these bacterial lifespans are on average more than 10 days?*