Homework # 3. Due to Wednesday, October 30, 11:00 am

(0) Construct a homeomorphism \( SO(4) \cong SO(3) \times S^3 \).\(^1\)

(1) Prove that Definitions HE-I and HE-III are equivalent.

(2) Give definition of a contractible space. Prove that \( \mathcal{E}(X, x_0) \) is contractible.

(3) Prove that a space \( X \) is contractible if and only if every map \( f : Y \to X \) is null-homotopic.

(4) Prove that a space \( X \) is contractible if and only if it is homotopy equivalent to a point.

(5) Prove that if a subspace \( A \) is a deformation retract of \( X \) then the inclusion \( A \to X \) is a homotopy equivalence.

(6) Let \( X, Y \) be pointed spaces. Let \( X \sim Y \). Prove that \( \Sigma X \sim \Sigma Y \) and \( \Omega X \sim \Omega Y \).

(7) Prove that a \( CW \)-complex compact if and only if it is finite.

(8) Construct a cellular decomposition of \( S^n, D^n, \mathbb{R}P^n, \mathbb{C}P^n, \mathbb{H}P^n \).

(9) Construct a cellular decomposition of the oriented 2-manifold of genus \( g \).

(10) Prove that a finite \( CW \)-complex can be embedded into Euclidian space of finite dimension.

\(^1\)If you have not done this in the homework # 2