TEST # 1  
Instructions: Please do all problems. Show all your work in detail.

1. (10*) Find the derivative of the following functions:
   (a) \( f(x) = \int_{\tan x}^{\sin x} \sqrt{1 + t^2} \, dt \)
   (b) \( g(x) = \int_{0}^{x} \sqrt{t^2 + \sin t} \, dt \)

2. (20*) Evaluate the following indefinite integrals:
   (a) \( \int \frac{7 + 3x}{4x^2 + 1} \, dx \)
   (b) \( \int \frac{6}{\sqrt{1 - 9x^2}} \, dx \)

3. (20*) Find the definite integrals:
   (a) \( \int_{0}^{\pi} |\sin 2x| \, dx \)
   (b) \( \int_{0}^{1} \frac{x}{x^2 + 2} \, dx \).

4. (25*) The region \( \mathcal{R}_{(c)} \) bounded by the curves \( y = 5, \ y = x + \frac{4}{x}, \) is rotated about the line \( x = -2. \) Sketch the region. Find the volume of the resulting solid \( \mathcal{S}_{(c)}. \)

5. (25*) A tank is filled with liquid (of density \( \rho \)) as it is shown on the picture below. Find the work required to pump the liquid out of the tank.

[Diagram of a tank with liquid inside, showing dimensions and the liquid level.]