# Time Value of Money

Definitions:

- Future Value: value of an amount at a particular time in the future.
    - Example: If a customer owes me $10,000 to be paid at the end of 5 years, the future value of the amount is $10,000.
- Present Value: value of that amount today.

Given an interest rate (r), and the number of periods to the payment (N), the following expression links the two values:

Future Value = $(1 + r)^N$ x Present Value

Or

Present Value = Future Value ÷ $(1 + r)^N$

Or

Present Value = Future Value x $1/(1+r)^N$

If we use a 10% interest rate, then the $10,000 to be received after 5 years would be worth:

$PV = FV \times 1/(1+r)^N$
$PV = 10{,}000 \times 1/(1.10)^5$

PV = $10,000 x 0.62092
PV = $6,209

This means that using a 10% rate, the right to receive $10,000 after 5 years is worth $6,209 today. Alternatively, if you invest $6,209 today at a 10% interest rate, you would have $10,000 after 5 years.

Note that $1.10^5 = 1.61051$ is the future value *factor* for n =5 and r = 10% from Table 1 on the inside of the front cover of the text.

$1/(1.10)^5 = 1/1.61051 = 0.62092$ is the present value factor for n = 5 and r = 10% from Table 3 on the inside of the back cover of the text.

# Annuity

An annuity is a special case of a present value problem. An annuity occurs if there are equal payments to be made over equal intervals.

For example, assume that you have the right to receive payments of $2,000 per year over the next 5 years. This is an annuity because the amount of each payment ($2,000) is equal and the period of time between each payment (one year) is equal.

Assume that we make this contract on January 1, 2001. The first payment will be made on December 31, 2001, the second on December 31, 2002, etc.

Therefore, we could form the following table:

| Payment Date | # of periods Invested | FV Factor | Payment Amount | Future Value |
|---|---|---|---|---|
| 31-Dec-01 | 4 | 1.4641 | 2,000 | 2,928 |
| 31-Dec-02 | 3 | 1.3310 | 2,000 | 2,662 |
| 31-Dec-03 | 2 | 1.2100 | 2,000 | 2,420 |
| 31-Dec-04 | 1 | 1.1000 | 2,000 | 2,200 |
| 31-Dec-05 | 0 | 1.0000 | 2,000 | 2,000 |
| Total | | 6.1051 | | 12,210 |

Note that 6.1051 is the future value factor for an annuity with n = 5 periods and a 10% interest rate. This factor can be found in Table 2 inside the front cover. Therefore, instead of forming the table, we can simply multiply the factor, 6.1051 times the payment $2,000 to get the future value $12,210.

We can do something similar with present value.

| Payment Date | # of periods until received | PV Factor | Payment Amount | Present Value |
|---|---|---|---|---|
| 31-Dec-01 | 1 | 0.9091 | 2,000 | 1,818 |
| 31-Dec-02 | 2 | 0.8264 | 2,000 | 1,653 |
| 31-Dec-03 | 3 | 0.7513 | 2,000 | 1,503 |
| 31-Dec-04 | 4 | 0.6830 | 2,000 | 1,366 |
| 31-Dec-05 | 5 | 0.6209 | 2,000 | 1,242 |
| Total | | 3.79079 | | 7,582 |

The present value factor for an annuity with n = 5 periods, and a 10% interest rate can be found in Table 4 inside the back cover. You can use the factor to find the present value by multiplying the factor, 3.79079 times the payment, $2,000 to get the present value $7,582.

# Interpretation

If I deposited $2,000 a year into an account at the end of each of the next 5 years and earned 10% interest on the deposits, at the end of 5 years I would have $12,210.

Using a discount rate of 10%, I would pay $7,582 for the right to receive payments of $2,000 at the end of each of the next five years.

Present value concepts are very important in accounting because amounts are often recorded at present value.

Let's say that we made a sale to a customer and in exchange the customer agreed to pay us $10,000 at the end of five years. Using the 10% interest rate, we would record the following entry:

Dr. Note receivable $6,209
    Cr. Sales Revenue $6,209

The difference between the $10,000 future value and the $6,209 present value ($3,791) is future interest we will collect on the note.

Therefore, at the end of the first year we would record:
Dr. Note receivable $621
    Cr. Interest revenue $621

# Example

Mike Hampton is a pitcher for the Colorado Rockies who signed a six-year contract that includes a signing bonus of $19 million. The $19 million is not paid until the six-years of the contract expire. In addition, the $19 million is spread one million a year over 19 years.

Present Value Factors for $1

| N | 7% | 8% | 9% | 10% | 11% |
|---|---|---|---|---|---|
| 1 | 0.93458 | 0.92593 | 0.91743 | 0.90909 | 0.90090 |
| 2 | 0.87344 | 0.85734 | 0.84168 | 0.82645 | 0.81162 |
| 3 | 0.81630 | 0.79383 | 0.77218 | 0.75131 | 0.73119 |
| 4 | 0.76290 | 0.73503 | 0.70843 | 0.68301 | 0.65873 |
| 5 | 0.71299 | 0.68058 | 0.64993 | 0.62092 | 0.59345 |
| 6 | 0.66634 | 0.63017 | 0.59627 | 0.56447 | 0.53464 |
| 7 | 0.62275 | 0.58349 | 0.54703 | 0.51316 | 0.48166 |
| 8 | 0.58201 | 0.54027 | 0.50187 | 0.46651 | 0.43393 |

Present Value Factors for an Annuity

| N | 7% | 8% | 9% | 10% | 11% |
|---|---|---|---|---|---|
| 16 | 9.44665 | 8.85137 | 8.31256 | 7.82371 | 7.37916 |
| 17 | 9.76322 | 9.12164 | 8.54363 | 8.02155 | 7.54879 |
| 18 | 10.05909 | 9.37189 | 8.75563 | 8.20141 | 7.70162 |
| 19 | 10.33560 | 9.60360 | 8.95011 | 8.36492 | 7.83929 |
| 20 | 10.59401 | 9.81815 | 9.12855 | 8.51356 | 7.96333 |

Questions:

- Using a 10% discount rate what is the “value” of the bonus?
- If he were to negotiate the bonus to $1.5 million per year, what would be the increase in the value of the bonus?
- If he were given a choice of receiving the current contract or receiving a bonus of $7 million after 3 years, which contract should he take?
- Would his decision change if he used a lower interest rate, like 7%?