Global structure theorems for unstable algebras over the Steenrod algebra

Unstable algebras over the Steenrod algebra $\mathcal{A}$ arise in topology as the cohomology of spaces, and in algebra as invariants under actions on polynomial algebras by subgroups of general linear groups. These incarnations from topology and algebra also overlap, as in certain classifying spaces for vector bundles, whose cohomology coincides with the symmetric algebra of invariants under an appropriate symmetric group. We show that a number of these have strikingly tractable global descriptions in terms of minimal sets of generators and relations within the category of unstable $\mathcal{A}$-algebras. These include various classifying spaces, and the invariants under actions of the general and special linear, upper triangular, and symmetric groups. In each case we begin with a specific unstable module described by a minimal set of generators and appropriate relations. We next form the free unstable $\mathcal{A}$-algebra on this module, and show that the desired unstable $\mathcal{A}$-algebra is realized in its category by imposing only a small set of algebra relations. For instance, the Dickson algebras (invariants under the full general linear group) can be presented in this way with only one algebra relation. Comparison of the structure theorems for various unstable algebras yields interesting results. For example, we prove that the mod two symmetric algebra (cohomology of BO) projects via $\mathcal{A}$-algebra maps onto each of the Dickson algebras; we can also project to obtain global descriptions for images in the connected covers of BO, and for the classifying spaces $BO(n)$ for finite dimensional vector bundles. Our methods apply equally well at even and odd primes; the results may differ, but the descriptions obtained shed light on their similarities and differences.

David Pengelley (davidp@nmsu.edu)
Mathematics, New Mexico State University, Las Cruces, NM 88003 USA
Tel: 505-646-3901=dept., 505-646-2723=my office; Fax: 505-646-1064
http://math.nmsu.edu/~davidp