The density of most substances falls by several per cent on melting (the increase in density in the transition from ice to water being exceptional), and it is paradoxical that such a small change in the molecular spacing is accompanied by such a dramatic change in the mobility of the material. Knowledge of the liquid state is still incomplete, but it appears that the arrangement of the molecules is partially ordered, with groups of molecules as a whole having mobility, sometimes falling into regular array with other groups and sometimes being split up into smaller groups. The arrangement of the molecules is continually changing, and, as a consequence, any force applied to the liquid (other than a bulk compression) produces a deformation which increases in magnitude for so long as the force is maintained. The manner in which some of the molecular properties of a liquid stand between those of a solid and a gas is shown in the following table. In the matter of the simplest macroscopic quantity, viz. density, liquids stand much closer to solids; and in the matter of fluidity, liquids stand wholly with gases.

	Intermolecular forces	Ratio of amplitude of random thermal movement of molecules to d_0	Molecular arrangement	Type of statistics needed
solid	strong	$\ll 1$	ordered	quantum
liquid	medium	of order unity	partially ordered	quantum + classical
gas	weak	$\gg 1$	disordered	classical

The molecular mechanism by which a liquid resists an attempt to deform it is not the same as that in a gas, although, as we shall see, the differential equation determining the rate of change of deformation has the same form in the two cases.

1.2. The continuum hypothesis

The molecules of a gas are separated by vacuous regions with linear dimensions much larger than those of the molecules themselves. Even in a liquid, in which the molecules are nearly as closely packed as the strong short-range repulsive forces will allow, the mass of the material is concentrated in the nuclei of the atoms composing a molecule and is very far from being smeared uniformly over the volume occupied by the liquid. Other properties of a fluid, such as composition or velocity, likewise have a violently non-uniform distribution when the fluid is viewed on such a small scale as to reveal the individual molecules. However, fluid mechanics is normally concerned with the behaviour of matter in the large, on a macroscopic scale large compared with the distance between molecules, and it will not often happen that the molecular structure of a fluid need be taken into account explicitly. We shall suppose, throughout this book, that the macroscopic behaviour of fluids is the same as if they were perfectly continuous in

structure; and physical quantities such as the mass and momentum associated with the matter contained within a given small volume will be regarded as being spread uniformly over that volume instead of, as in strict reality, being concentrated in a small fraction of it.

The validity of the simpler aspects of this *continuum hypothesis* under the conditions of everyday experience is evident. Indeed the structure and properties of air and water are so obviously continuous and smoothly-varying, when observed with any of the usual measuring devices, that no different hypothesis would seem natural.

When a measuring instrument is inserted in a fluid, it responds in some way to a property of the fluid within some small neighbouring volume, and provides a measure which is effectively an average of that property over
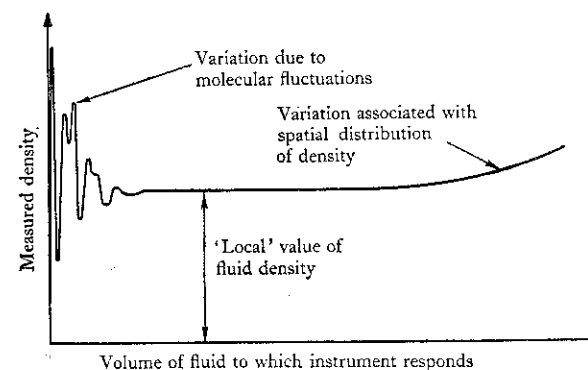

Figure 1.2.1. Effect of size of sensitive volume on the density measured by an instrument.

the 'sensitive' volume (and sometimes also over a similar small sensitive time). The instrument is normally chosen so that the sensitive volume is small enough for the measurement to be a 'local' one; that is, so that further reduction of the sensitive volume (within limits) does not change the reading of the instrument. The reason why the particle structure of the fluid is usually irrelevant to such a measurement is that the sensitive volume that is small enough for the measurement to be 'local' relative to the macroscopic scale is nevertheless quite large enough to contain an enormous number of molecules, and amply large enough for the fluctuations arising from the different properties of molecules to have no effect on the observed average. Of course, if the sensitive volume is made so small as to contain only a few molecules, the number and kind of molecules in the sensitive volume at the instant of observation will fluctuate from one observation to another and the measurement will vary in an irregular way with the size of the sensitive volume. Figure 1.2.1 illustrates the way in which a measurement of density of the fluid would depend on the sensitive volume of the instrument.

We are able to regard the fluid as a continuum when, as in the figure, the measured fluid property is constant for sensitive volumes small on the macroscopic scale but large on the microscopic scale.

One or two numbers will indicate the great difference between the length scale representative of the fluid as a whole and that representative of the particle structure. For most laboratory experiments with fluids, the linear dimensions of the region occupied by the fluid is at least as large as 1 cm and very little variation of the physical and dynamical properties of the fluid occurs over a distance of 10^{-3} cm (except perhaps in special places such as in a shock wave); thus an instrument with a sensitive volume of 10^{-9} cm^3 would give a measurement of a local property. Small though this volume is, it contains about 3×10^{10} molecules of air at normal temperature and pressure (and an even larger number of molecules of water) which is large enough, by a very wide margin, for an average over the molecules to be independent of their number. Only under extreme conditions of low gas density, as in the case of flight of a missile or satellite at great heights above the earth's surface, or of very rapid variation of density with position, as in a shock wave, is there difficulty in choosing a sensitive volume which gives a local measurement and which contains a large number of molecules.

Our hypothesis implies that it is possible to attach a definite meaning to the notion of value 'at a point' of the various fluid properties such as density, velocity and temperature, and that in general the values of these quantities are continuous functions of position in the fluid and of time. On this basis we shall be able to establish equations governing the motion of the fluid which are independent, so far as their form is concerned, of the nature of the particle structure—so that gases and liquids are treated together—and indeed, independent of whether any particle structure exists. A similar hypothesis is made in the mechanics of solids, and the two subjects together are often designated as continuum mechanics.

Natural though the continuum hypothesis may be, it proves to be difficult to deduce the properties of the hypothetical continuous medium that moves in the same way as a real fluid with a given particle structure. The methods of the kinetic theory of gases have been used to establish the equations determining the 'local' velocity (defined as above) of a gas, and, with the help of simplifying assumptions about the collisions between molecules, it may be shown that the equations have the same form as for a certain continuous fluid although the values of the molecular transport coefficients (see §1.6) are not obtained accurately. The mathematical basis for the continuum treatment of gases in motion is beyond our scope, and it is incomplete for liquids, so that we must be content to make a hypothesis. There is ample observational evidence that the common real fluids, both gases and liquids, move as if they were continuous, under normal conditions and indeed for considerable departures from normal conditions, but some of the properties of the equivalent continuous media need to be determined empirically.

1.3. Volume forces and surface forces acting on a fluid

It is possible to distinguish two kinds of forces which act on matter in bulk. In the first group are long-range forces like gravity which decrease slowly with increase of distance between interacting elements and which are still appreciable for distances characteristic of natural fluid flows. Such forces are capable of penetrating into the interior of the fluid, and act on all elements of the fluid. Gravity is the obvious and most important example, but two other kinds of long-range force of interest in fluid mechanics are electromagnetic forces, which may act when the fluid carries an electric charge or when an electric current passes through it, and the fictitious forces, such as centrifugal force, which appear to act on mass elements when their motion is referred to an accelerating set of axes. A consequence of the slow variation of one of these long-range forces with position of the element of fluid on which it is acting is that the force acts equally on all the matter within a small element of volume and the total force is proportional to the size of the volume element. Long-range forces may thus also be called volume or body forces.

When writing equations of motion in general form, we shall designate the total of all body forces acting at time t on the fluid within an element of volume δV surrounding the point whose position vector is $\mathbf{x}$ by

$$\mathbf{F}(\mathbf{x}, t)\rho\,\delta V; \qquad (1.3.1)$$

the factor ρ has been inserted because the two common types of body force per unit volume—gravity and the fictitious forces arising from the use of accelerating axes—are in fact proportional to the mass of the element on which they act. In the case of the earth's gravitational field the force per unit mass is

$$\mathbf{F} = \mathbf{g},$$

the vector $\mathbf{g}$ being constant in time and directed vertically downwards.

In the second group are short-range forces, which have a direct molecular origin, decrease extremely rapidly with increase of distance between interacting elements, and are appreciable only when that distance is of the order of the separation of molecules of the fluid. They are negligible unless there is direct mechanical contact between the interacting elements, as in the case of the reaction between two rigid bodies, because without that contact none of the molecules of one of the elements is sufficiently close to a molecule of the other element. The short-range forces exerted between two masses of gas in direct contact at a common boundary are due predominantly to transport of momentum across the common boundary by migrating molecules. In the case of a liquid the situation is more complex because there are contributions to the short-range or contact forces from transport of momentum across the common boundary by molecules in oscillatory motion about some quasi-stationary position and from the forces between molecules on the two