JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 112, F02036, doi:10.1029/2006JF000701, 2007

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Comment on "A quantitative framework for interpretation of basal ice facies formed by ice accretion over subglacial sediment" by Poul Christoffersen et al.

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Received 28 September 2006; revised 28 February 2007; accepted 8 May 2007; published 28 June 2007.

Citation: Rempel, A. W., J. S. Wettlaufer, and M. G. Worster (2007), Comment on ''A quantitative framework for interpretation of basal ice facies formed by ice accretion over subglacial sediment'' by Poul Christoffersen et al., *J. Geophys. Res.*, *112*, F02036, doi:10.1029/2006JF000701.

[1] Frost heave is the term given to the deformation at the Earth's surface that is caused by the formation and growth of segregated ice lenses in permafrost and seasonally frozen ground. Efforts to understand and predict the characteristics of solidification in porous media led *O'Neill and Miller* [1985] to produce the first comprehensive and tractable model for the thicknesses, spacings and growth rates of sediment-free ice lenses. A central feature of the model involved an assessment of the balance of forces within a frozen fringe of partially ice-saturated sediments beneath the lowest segregated ice interface. When *O'Neill and Miller* [1985] formulated their "rigid-ice" model, it was not yet clear how exactly to calculate the net force transmitted by sediment particles to the ice. As a consequence, the rigid-ice model is only approximately correct in its treatment of the force balance that describes how lenses grow, and an ad hoc formulation was needed to prescribe when new lenses would form (always somewhere within the frozen fringe). Accurate predictions could nevertheless be obtained by tuning the model to reproduce the results of available empirical studies. We now understand that the forces transmitted by the sediment particles to the ice are a consequence of interfacial premelting, which is a basic aspect of phase behavior that ice shares in common with all other classes of solids [see, e.g., *Wettlaufer and Worster*, 2006; *Dash et al.*, 2006]. *Rempel et al.* [2004] demonstrated how to calculate the net effects of these interactions, correctly formulate the force balance constraint that describes how lenses grow, and remove the need for an ad hoc lens initiation criterion. Both formulations prescribe that a new lens forms when the particle-particle stress vanishes: it is the partition of stresses between particles, ice and water, on which the lens initiation is based, that is additionally prescribed by *O'Neill and Miller* [1985], but determined from fundamental force balances by *Rempel et al.* [2004]. In common with the *O'Neill and Miller* [1985] model, certain properties of the porous media host are critical for making quantitative predictions, most importantly, knowledge of the manner in which the ice saturation of the sediments and the permeability to fluid flow vary with temperature. Figure 1 illustrates the generally close agreement between the predictions of the two models for the steady state thickness of a subglacial layer of frozen-on sediment as a function of the rate of melting or freezing.

[2] Though seen in recovered ice cores and at glacier termini in the past, some of the most stunning glaciological examples of the alternating bands of debris free and debris-rich ice that frost heave would be expected to produce were revealed beneath the Kamb Ice Stream by the pioneering borehole camera work of *Carsey et al.* [2002]. In a recent paper *Christoffersen et al.* [2006] again focus attention on these fascinating basal layers and apply the model of *Christoffersen and Tulaczyk* [2003] in an attempt to explain some interesting new findings produced by detailed image analysis. In justifying their modeling approach (see especially section 2 and Appendix A of *Christoffersen et al.* [2006]), several erroneous and misleading references are made to the work described by *Rempel et al.* [2004]. Here we lay bare these misconceptions and show how they influence the modeling framework that *Christoffersen et al.* [2006] employ.

[3] In section 2, *Christoffersen et al.* [2006] imply that the frost heave model of *O'Neill and Miller* [1985] is more amenable to their application than the "microphysically correct" approach of *Rempel et al.* [2004] because the older frost heave model does not rely on knowledge of the spatial distribution of ice saturation. As noted above, there is no truth to this; in fact, the model originally developed by *Christoffersen and Tulaczyk* [2003] and revisited here contains sufficient modifications from that of *O'Neill and Miller* [1985] that this essential aspect of the underlying physics has been ignored. In their appendix, *Christoffersen et al.* [2006] lament that they "are not aware of any observational constraints on spatial distribution of [the ice saturation] S_s." The ice saturation as a function of temperature is the required function, or its complement, the water

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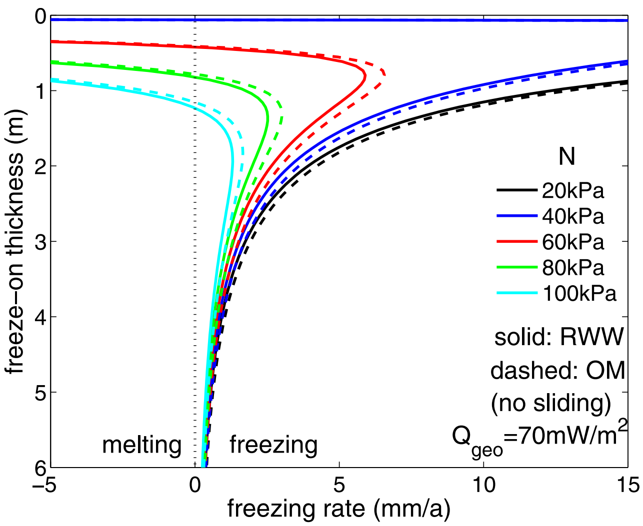

Figure 1. Steady state thickness of subglacial frozen sediments as a function of freezing rate (negative values correspond to melting) for the values of effective stress N given in the legend. Solid curves are the predictions of the frost heave model by *Rempel et al.* [2004], and dashed curves are the predictions of the "rigid-ice" model described by *O'Neill and Miller* [1985]. All calculations shown are for a geothermal input of Q_{geo} = 70 mW/m^2, conductive heat transport, distributed latent heat release resulting from ice advection at the freezing rate, and no work due to subglacial sliding. Values to the left of the vertical dotted line correspond to the predicted thickness of subglacial frozen sediments during steady state melting. For the soil properties used here, extracted from empirical data for Chena silt [*Andersland and Ladanyi*, 2004], a stable steady state rate of melting or freezing can occur with no frozen-on sediments when N < 38 kPa. For larger N, liquid water must percolate through a layer of partially frozen sediments at the glacier base. For given N, the steady state freezing rate attains a maximum that is higher for smaller N. Heat removal that is fast enough to promote more rapid freezing entails transient behavior with the freeze-on thickness increasing as the rate of water flow to the glacier base at the top of the frozen sediments decreases. A family of unstable steady states that characterizes this realm, in which freezing can produce lenses of clean ice interspersed with sediment-rich bands, is shown with freezing rate decreasing as freeze-on thickness increases in the lower portion of the plot. Note that the basal heat flux into the glacial ice at the top of the frozen sediment layer is equal to Q_{geo} = 70 mW/m^2 only when no freezing or melting occurs. Latent heat effects modify this so that the heat flux into the glacial ice above the frozen sediment layer that corresponds to melting at 5 mm/a is approximately 20 mW/m^2, and the basal heat flux that corresponds to freezing at 15 mm/a is 220 mW/m^2.

content, which is the form used by *O'Neill and Miller* [1985], shown schematically in their Figure 1, and plotted for a particular model soil in their Figure 2. The importance of changes in ice saturation with temperature to the permafrost community has motivated a considerable body of work (e.g., see compilations by *Andersland and Ladanyi* [2004] and *Nixon* [1991] and references therein). In Table 2-6 of *Andersland and Ladanyi* [2004], ratios of water mass to dry mass are reported for a range of porous media in the form $w_u = A/(T_m - T)^\beta$ where A is chosen to fit the empirical data they reference when $T_m - T$ is the temperature departure from bulk melting, measured in degrees centigrade: S_s is easily extracted from this. Figure C-14 of *Andersland and Ladanyi* [2004] contains a log-log plot of hydraulic conductivity as a function of $T_m - T$ for a number of porous media: the permeability is easily extracted from this. A theoretical understanding of the controls on S_s, based on the effects of surface energy and interfacial premelting in monodispersed powders compares favorably with experimentally determined values [see *Dash et al.*, 2006, and references therein]. Although the essential physics would not differ from these other settings, we are not aware of any measurements of ice saturation and permeability in glacial tills; this is an important deficiency, and *Christoffersen et al.* [2006] are correct to alert the community to this need.

[4] Thus we are led to ask how the model of *Christoffersen and Tulaczyk* [2003] need not represent the dependence of ice saturation or permeability on temperature, but in fact "predict" lens formation at locations that are unconnected by pore ice and thereby are unable to transmit stresses to the overlying segregated ice layer? The answer begins with equation (1a) of that paper, where they note the presence of a pressure difference between the ice and liquid that is caused by the effects of surface energy over the curved ice-water interface. In equation (1b), they assume that the ice-water interface "complies with the particle surfaces" and so the curvature of the ice-water interface can be equated with the (positive) specific surface area (*SSA*) of the sediments. This yields a value that is not directly related to temperature and they proceed to build their model upon this foundation. As can be seen by substituting the early part of equation (1b) into equation (9), new ice lenses are assumed to form when the liquid water pressure exceeds the vertical overburden. In their model, the pore ice does not influence the force balance or fluid supply until after elevated water pressure drives the sediment grains apart and an ice lens appears. In fact, the treatment of pore ice starts with a sign error. Where the ice surface runs parallel to the surfaces of sediment particles, on average the mean curvature of the ice surface is negative and so can be identified with -*SSA*. The curvature of the ice-liquid interface only produces the required positive pressure difference between the ice and liquid phases in locations where it is distant from the sediment particles and oriented so that the center of curvature is within the ice. The curvature in such regions depends on temperature but is not directly related to *SSA*. We note that the specific surface area is important for other reasons, since it is related to the area over which interfacial premelting occurs and (temperature-dependent) forces are transmitted between the sediment particles and the ice. It is these forces that are responsible for the positive pressure difference between the ice and liquid phases in locations adjacent to particle surfaces where the ice-water interface has negative curvature.

[5] In summary, the models proposed by *O'Neill and Miller* [1985] and by *Rempel et al.* [2004] are essentially identical, particularly in their reliance on knowledge of the

spatial distribution of ice saturation, the latter simply being more rigorous with regard to the underlying physical interactions. In contrast, the underlying physical basis of the *Christoffersen and Tulaczyk* [2003] model is, at best, unclear and completely distinct from that of the *O'Neill and Miller* [1985] model on which it purports to be based.

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