Math 251 Midterm 2 NAME:_____
F 22 Nov. 2002 Student ID:_____

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. Closed book, except for a graphing calculator and a 3×5 file card. Note: you may not use the calculator as a substitute for calculus.
3. The point values are as indicated in each problem; total 100 points.
4. Write all answers on the test paper. Use the space below the extra credit problems for long answers or scratch work.
5. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
6. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
7. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.
8. Final answers must always be simplified (just as in homework), unless otherwise specified.
9. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
10. Time: 50 minutes.

1	2	3	4	5	6	7	TOTAL	EC
1	9	15	20	10	25	20	100	

1. (1 point) True or false: L'Hospital's rule was included in the course for the purpose of confusing students who are having trouble remembering the quotient rule.

2. (9 points) Let f be a function such that $f'(x) = -8f(x)$. Find the derivative of the function $g(x) = \cos(f(x) + x)$. (Your answer might involve the function f.)

3. (15 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = (3x^2 - 5x - 5)\, e^{-x-7}$ takes its absolute minimum and maximum values on the interval $[2, 10]$.

Hint: $f'(x) = (-3x^2 + 11x)\, e^{-x-7}$.

4. (10 points/part) Evaluate the following limits. (Give exact values.)

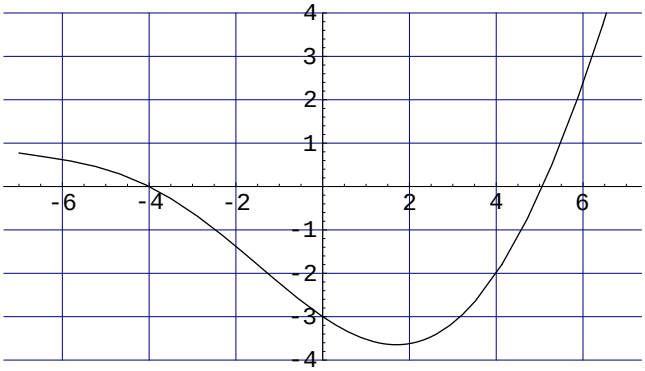
(a) $\lim_{x \to 0} \dfrac{e^{k \sin(x)} - 1}{\sin(x)}$, where k is a nonzero constant.

(b) $\lim_{x \to 2} \dfrac{x}{e^{3x}}$

5. (10 points) If $x^7 y = x + \cos(ky) + \pi^3$, where k is a constant, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

6. (25 points) A 13 foot ladder leans against a vertical wall in a room with a high ceiling and level floor. Because the floor is slippery, the foot of the ladder is sliding away from the wall. When the foot is 5 feet from the wall, it is sliding away at 3 feet per hour. At this time, how fast is the top of the ladder sliding down the wall?

7. (5 points/part) The picture below is the graph of the *DERIVATIVE* $y = w'(x)$ for a certain function w. **CAUTION:** You are given the graph of the *derivative* $w'(x)$, *not* the graph of $w(x)$, but you are asked questions about $w(x)$.



(a) Find all a in $(-6, 6)$ such that w has a local maximum at a. Give reasons for your choices, or explain why there are none.

(b) Suppose we are given that $w(4) = -10$. Use the linear approximation to estimate $w(3.8)$.

(c) Is w concave up or concave down at $x = 0$, or does w (nearly) have an inflection point at $x = 0$, or is there not enough information provided to determine this? Why?

(d) Let $f(x) = w(9 - x^2)$. Find $f'(-3)$, or explain why there is not enough information to do so.

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.)

EC1. (7 extra credit points) Find a function f such that $f'(x) = x\sin(x^2)$ for all x. Check your function to be sure its derivative really is what you think it is.

EC2. (15 extra credit points) Let g be a continuous function on $(-1, 1)$ such that $g(0) = 0$ and $g'(0) = 1$. Suppose that g'' exists and is continuous on $(-1, 1)$. Define

$$f(x) = \begin{cases} g(x)/x & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Show that $f'(0)$ exists, and prove a formula for it in terms of a suitable derivative of g.