Math 251 Final Exam NAME:____________________

Friday 13 Dec. 2002 Student ID:____________________

Grades may be posted outside my office door or on the web (or both). If you want your grade posted, then fill out the items below:

Please post my grade under the following code: ____________________

Note: Do not use more than 6 digits of your social security number; otherwise, you run the risk of identity theft. Also, no obscenities or political slogans. No non-ASCII characters in codes for web postings.

Please post my grade outside the office door. _____ (Mark "X" if yes.)

Please post my grade on the web. _____ (Mark "X" if yes.)

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. Closed book, except for a graphing calculator and a 3×5 file card. Note: you may not use the calculator as a substitute for calculus.
3. The point values are as indicated in each problem; total 235 points.
4. Write all answers on the test paper. Use the space below the extra credit problems for long answers or scratch work.
5. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
6. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
7. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.
8. Final answers must always be simplified (just as in homework), unless otherwise specified.
9. Time: 120 minutes. (Expected to be extended.)

DO NOT WRITE BELOW THIS LINE

1	2	3	4	5	6	7	8	9	10	TOTAL
1	40	12	45	15	18	35	16	18	35	235

Extra Credit:

1. (1 point) True or false: Final exams on Friday are #[%&(?]#) [expletive deleted]!!

2. (10 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give reasons in all cases.

(a) $\lim\limits_{x\to\infty} \dfrac{\sqrt{3x^2+2}}{9x+7109}$.

(b) $\lim\limits_{x\to 7^+} \dfrac{f(x)}{x-7}$, given that $\lim\limits_{x\to 7} f(x) = -2$.

(c) $\lim\limits_{x\to 0} \dfrac{f(x)-7}{\sin(3x)}$, given that $f(0)=7$, that $f'(0)=2$, and that f' is continuous at 0.

(d) $\lim\limits_{x\to 0} \dfrac{f(x)-7}{\cos(3x)}$, given that $f(0)=7$, that $f'(0)=2$, and that f' is continuous at 0.

3. (12 points) Let f be a function such that $f'(s) = \left(3s^2+\sqrt{12}\right)^{108} + 8$. Find the derivative of the function $h(x) = \ln(x - f(x))$. (Your answer might involve the function f.)

4. (45 points) An open rectangular box (no top) is to have a base that is twice as long as it is wide. 96 square feet of material are available to make the box. Find the dimensions of the box which maximizes the volume.

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

5. (5 points/part) Let $T(t)$ be the temperature at time t at the Eugene airport. Assume that t is measured in hours after midnight on 31 Dec. 2000, and that $T(t)$ is measured in °C.

(a) What are the units of $T'(t)$?

(b) Do you expect $T'(11)$ to be positive or negative? Why?

(c) Explain the practical significance of the statement $T'(21) = -2$.

6. Let f and g be functions such that:

$$f(3) = -2, \quad f'(3) = 11, \quad g(3) = -1, \quad \text{and} \quad g'(3) = 3$$

and

$$f(-1) = 7, \quad f'(-1) = -3, \quad g(-1) = 3, \quad \text{and} \quad g'(-1) = -1.$$

Let $h(x) = f(g(x))$.

(a) (2 points) Find $h(-1)$. (You will not need to use all the information provided.)

(b) (8 points) Find $h'(-1)$. (You will not need to use all the information provided.)

(c) (8 points) Use the linear approximation to estimate $h(-1.1)$.

7. (35 points) Let $g(x) = \frac{3}{10}x^5 + 11x^4 + 120x^3 - 6656x + 36000$. Produce graphs (more than one, if necessary) of $y = g(x)$ which reveal all the important features of the function. In particular, estimate the intervals of increase and decrease, critical numbers, extreme values, intervals of concavity, and inflection points, either using graphs of the first and second derivatives of the function, or directly from the formulas for these derivatives. (Your graphs must be shown on the test paper.)

Note: More credit will be given for the features that are harder to find.

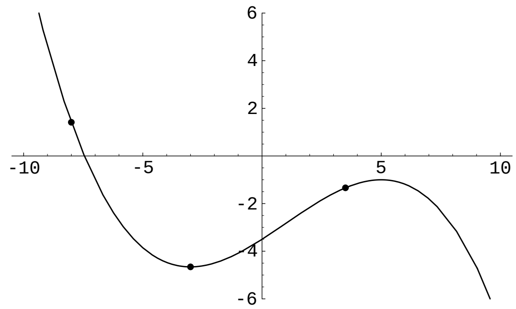
Hint: Here are the first and second derivatives in partially factored form:

$$g'(x) = \frac{1}{2}(x+8)(3x^3 + 64x^2 + 208x - 1664) \quad \text{and} \quad g''(x) = 6x(x+10)(x+12).$$

The only real root of $3x^3 + 64x^2 + 208x - 1664 = 0$ is approximately 3.53039.

8. (16 points) Find $\frac{dy}{dx}$ if $\frac{x}{y} = \tan(x - y) - \cos(1)$. (You must solve for $\frac{dy}{dx}$.)

9. (6 points/part) The picture below is the graph of $y = h(x)$ for a certain function h. (This is a graph of the function, *not* its derivative.)



For each value of x listed, answer the following three questions. Give reasons for your answers.

(1) Is $h(x)$ positive, negative, or near zero, or is there not enough information provided to determine this?

(2) Is $h'(x)$ positive, negative, or near zero, or is there not enough information provided to determine this?

(3) Is $h''(x)$ positive, negative, or near zero, or is there not enough information provided to determine this?

(The dots on the graph indicate the points on the graph corresponding to the given values of x.)

(a) $x = -8$.

(b) $x = -3$.

(c) $x = 3.5$.

10. (35 points) A street light is mounted on the top of a 15 foot tall pole. A 6 foot tall woman is walking away from the pole along a straight path. When she is 40 feet from the pole, she is walking 5 feet per second. How fast is the tip of her shadow moving at this time? Be sure to include correct units.

(Extra credit on next page.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.) Write answers in the space after all the extra credit problems, or on the backs of the pages.

EC1. (25 extra credit points) This is a more real applied problem than most we have seen.

On a computer disk, the amount of information (number of bytes) stored on a track that goes once around the disk is the same whether the track is near the center or near the edge. (This is required by the mechanics of reading the disk, because the disk must rotate at a fixed speed.) Let b be the maximum number of bytes per centimeter one can put on a track, let R be the radius of the disk, and let a be the number of tracks per radial centimeter. On the disk, there will be a central circular area with no information storage. Determine what the radius of this area should be to maximize the total amount of information on the disk.

Be sure to verify that your maximum or minimum really is what you claim it is.

Note: I have been told that the description in this problem is correct only for very cheap media, and that otherwise it is worth the effort of building drives that can handle constant linear density.

EC2. Define

$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases} .$$

(a) (5 extra credit points) Prove that f is continuous at 0.
(b) (15 extra credit points) Prove that $f'(0) = 0$.
(c) (25 extra credit points) Prove that $f''(0) = 0$.