MATH 251 WINTER 2008: SAMPLE MIDTERM 2 QUESTIONS

At least 80% of the points on the real exam will be modifications of problems from Midterms 1 and 2 from the last time I taught the course, the problems below, the sample problems for Midterm 1, the real Midterm 1 (all four versions, including those given to the other section), homework problems (particularly written homework) from the entire course, and problems from the sample and real Midterms 0. Note, though, that the exact form of the functions to be differentiated and of the limits to be computed could vary substantially, and the methods required to do them might occur in different combinations.

Standard formulas, such as for the area and circumference of a circle, area and perimeter of a rectangle, and volume and surface area of a sphere, cylinder, box, or hemisphere, will not be provided on the exam. If you can't remember these, put them on your note card.

There are many variations possible on the graph problems. What you see on the real exam will probably not be same as the ones given here.

The instructions are the same as for Midterm 1. You might actually need a calculator for problems asking where global minimums and maximums occur. As for Midterm 1, you will be allowed *one* 3×5 file card.

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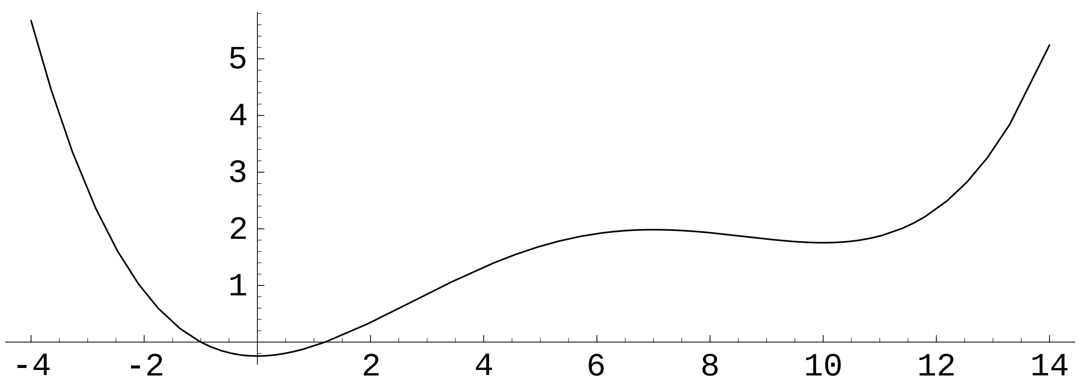
1. (15 points) Find the exact values of x at which the function $h(x) = \left(8 + 8x - 7x^2\right) e^{-x}$ takes its absolute minimum and maximum on the interval $[1, 5]$.

Note: The problem does not ask you to find the exact maximum and minimum values of h on the interval, only the exact values of x at which they occur.

2. (5 points) The picture below shows the graph of $y = f(x)$ for a particular function f.



Use this graph to find a number x in the interval shown such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$, being sure to explain why your choice of x satisfies these conditions. If no such x exists, explain why not.

3. (15 points) Let $f(x) = e^{\sin(x)+ax}$, where a is a constant. Find $f''(x)$.

4. (8 points/part) Find $\dfrac{dy}{dx}$:

(a) $y = \dfrac{kx}{\arctan(x)} + \dfrac{\sqrt{\pi}}{2}$, where k is a constant.

(b) $y = \dfrac{g(5x)}{2} + \ln(17)$, where $g'(x) = \cos(x^2)$. (Your answer might involve the function g.)

5. (8 points) Suppose the functions f and g satisfy the following:

$$f(x) = x^3 + x + 2, \quad f(g(x)) = x^2, \quad \text{and} \quad g(3) = -1.$$

Use the chain rule to find $g'(3)$.

6. (8 points/part) Find the exact values of the following limits, or explain why they do not exist or there is not enough information to evaluate them.

(a) $\lim_{x \to 0} \frac{x^3}{\sin(7x^3)}$

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$.

(c) $\lim_{x \to \infty} \frac{\sqrt{2x^2 + 1}}{7x + 1109}$.

(d) $\lim_{x \to 3^+} \frac{f(x)}{x - 7}$, given that $f(3) = 2$, that $f'(3) = 9$, and that f' is continuous at 3.

(e) $\lim_{x \to 3} \frac{f(x) - 2}{x - 3}$, given that $f(3) = 2$, that $f'(3) = 9$, and that f' is continuous at 3.

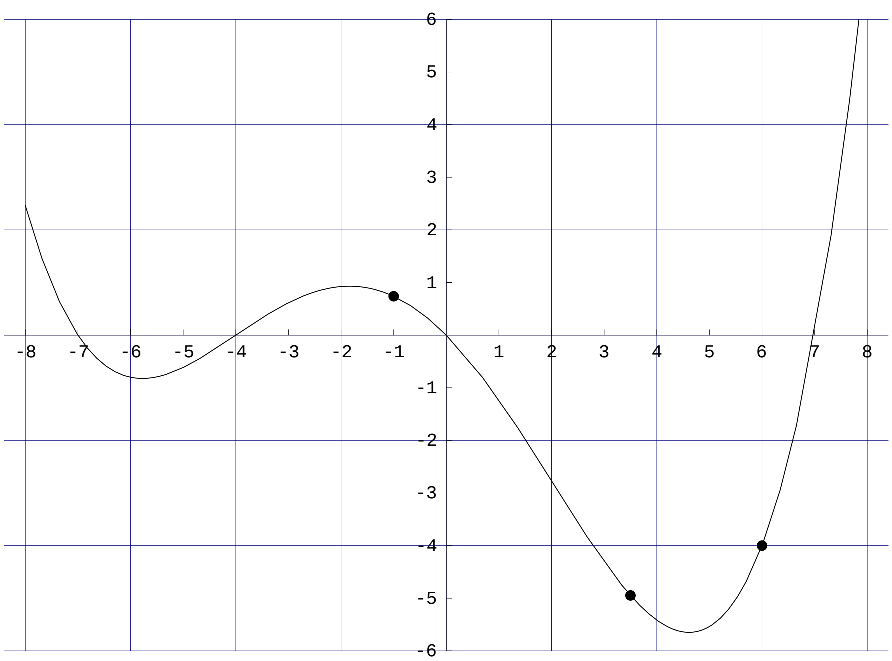
7. (5 points/part) The function $f(x)$ satisfies the following three properties:

$$f(2) = -3, \quad f'(2) = 0.40, \quad \text{and} \quad f''(2) = 1.1.$$

(a) Use the linearization (tangent line approximation) to estimate the value of $f(1.96)$.

(b) Is your answer in part (a) too big or too small? Explain with a picture.

8. (5 points/part) The picture below is the graph of the *DERIVATIVE* $y = f'(x)$ for a certain function f. **CAUTION:** You are given the graph of the *derivative* $f'(x)$, *not* the graph of $f(x)$, but you are asked questions about $f(x)$. The points referred to in Parts (a), (b), and (d) are marked on the graph with dots.



(a) Is f increasing, decreasing, or nearly flat at $x = -1$, or is there not enough information provided to determine this? Why?

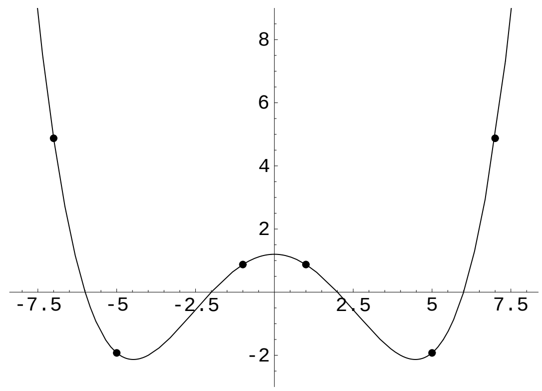
(b) Is f concave up or concave down at $x = 3.5$, or does f (nearly) have an inflection point at $x = 3.5$, or is there not enough information provided to determine this? Why?

(c) At which values of x in $[-8, 8]$ (the interval shown) does f have a local minimum? Explain.

9. (25 points) John Doe, the famous environmental vandal, has blown up an offshore oil rig. As a result, crude oil is escaping from the well and forming a circular oil slick on the ocean. At a certain time, 20 thousand cubic

meters of oil had escaped, and it was continuing to escape at the rate of 2 thousand cubic meters per day. At the same time, the oil slick was 6 kilometers in diameter. Assuming the oil slick has uniform thickness which remains constant, how fast is the diameter increasing? (Be sure to include the correct units in your answer.)

10. (6 points/part) The picture below is the graph of $y = f(x)$ for a certain function f:



For each value of x listed, answer the following three questions. Give reasons for your answers.

(1) Is $f(x)$ positive, negative, or near zero, or is there not enough information provided to determine this?

(2) Is f increasing, decreasing, or nearly flat at x, or is there not enough information provided to determine this?

(3) Is f concave up or concave down at x, or does f (nearly) have an inflection point at x, or is there not enough information provided to determine this?

(The dots on the graph indicate the points on the graph corresponding to the given values of x.)

(a) $x = -7$.

(b) $x = -5$.

(c) $x = -1$.

(d) $x = 1$.

(e) $x = 5$.

(f) $x = 7$.

11. (15 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = \frac{1}{2}x^4 - \frac{11}{3}x^3 + 7x^2 - 5$ takes its absolute minimum and maximum on the interval $[1, 5]$.

12. (12 points) Prove that there exists a unique real solution to the equation $x^9 + 2x + 2 = 0$. Give a complete justification for any theorems that you use, in particular being sure to check that the hypotheses hold.