Math 251 (11:00 am section) Midterm 2 NAME:_____
F 29 Feb. 2008 Student ID:_____
GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. Closed book, except for a graphing calculator and a 3×5 file card. Note: you may not use the calculator as a substitute for calculus. Calculators able to calculate derivatives or do other symbolic computations are prohibited.
3. No cell phones, laptops, or any other electronic device other than the graphing calculator referred to above.
4. The point values are as indicated in each problem; total 100 points.
5. Write all answers on the test paper. Use the extra sheet at the back for long answers or scratch work.
6. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
7. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
8. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.
9. Final answers must always be simplified (just as in homework), unless otherwise specified.
10. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
11. Time: 50 minutes.

1	2	3	4	5	6	7	TOTAL	EC
1	12	18	25	18	15	11	100	

1. (1 point) True or false: Related rates problems were invented by the devil.

2. (12 points.) Find $\frac{dy}{dx}$ if $xy = \sin(x + y) + \sin(6)$. (Use implicit differentiation. You must solve for $\frac{dy}{dx}$.)

3. (9 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give reasons in all cases.

(a) $\lim_{x \to 0} \frac{kx}{\tan(x^2 - 3x)}$, where k is a nonzero constant.

(b) $\lim_{x \to -3} \frac{x^2 - x + 19}{x - 3}$.

4. (25 points) Interstate 15 and Interstate 10 meet at a right angle near Rancho Cucamonga (CA). At midnight one night, Professor Greenbottle was driving south on Interstate 15 at 50 mph, and was 4 miles south of the intersection. At the same time, a cocaine kingpin was driving west on Interstate 10 at 80 mph, and was 3 miles east of the intersection. Were Professor Greenbottle and the cocaine kingpin getting closer together or further apart? At what rate? (Be sure to include the correct units.)

5. (18 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = 4x^3 - 33x^2 + 30x$ takes its absolute minimum and maximum on the interval $[-1, 1]$.

6. Let f and g be functions such that:

$$g(1) = 2, \quad g'(1) = 1, \quad f(1) = 17, \quad \text{and} \quad f'(1) = 6$$

and

$$g(2) = 1, \quad g'(2) = 2, \quad f(2) = 3, \quad \text{and} \quad f'(2) = -1.$$

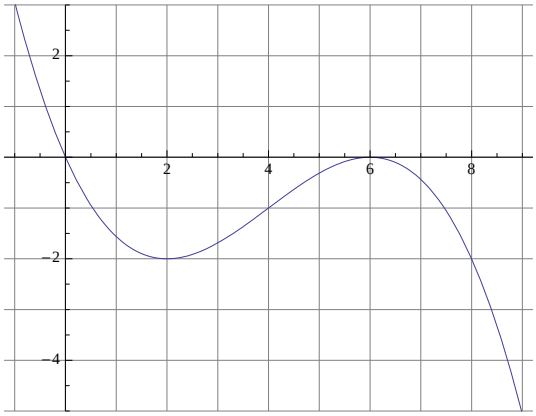
Let $h(x) = f(g(x))$.

(a) (2 points) Find $h(2)$. (You will not need to use all the information provided.)

(b) (7 points) Find $h'(2)$. (You will not need to use all the information provided.)

(c) (6 points) Use the linear approximation to estimate $h(1.96)$.

7. The picture below is the graph of the *DERIVATIVE* $y = f'(x)$ for a certain function f. **CAUTION:** You are given the graph of the *derivative* $f'(x)$, *not* the graph of $f(x)$, but you are asked questions about $f(x)$.



(a) (7 points.) Find and identify all local minimums and local maximums of f in the interval $(-1, 9)$. Justify your answer. (Note that you are being asked about f, **not** the function shown in the graph, which is f'.)

(b) (4 points.) Suppose $f(4) = 0$. Is $f(3)$ positive, negative, or is there not enough information to decide? Justify your answer. (Note that you are being asked about f, **not** the function shown in the graph, which is f'.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.)

Do these problems on the extra sheet at the back.

EC1. (5 extra credit points/part) For each of the following parts, find a function f whose derivative is as given. Check your function to be sure its derivative really is what you think it is. (They are arranged in what I believe to be increasing order of difficulty, but (c) might be harder than (d).)

(a) $f'(x) = \sin(7x)$.

(b) $f'(x) = xe^{-x^2}$.

(c) $f'(x) = \dfrac{e^x}{1 + e^{2x}}$.

(d) $f'(x) = x\sin(x)$.