

MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 5.

This sheet is part of the homework for Week 5, and is due in class on Friday 8 February 2008.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WebAssign), give exact answers (not decimal approximations; again, unlike WebAssign), and use correct notation. Some of the grade will be based on correctness of notation in the work shown.

1. Let n be a constant. Find the derivative of the function $g(x) = x^n + n^x$.

Solution: Rewrite the second term as $e^{\ln(n) \cdot x}$. Then differentiate, using the chain rule on the second term:

$$g'(x) = nx^{n-1} + \ln(n)e^{\ln(n) \cdot x}.$$

This can be rewritten as

$$g'(x) = nx^{n-1} + \ln(n)n^x,$$

but that is not necessary.

2. Let b be a function such that $b'(x) = -5b(x) + 1$. Find the derivative of the function $f(x) = \sin(b(x))$. (Your answer might involve the function b .)

Solution: Use the chain rule:

$$f'(x) = \cos(b(x))b'(x) = \cos(b(x))(-5b(x) + 1).$$

3. Let f be a function such that $f'(x) = (3x^2 + \pi^2)^{237}$. Find the derivative of the function $w(x) = f(\sqrt{19} + \cos(x))$. (Your answer might involve the function f .)

Solution: Use the chain rule:

$$\begin{aligned} w'(x) &= f'(\sqrt{19} + \cos(x)) \cdot \frac{d}{dx}(\sqrt{19} + \cos(x)) \\ &= (2(\sqrt{19} + \cos(x))^2 + \pi^2)^{237} [-\sin(x)] = -\sin(x)(2(\sqrt{19} + \cos(x))^2 + \pi^2)^{237}. \end{aligned}$$

4. Let f be a function such that $f'(x) = e^x + f(x)$. Find the derivative of the function $g(z) = f(7z)$. (Your answer might involve the function f .)

Solution: Use the chain rule:

$$g'(z) = f'(7z) \cdot \frac{d}{dz}(7z) = [e^{7z} + f(7z)] \cdot 7 = 7[e^{7z} + f(7z)].$$

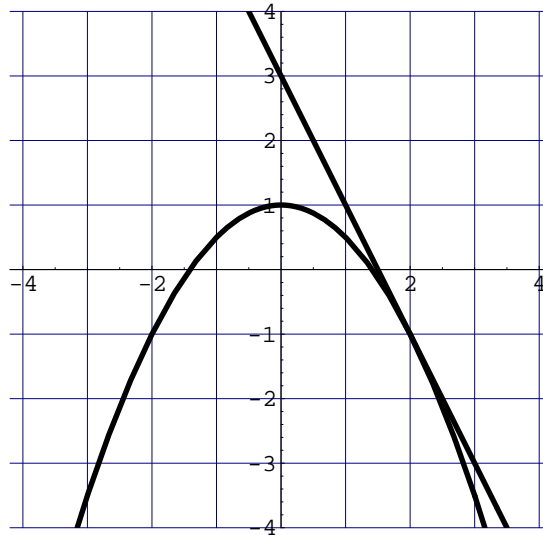
5. Let f be a function such that $f'(t) = (t^3 + 7)^{72} + t$. Find the derivative of the function $b(x) = \cos(xf(x))$. (Your answer might involve the function f .)

Solution: Use the chain and product rules:

$$\begin{aligned} b'(x) &= -\sin(xf(x)) \cdot \frac{d}{dx}(xf(x)) = -\sin(xf(x))(f(x) + xf'(x)) \\ &= -\sin(xf(x))(f(x) + x[(x^3 + 7)^{72} + x]) = -\sin(xf(x))(f(x) + x(x^3 + 7)^{72} + x^2). \end{aligned}$$

(Continued on back or on next page.)

6. The picture below shows the graph of a function $y = f(x)$ and a tangent line to the graph.



Let w be the function $w(x) = [f(x)]^3$. Find $w'(2)$.

Solution: By the chain rule, we have $w'(x) = 3[f(x)]^2 f'(x)$. Therefore $w'(2) = 3[f(2)]^2 f'(2)$.

We can read off $f(2) = -1$ from the graph.

We now need $f'(2)$. Examining the graph, we see that the tangent line goes through the points $(2, -1)$ and $(0, 3)$. Therefore its slope is $f'(2) = \frac{3 - (-1)}{0 - 2} = -2$.

Now substitute:

$$w'(2) = 3[f(2)]^2 f'(2) = 3[-1]^2 \cdot (-2) = -6.$$

For reference, the function in the graph is $f(x) = 1 - \frac{1}{2}x^2$.