MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 4.

This sheet is part of the homework for Week 4, and is due in class on **Wednesday** 30 January 2008.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WebAssign), give exact answers (not decimal approximations; again, unlike WebAssign), and use correct notation. Some of the grade will be based on correctness of notation in the work shown.

1. Differentiate the following functions:

a. $g(x) = 2x^k - \dfrac{a}{x} - b\sqrt{x} + \dfrac{3}{7} - \pi^2$, where a, b, and k are constants.

Solution: Before doing so, we rewrite the function to make it easy to differentiate:

$$g(x) = 2x^k - ax^{-1} - bx^{1/2} + \frac{3}{7} - \pi^2.$$

Then

$$g'(x) = 2kx^{k-1} - a(-1)x^{-2} - b\left(\tfrac{1}{2}\right)x^{1/2-1} = 2kx^{k-1} + ax^{-2} - \tfrac{1}{2}bx^{-1/2}.$$

The expressions $\frac{3}{7}$ and π^2 are *constants*, so their derivatives are zero. (I have seen people waste lots of time using the quotient rule to differentiate expressions like $\frac{3}{7}$.)

b. $h(x) = -xf(x)$, where f is a function such that $f'(x) = e^{-x^2} + \sqrt{3}$. (Your answer might involve the function f.)

Solution: Use the product rule:

$$h'(x) = -\left(f(x) + xf'(x)\right) = -\left(f(x) + x\left(e^{-x^2} + \sqrt{3}\right)\right) = -f(x) - xe^{-x^2} - \sqrt{3}\,x.$$

c. $h(t) = t + \sqrt{2} - cg(t)$, where g is a function such that $g'(x) = e^{x^2} - e^x$ and c is a constant. (Your answer might involve the function g.)

Solution:

$$h'(t) = 1 - cg'(t) = 1 - c\big(e^{t^2} - e^t\big) = 1 - ce^{t^2} + ce^t.$$

Note that $\sqrt{2}$ is a *constant*, so its derivative is zero. Also note that $1 - ce^{x^2} + ce^x$ is not correct. This expression is $h'(x)$, but the problem asked for $h'(t)$.

2. In each part, either draw the graph of a function satisfying the given conditions, or explain why no such function exists.

a. f is differentiable at -1 but not continuous at -1.

Solution: No such function exists, because if the function f is differentiable at a, then it is automatically continuous at a.

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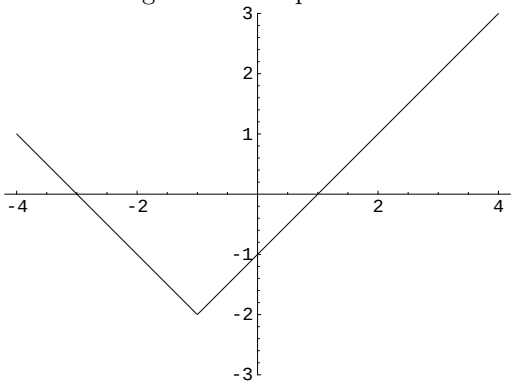
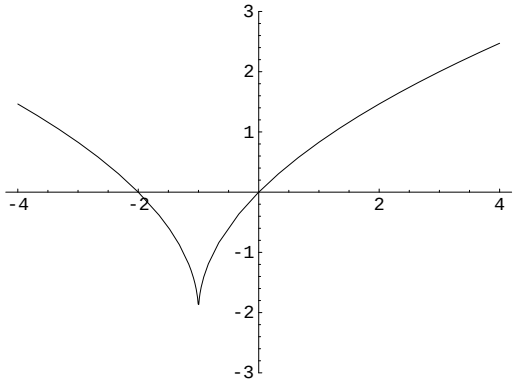
b. f is continuous at -1 but not differentiable at -1.

Solution: Here are the graphs of two possible solutions. The one on the left has a corner at $x = -1$, and the one on the right has a cusp at $x = -1$.

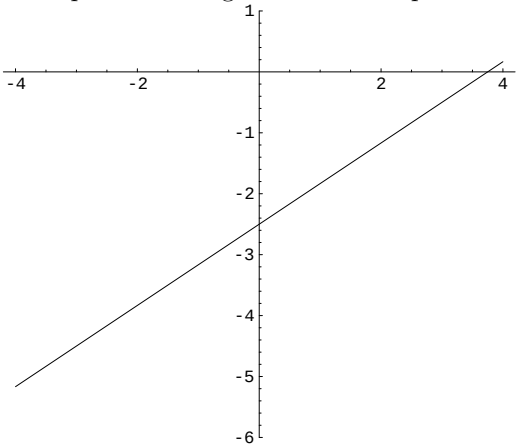


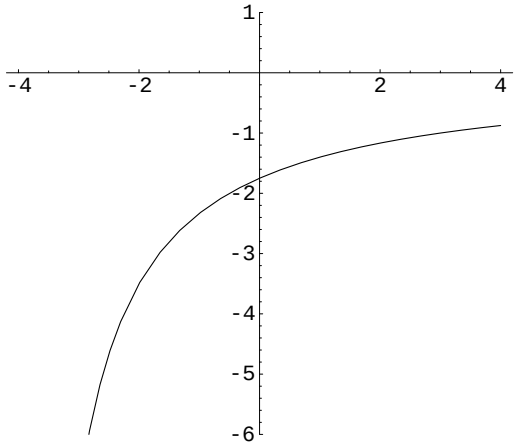


The function on the left is $f(x) = |x + 1| - 2$, and the function on the right is $f(x) = 2\sqrt{|x + 1|} - 2$. (Note, however, that the problem does not require you to give a formula for your function.)

c. $f(1) < 0$ and $f'(1) > 0$.

Solution: Here are the graphs of two possible solutions. In both of them, you can see that $f(1) < 0$, but that the slope of the tangent line at 1 is positive.

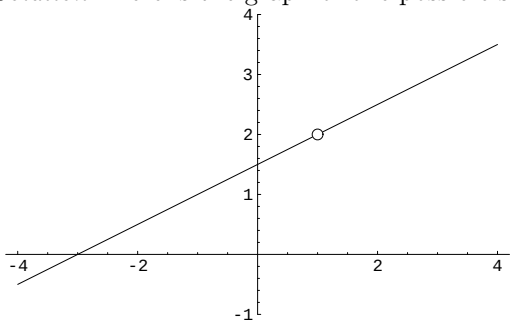




The function on the left is $f(x) = \frac{2}{3}x - \frac{5}{2}$, and the function on the right is $f(x) = -\frac{7}{x+4}$. (Note, however, that the problem does not require you to give a formula for your function.)

d. f is not defined at 1, but $\lim_{x\to 1} f(x)$ exists.

Solution: Here is the graph of one possible solution:



In this example, $\lim_{x\to 1} f(x) = 2$. The function is defined by $f(x) = \frac{1}{2}x + \frac{3}{2}$ for $x \neq 1$. (Note, however, that the problem does not require you to give a formula for your function.)