MATH 251 (Phillips), Winter 2008 NAME: NAME: SOLUTIONS

Graphical limit problems

Instructions: For each problem,

(1) Make your best guess as to what you think the limit should be. Answers might be $\pm\infty$ or that the limit does not exist. On the ones with no hints, write "?" if you truly have no idea.
(2) Use a graphing calculator or computer to try to graphically determine the limit as accurately as you can. Draw the graph in the space below the problem, being sure to give scales on the axes.
(3) Write the numerical answer from Part (2).

However, if you give a reason why your guess must be correct, and you get it right, you need not do Parts (2) and (3).

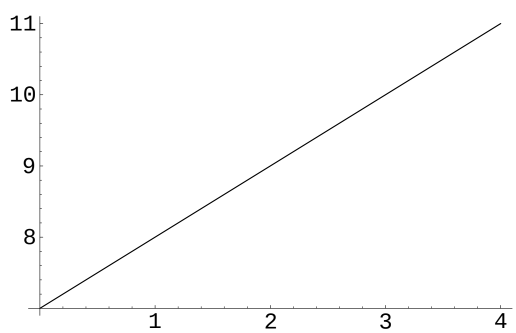
Notes on this solution sheet: I only expected you to provide one graph, but on most problems I have provided two: one 4 units wide on the horizontal scale, and one 0.02 units wide. The graphs were made using Mathematica, almost all using its automatic placement of axes and choice of scale. Usually neither axis is in the conventional location. The numerical answers are as I read them off the second graph. The number of decimal places shown is the accuracy I think I got from the graph. (Note that the last digit is not always correct.)

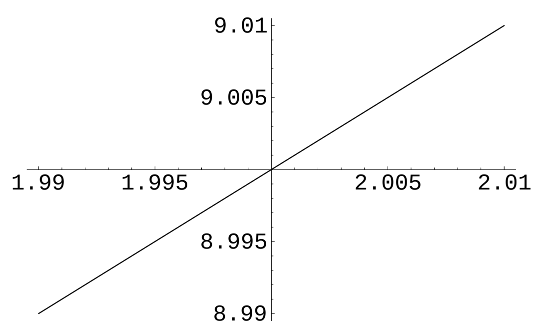
I don't necessarily expect people to have made guesses on problems with no hints.

1. $\lim_{x\to 2}(x+7)$. (For your guess, think about what $x+7$ will be when x is close to 2.)

My suggested guess: 9, because when x is close to 7 then $x+2$ should be close to 9.

Graphs:





Estimated value from graphs: 9.000

Comments: The correct limit is in fact 9.

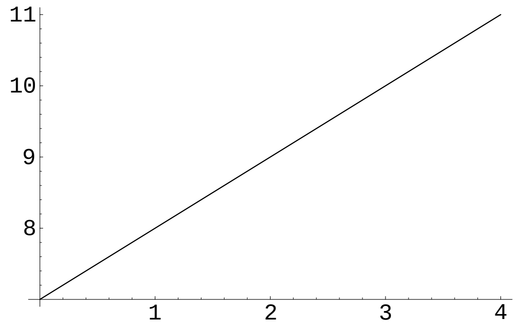
2. $\lim_{x\to 2}\dfrac{x^2+5x-14}{x-2}$. (For your guess, simplify the fraction and compare with $\lim_{x\to 2}(x+7)$.)

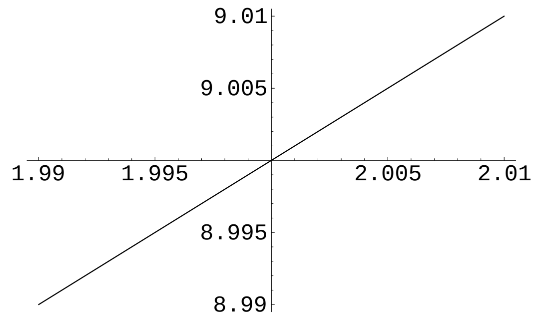
My suggested guess: 9, because of the simplification

$$\frac{x^2+5x-14}{x-2} = \frac{(x+7)(x-2)}{x-2} = x+7$$

for $x \neq 2$, and using $\lim_{x\to 2}(x+7) = 9$.

Graphs:





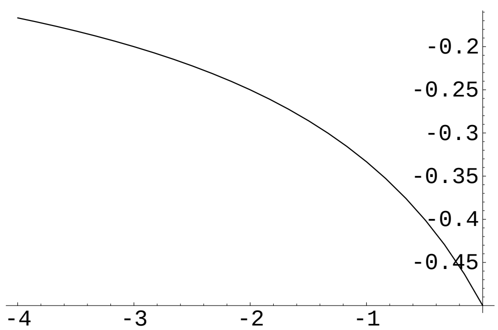
Estimated value from graphs: 9.000

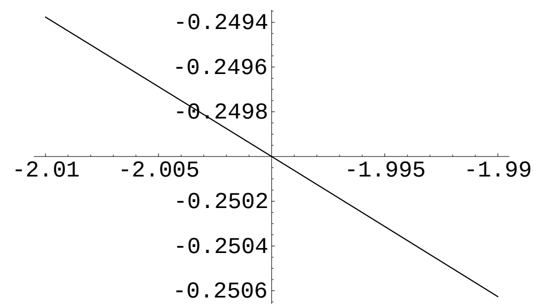
Comments: The correct limit is in fact 9.

3. $\lim_{x\to-2} \dfrac{1}{x-2}$ (For your guess, think about what $\dfrac{1}{x-2}$ will be when x is close to -2.)

My suggested guess: $-\frac{1}{4}$, because when x is close to -2 then $x-2$ should be close to -4, so $\dfrac{1}{x-2}$ should be close to $-\frac{1}{4}$.

Graphs:





Estimated value from graphs: -0.25000

Comments: The correct limit is in fact $-\frac{1}{4} = -0.25$.

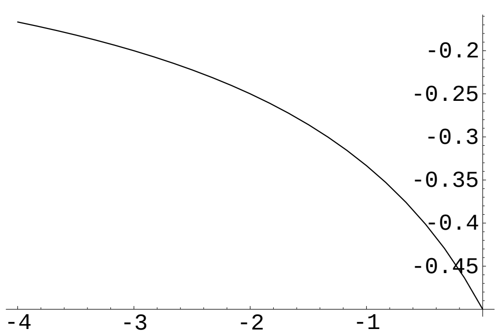
4. $\lim_{x\to-2} \dfrac{x+2}{x^2-4}$ (For your guess, simplify the fraction and compare with $\lim_{x\to-2} \dfrac{1}{x-2}$.)

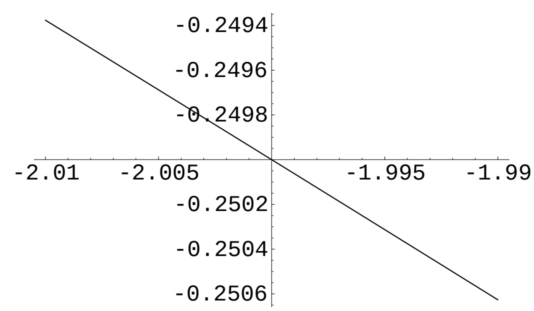
My suggested guess: $-\frac{1}{4}$, because of the simplification

$$\frac{x+2}{x^2-4} = \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$$

for $x \neq -2$, and using $\lim_{x\to-2} \dfrac{1}{x-2} = -\dfrac{1}{4}$.

Graphs:





Estimated value from graphs: -0.25000

Comments: The correct limit is in fact $-\frac{1}{4} = -0.25$.

5. $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$

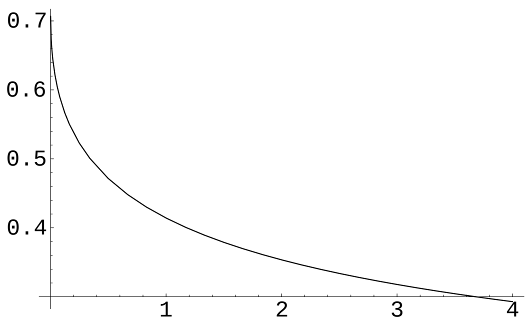
My suggested guess: This one is much harder. Here is the trick, which we will see soon, but which I don't expect you to have gotten on this assignment:

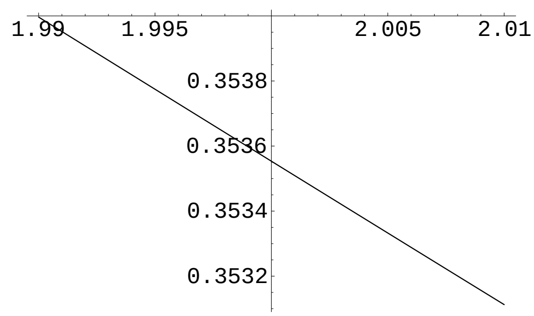
$$\frac{\sqrt{x} - \sqrt{2}}{x - 2} = \frac{\left(\sqrt{x} - \sqrt{2}\right)\left(\sqrt{x} + \sqrt{2}\right)}{(x - 2)\left(\sqrt{x} + \sqrt{2}\right)} = \frac{x - 2}{(x - 2)\left(\sqrt{x} - \sqrt{2}\right)} = \frac{1}{\sqrt{x} + \sqrt{2}}.$$

Now when x is close to 2, then $\sqrt{x}$ should be close to $\sqrt{2}$, so we expect the limit to be

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} = \lim_{x \to 2} \frac{1}{\sqrt{x} + \sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

Graphs:





Estimated value from graphs: 0.35356

Comments: The correct limit is in fact $\dfrac{1}{2\sqrt{2}} \approx 0.3535533906$.

6. $\lim_{x \to 2} \dfrac{x - 2}{\sqrt{x} - \sqrt{2}}$. (For your guess, compare with $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$.)

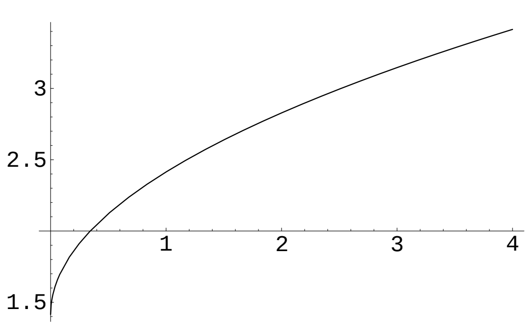
My suggested guess: Having gotten

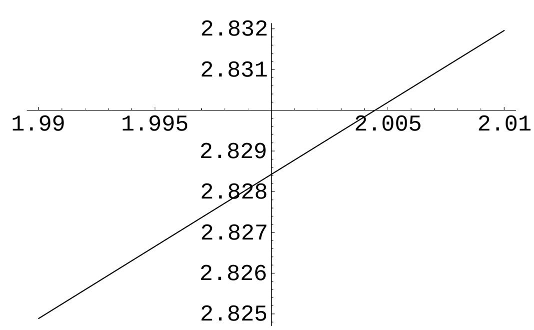
$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} \approx 0.35356,$$

we should expect

$$\lim_{x \to 2} \frac{x - 2}{\sqrt{x} - \sqrt{2}} = \frac{1}{\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}} \approx \frac{1}{0.35356} \approx 2.82824.$$

Graphs:





Estimated value from graphs: 2.8285

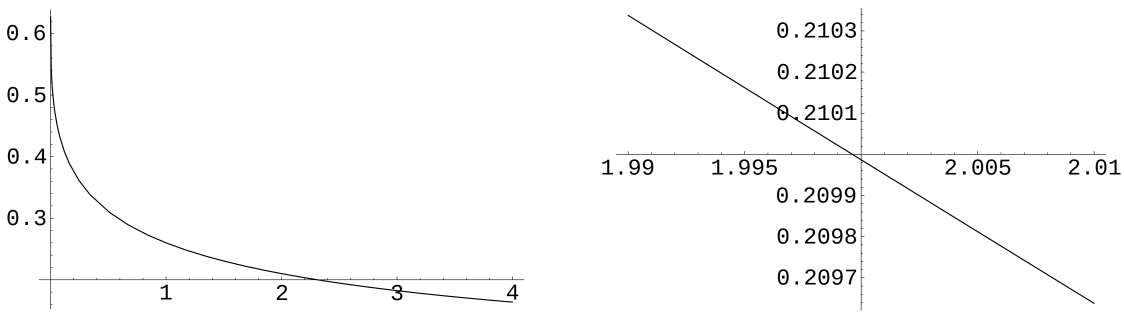
Comments: The correct limit is in fact

$$\frac{1}{\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{x-2}} = \frac{1}{1/(2\sqrt{2})} = 2\sqrt{2} \approx 2.8282427125.$$

7. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{x}-\sqrt[3]{2}}{x-2}$.

My suggested guess: This one is very hard. Here is an outline of the trick, which however I don't expect you to have gotten on this assignment. Multiply numerator and denominator by $x^{2/3}+2^{1/3}x^{1/3}+2^{2/3}$, simplify, and cancel $x-2$ from numerator and denominator. This gives you something which is defined when $x=2$. Substitute $x=2$ to get $\dfrac{1}{3\sqrt[3]{4}}$.

Graphs:



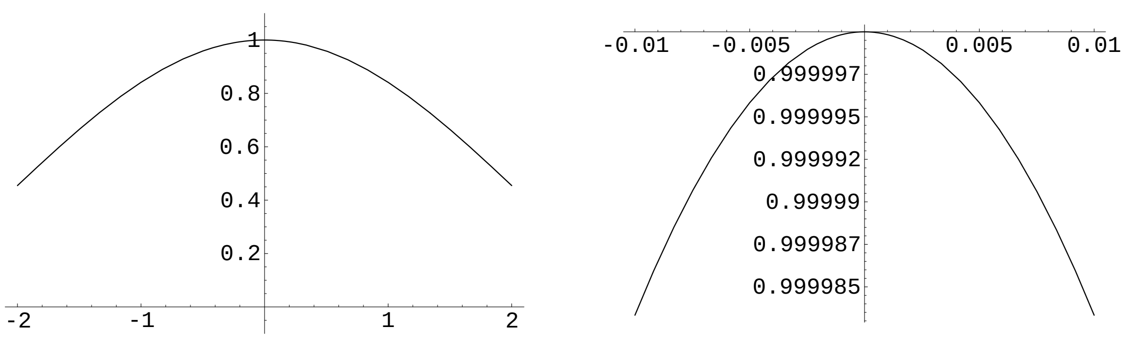
Estimated value from graphs: 0.020998

Comments: The correct limit is in fact $\dfrac{1}{3\sqrt[3]{4}} = \dfrac{1}{3\cdot 2^{2/3}} \approx 0.2099868416.$

8. $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$

My suggested guess: ?

Graphs:



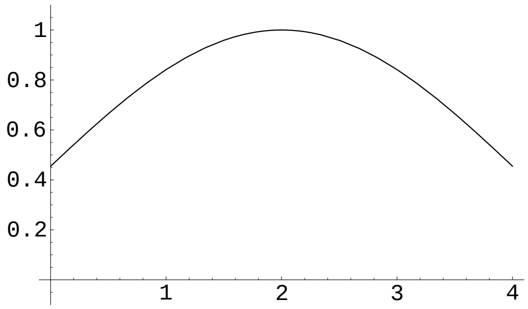
Estimated value from graphs: 1.000000.

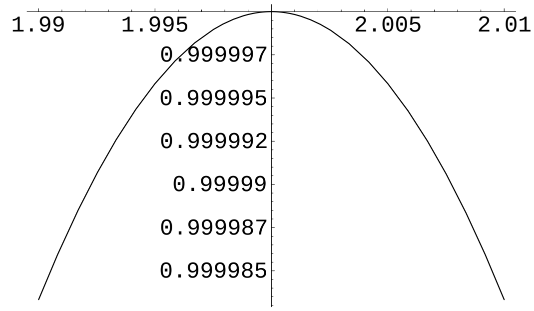
Comments: The correct limit is in fact 1.

9. $\lim\limits_{x\to 2} \dfrac{\sin(x-2)}{x-2}$. (For your guess, compare with $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$.)

My suggested guess: 1. We have found that $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x}$ appears to be equal to 1. Now $\dfrac{\sin(x-2)}{x-2}$ for x close to 2 is the same as $\dfrac{\sin(x)}{x}$ for x close to 0. (Set $t=x-2$. Then $\dfrac{\sin(x-2)}{x-2} = \dfrac{\sin(t)}{t}$, and having x close to 2 is the same as having $x-2$ close to 0, that is, having t close to 0.)

Graphs:





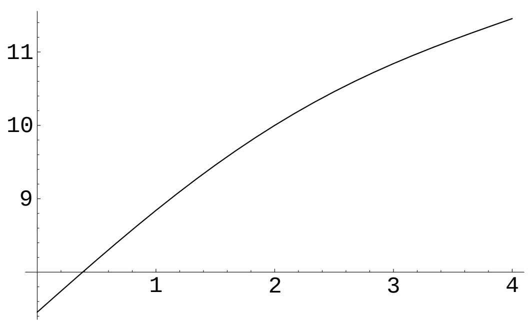
Estimated value from graphs: 1.000000.

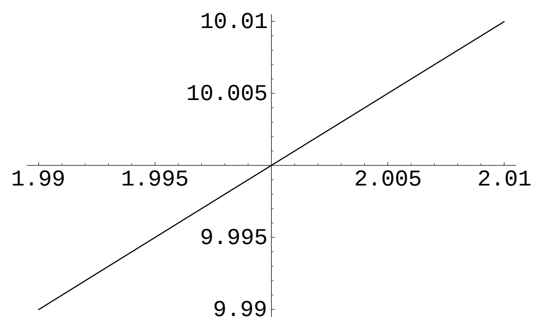
Comments: The correct limit is in fact 1. Please compare the graphs with those in Problem 8.

10. $\lim_{x\to 2}\left(\dfrac{x^2+5x-14}{x-2}+\dfrac{\sin(x-2)}{x-2}\right)$. (For your guess, compare with $\lim_{x\to 2}\dfrac{x^2+5x-14}{x-2}$ and $\lim_{x\to 2}\dfrac{\sin(x-2)}{x-2}$.)

My suggested guess: $9+1=10$. In previous problems, we saw that when x is close to 2, then $\dfrac{x^2+5x-14}{x-2}$ is close to 9 and $\dfrac{\sin(x-2)}{x-2}$ is close to 1, so their sum should be close to 10.

Graphs:





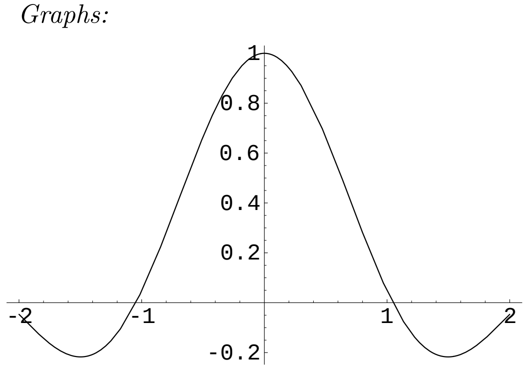
Estimated value from graphs: 10.000

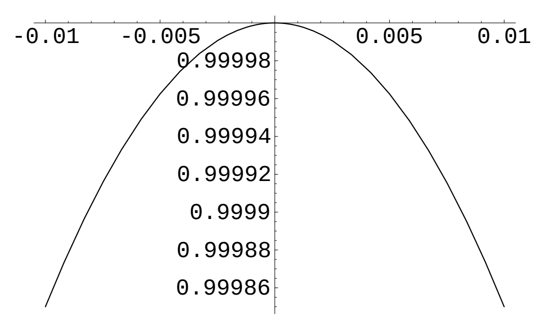
Comments: The correct limit is in fact 10.

11. $\lim_{x\to 0}\dfrac{\sin(3x)}{3x}$. (For your guess, compare with $\lim_{x\to 0}\dfrac{\sin(x)}{x}$.)

My suggested guess: 1. We have found that $\lim_{x\to 0}\dfrac{\sin(x)}{x}$ appears to be equal to 1. Now $\dfrac{\sin(3x)}{3x}$ for x close to 0 is the same as $\dfrac{\sin(x}{x}$ for x close to 0. (Set $t=3x$. Then $\dfrac{\sin(3x)}{3x}=\dfrac{\sin(t)}{t}$, and having x close to 0 is the same as having $3x$ close to 0, even though not quite as close, that is, having t close to 0.)

Graphs:

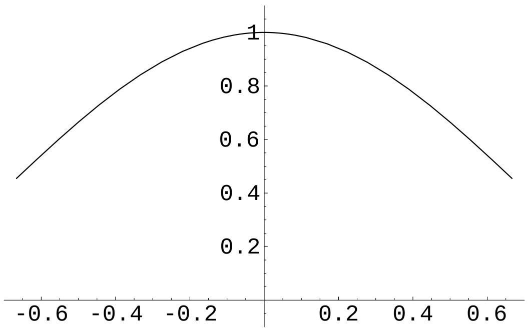


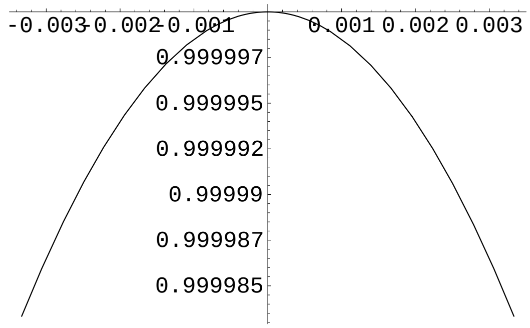


Estimated value from graphs: 1.000000.

Comments: The correct limit is in fact 1.

Here are graphs at slightly different scales; please compare them with those in Problems 8 and 9:

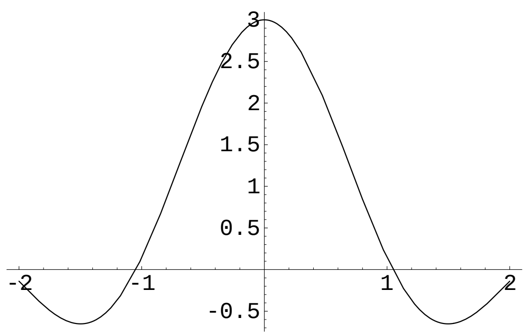


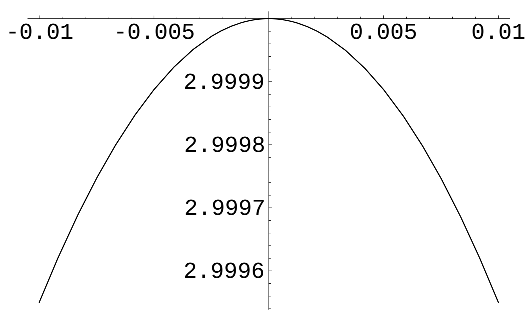


12. $\lim_{x\to 0} \dfrac{\sin(3x)}{x}$. (For your guess, compare with $\lim_{x\to 0} \dfrac{\sin(3x)}{3x}$.)

My suggested guess: $3 \cdot \lim_{x\to 0} \dfrac{\sin(3x)}{3x} = 3$.

Graphs:





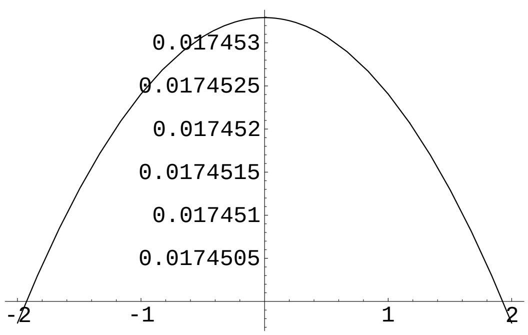
Estimated value from graphs: 3.00000.

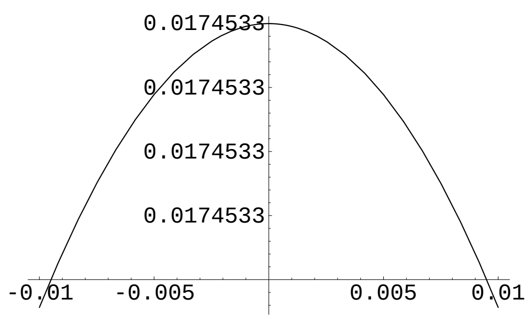
Comments: The correct limit is in fact 3.

13. $\lim_{x\to 0} \dfrac{S(x)}{x}$, where $S(x)$ is the sine of the angle x *degrees*. (Set your calculator in degree mode, but be sure to set it back to radians after doing this problem!)

My suggested guess: Since $S(x) = \left(\dfrac{\pi x}{180}\right)$, I will follow the method for $\lim_{x\to 0} \dfrac{\sin(3x)}{x}$ and guess $\dfrac{\pi}{180}$.

Graphs (sorry about the bad vertical scale on the second one):





Estimated value from graphs: 0.0174533.

Comments: The correct limit is in fact $\dfrac{\pi}{180} \approx 0.01745329252$.