Math 251 Final Exam NAME:____________________
Wednesday 19 March 2008 Student ID:____________________

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. Closed book, except for a graphing calculator and a 3×5 file card. Note: you may not use the calculator as a substitute for calculus. Calculators able to calculate derivatives or do other symbolic computations are prohibited.
3. No cell phones, laptops, or any other electronic device other than the graphing calculator referred to above.
4. The point values are as indicated in each problem; total 200 points.
5. Write all answers on the test paper. Use the extra sheet at the back for long answers or scratch work.
6. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
7. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
8. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.
9. Final answers must always be simplified (just as in homework), unless otherwise specified.
10. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
11. Time: 120 minutes.

DO NOT WRITE BELOW THIS LINE

1	2	3	4	5	6	7	8	TOTAL	EC
1	36	14	44	20	20	30	35	200	

1. (1 point) What do you think math professors do on spring break?

2. (12 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give reasons in all cases.

(a) $\lim_{x \to 0} \dfrac{\cos(3x) - 1}{7x^2}$.

(b) $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 - 5x - 6}$.

(c) $\lim_{x \to \infty} \dfrac{7x + 2}{\sqrt{2x^2 + 7189}}$.

3. (14 points) If $\sin(xy) = x + y + \ln(2)$, find $\dfrac{dy}{dx}$. (Use implicit differentiation. You must solve for $\dfrac{dy}{dx}$.)

4. (44 points) Professor Malvixx wants to build a walled rectangular enclosure. One side will be part of a long already existing wall which runs east-west, leaving three walls to build. The enclosure is to be divided in half by a fourth wall which is perpendicular to the existing wall. (She wants to put a Crumple-Horned Snorkack in one half, and a Spiral-Horned Snorkack in the other half.) If the total area is to be 1200 square meters, what is the shortest possible total length of wall needed to build such an enclosure?

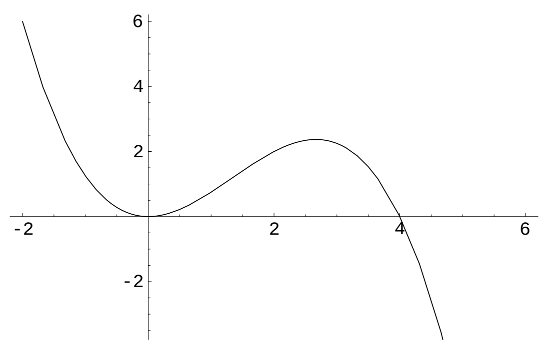
Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

5. (10 points/part)

(a) Let h be a function such that $h'(x) = \dfrac{\sin(x)}{x} + x$. Find the derivative of the function $f(x) = h(e^x) + \pi^2$. (Your answer might involve the function h.)

(b) Let c be a constant. Let $g(x) = x\tan(cx^2 + x)$. Find $g'(x)$.

6. (5 points/part) The picture below shows the graph of $y = f(x)$ for a particular function f. (This is a graph of the function, *not* its derivative.)



For each part, use this graph to find at least one number x in the interval shown satisfying the stated conditions, or explain why no such number x exists.

(a) $f(x) > 0$ and $f'(x) < 0$.

(b) $f'(x) < 0$ and $f''(x) > 0$.

(c) $x < 0$ and $f''(x) < 0$.

(d) $f(x) = 0$, $f'(x) = 0$, and $f''(x) \geq 0$.

7. (30 points) A spherical balloon was being slowly blown up, starting at noon one day. At 12:10 pm, its radius was 20 inches, and was increasing at 5 inches per minute. Was its surface area increasing or decreasing? At what rate? (Be sure to include the correct units in your answer.)

8. (35 points) Let $g(x) = -\frac{1}{7}x^7 - \frac{17}{4}x^6 - \frac{433}{10}x^5 - 153x^4$. Use your calculator to produce graphs (more than one, if necessary) of $y = g(x)$ which reveal all the important features of the function. In particular, estimate the intervals of increase and decrease, critical numbers, extreme values, intervals of concavity, and inflection points, either using graphs of the first and second derivatives of the function, or directly from the formulas for these derivatives. (Your graphs must be shown on the test paper.)

Hint: Here are the first and second derivatives in partially factored form:

$$g'(x) = -x^3(x+8)(x+9)\left(x+\frac{17}{2}\right) \quad \text{and} \quad g''(x) = -\frac{1}{2}x^2(12x^3+255x^2+1732x+3672).$$

The roots of $12x^3 + 255x^2 + 1732x + 3672 = 0$ are approximately -4.23039, approximately -8.22163, and approximately -8.79798.

(Extra credit on next page.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.) Write answers in the space after all the extra credit problems, or on the backs of the pages.

EC. Use the methods of calculus to prove the following statements:

(a) (2 extra credit points) $e^x > 1$ for all $x > 0$.

(b) (5 extra credit points) $e^x > 1 + x$ for all $x > 0$.

(c) (8 extra credit points) $e^x > 1 + x + \frac{1}{2}x^2$ for all $x > 0$.

(d) (25 extra credit points) For every positive integer n,

$$e^x > 1 + x + \frac{x^2}{2} + \cdots + \frac{x^n}{n!}$$

for all $x > 0$.