

**MATH 243 (PHILLIPS, SPRING 2008): MIDTERM 1 (GREEN)  
SOLUTIONS**

Multiple choice: 3 points/part; 21 points total. Circle the letter of the best answer.

MC1. A researcher wants to study the relationship between color and a certain genetic mutation in Rocky Mountain spotted frogs. He chooses a stream which looks like good frog habitat and collects for study all the frogs he can net on this stream. The frogs he collected form:

- a. A voluntary response sample.
- a. A convenience sample.
- c. A stratified random sample.
- d. A simple random sample.
- e. A population.

MC2. As part of an investigation of the mean Math SAT score of University of Oregon students, the Registrar's office selects 100 students from the University of Oregon, and looks up their Math SAT scores in their application materials. What is the population?

- a. The Math SAT scores of the 100 selected students.
- b. The Math SAT scores of all University of Oregon students.
- c. Everybody who took the Math SAT.
- d. All University of Oregon students.
- e. The 100 selected students.
- f. Impossible to tell from the information given.
- g. None of the above.

MC3. The town of Hicksville contains 20 tax lots. The mean of their assessed values is \$80,000, the median is \$60,000, the standard deviation of the assessed values is \$35,000, and the range of the assessed values is \$200,000. The total assessed value of taxable property in Hicksville is:

- a. \$700,000
- b. \$1,200,000
- c. \$1,600,000
- d. \$4,000,000
- e. Impossible to tell from the information given.
- f. None of the above.

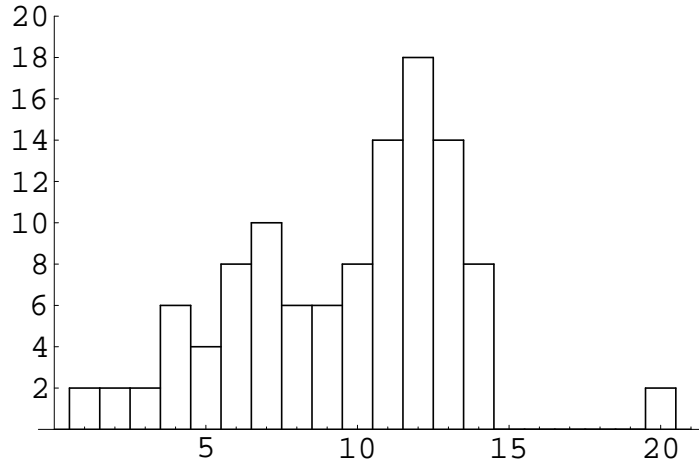
(Multiply the mean by the number of tax lots.)

MC4. A magician has a coin which is biased in such a way that the probability of getting heads when the coin is tossed is  $1/3$ . Which of the following statement gives a valid interpretation of this probability?

- a. Every third toss of this coin will be heads.

- b. In any three consecutive tosses of this coin, exactly one will be heads.
- c. In any three consecutive tosses of this coin, at least one will be heads.
- d. If the coin is tossed a very large number of times, the proportion of the tosses which are heads will be very close to  $\frac{1}{2}$ .
- e. If the coin is tossed a very large number of times, the proportion of the tosses which are heads will be very close to  $\frac{1}{3}$ .
- f. The magician is intending to cheat in a game involving coin tosses.

MC5. Consider the following histogram (made from integer data):



The distribution of the data plotted is:

- a. Roughly symmetric with no outliers.
- b. Roughly symmetric with one or more outliers.
- c. Skewed to the right with no outliers.
- d. Skewed to the right with one or more outliers.
- e. Skewed to the left with no outliers.
- f. Skewed to the left with one or more outliers.

It is skewed in the direction of the long tail.

MC6. Let  $\mu$  be the mean daily number of spam messages Prof. Zhang receives this year. Prof. Zhang counts the spam each day for a period of several weeks. Assume the days on which he counts can be treated as a simple random sample of all days in this year. The following 90% confidence interval for the mean  $\mu$  is computed:  $45 \leq \mu \leq 76$ . Which of the following statements gives a valid interpretation of this interval?

- a. On 90% of the days in the sample, Prof. Zhang received between 45 and 76 spam email messages.
- b. On 90% of the days this year, Prof. Zhang received between 45 and 76 spam email messages.
- c. If the procedure were repeated many times, approximately 90% of the resulting confidence intervals would contain the mean daily number of spam messages Prof. Zhang receives in the days in the sample.

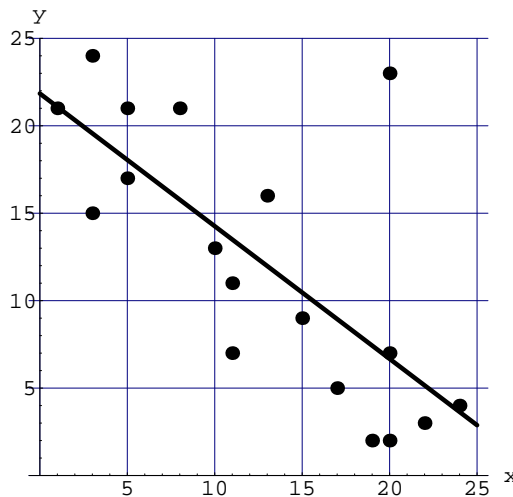
- d. If the procedure were repeated many times, approximately 90% of the resulting confidence intervals would contain the mean daily number of spam messages Prof. Zhang receives this year.
- e. 90% of all email Prof. Zhang receives is spam.
- f. Prof. Zhang should stop using email.

Choices (a) and (b) are not related to the meaning of a confidence interval. If a large sample is used, the confidence interval will contain much less than 90% of the data. Choice (c) is incorrect because *every* confidence interval, not just 90% of them, contains the mean of the sample it was derived from.

MC7. Two variables in a study are said to be confounded if what?

- a. The statistician conducting the study is incompetent.
- b. One cannot separate their effects on a response variable.
- c. They are highly correlated.
- d. They do not have a normal distribution.
- e. One of them is a placebo.
- f. Both are lurking variables.

1. (4 points/part; total 12 points.) Consider the following scatterplot and regression line.



a. What is the approximate correlation between the variables? Circle one.

2.00      1.00      0.70      0       -0.70      - 1.00      - 2.00

*Solution:* The correlation is clearly negative and not close to zero, but the points are not so close to the regression line that it can be close to  $-1$  either. (The points plotted actually all have integer coordinates, and the correlation is  $-0.721937$ , giving  $r^2 \approx 0.521193$ .)

b. Give the approximate coordinates of any outliers. If there are none, write "NONE".

*Solution:* (20, 23). (Note that (23, 20) is not correct.)

c. The residual for the observation with approximate coordinates  $(17, 5)$  is closest to: (circle one)

−12      − 8       −4      0      4      8      12

*Solution:* The corresponding point on the regression line is about  $(17, 9)$ , so the residual, which is the observed value minus the predicted value, is about  $5 - 9 = -4$ .

2. (2 points/part; 10 points total.) For a biology project, a student measures the tail length (in inches) and the weight (in ounces) of each of 12 mice of the same variety. What units of measurement do each of the following have? (Write “none” if appropriate.)

a. The median of the weights.

*Solution:* Ounces.

b. The standard deviation of the lengths of the tails.

*Solution:* Inches.

c. The  $z$ -score of the weight of the second mouse.

*Solution:* No units. (We also accept “standard deviations”.)

d. The correlation between tail length and weight.

*Solution:* No units.

e. The damages awarded in a lawsuit after the student was bitten by a mouse with rabies.

*Solution:* My best guess is millions of dollars.

3. (3 points/part; 18 points total. Work need not be shown.) The five number summary of the final exam scores in Professor Zhang’s Math 315 is:

4   45   60   75   98.

The questions below are about the original data, which consists of one number for each of the 40 students in the class. For each of the following statements, circle “A” if the statement must be true, circle “S” if the statement might or might not be true, and circle “N” if the statement cannot be true. In other words, under the stated circumstances the statement is **A**lways true, **S**ometimes true, or **N**ever true.

A   S   N   (a) The median score is 60.

By definition, the middle number of the five number summary is the median.

A   S    N   (b) About 30 students had scores of 75 or better.

At least half the students had scores of 60 or worse.

A   S    N   (c) The standard deviation of the scores is  $-15$ .

Standard deviations are never negative.

A S N (d) At least one student received a score of 60.

Since the number of scores is even, the median is half way between the two middle scores. Maybe the 20th best score was 62 and the next one down was 58.

A S N (e) The score 4 is an outlier.

The  $1.5 \cdot \text{IQR}$  rule is only a test for *suspected* outliers.

A S N (f) If Professor Zhang multiplies all the scores by 2 (since the final exam counts twice as much as the midterm), the new first quartile will be 90.

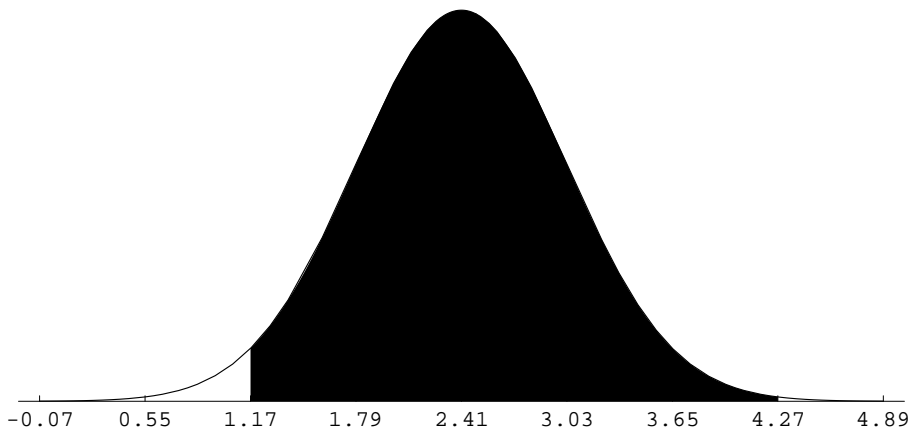
Doubling all the scores also doubles all the numbers in the five number summary.

4. (4 points/part; total 8 points.) The weights of zucchinis harvested in a certain California valley are approximately normally distributed with mean 2.41 pounds and standard deviation 0.62 pounds. A truckload of zucchinis is taken to market.

In each of the following parts, show your work, and draw a picture of the appropriate normal curve with the relevant points on the horizontal axis clearly marked and with the appropriate area shaded and clearly identified. The curve must look reasonable.

a. Approximately what percentage of the zucchinis weigh between 1.17 and 4.27 pounds? (You must make your method clear and show 4 significant digits.)

*Solution:* Here is the picture:



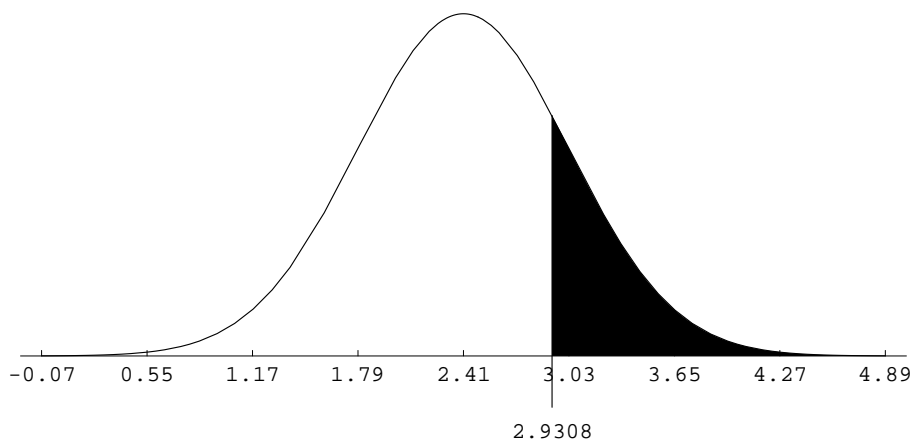
The labels on the horizontal axis are integer multiples of the standard deviation away from the mean. The shaded area corresponds to weights between 1.17 and 4.27 pounds. If you look closely, you will see that it stops at 4.27. The rule of thumb is applicable, because 1.17 is two standard deviations below the mean and 4.27 is three standard deviations above the mean. Accordingly, the unshaded area is about  $\frac{1}{2}(1-0.95) + \frac{1}{2}(1-0.997) = 0.0265$ , and the desired area is about  $1-0.0265 = 0.9735$ , or 97.35%.

If you use Table A instead, look up  $z = 3$  to get about 0.9987 for the area to the left of 4.27, and look up  $z = -2$  to get about 0.0228 for the area to the left of 1.17. Subtract to get about 97.59%.

b. Approximately 20% of the zucchinis weigh more than \_\_\_\_\_ pounds.

*Solution:* I want the number  $x$  of pounds such that 80% of the zucchinis weigh less than  $x$  pounds. The closest number to 0.80 which occurs in Table A is 0.7995, for  $z = 0.84$ . So  $x \approx 2.41 + (0.62)(0.84) = 2.9308$ .

Here is the picture. The shaded area corresponds to weights above 2.9308 pounds, and has area about 0.2.

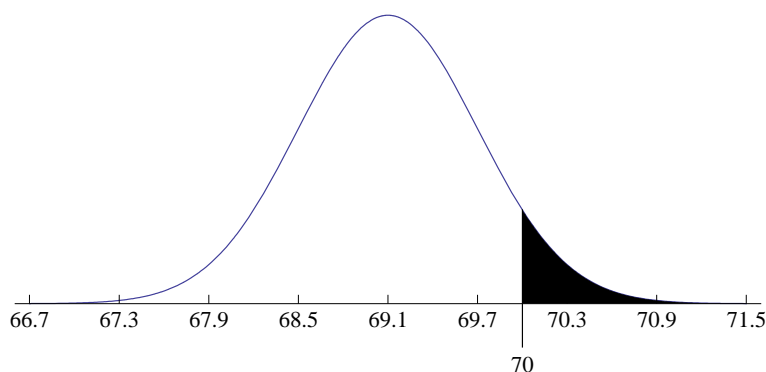


5. (7 points/part) Repeated measurements of the concentration of helium in a gas with a particular scientific instrument vary normally with standard deviation 1.8 milligrams/liter (mg/l) and with mean equal to the true concentration.

a. The true concentration of helium in one gas is 69.1 mg/l. If the concentration is measured 9 times with this instrument, what is the probability that the mean of the measurements is at least 70 mg/l? Illustrate your answer with a graph in which the appropriate areas are labelled.

*Solution:* The sampling distribution is normal with mean 69.1 mg/l (the same as the mean of the original distribution), and standard deviation  $1.8/\sqrt{9} = 0.6$  mg/l. To use Table A, compute the  $z$ -score of 70 mg/l in the sampling distribution:  $z = (70 - 69.1)/0.6 = 1.5$ . Look up in Table A to get about 0.9332. Since we want the probability that  $\bar{x} \geq 70$ , the correct answer is about  $1 - 0.9332 = 0.0668$ . (My computer gave about 0.0668072.)

Here is the graph. The labels are at integer numbers of sampling standard deviations above and below the mean. The shaded area represents the required probability.



b. For another gas, four measurements of the concentration of helium with this instrument gave  $\bar{x} = 51$  mg/l. Construct a 90% confidence interval for the true concentration of helium in this gas. Be sure to show the critical value you used, and how you used it.

*Solution:* The population standard deviation is known, so we use the one sample  $z$  procedure. The confidence interval is  $\bar{x} \pm z^* \sigma / \sqrt{n}$ . We have  $\bar{x} = 51$ ,  $\sigma = 1.8$ ,  $n = 4$ , and the appropriate  $z^*$  for 90% confidence is 1.645 (from Table C). So the confidence interval is

$$51 \pm (1.645)(1.8)/\sqrt{4} = 51 \pm 1.4805,$$

in mg/l.

The confidence interval can also be rewritten as (49.5195, 52.4805), in mg/l.

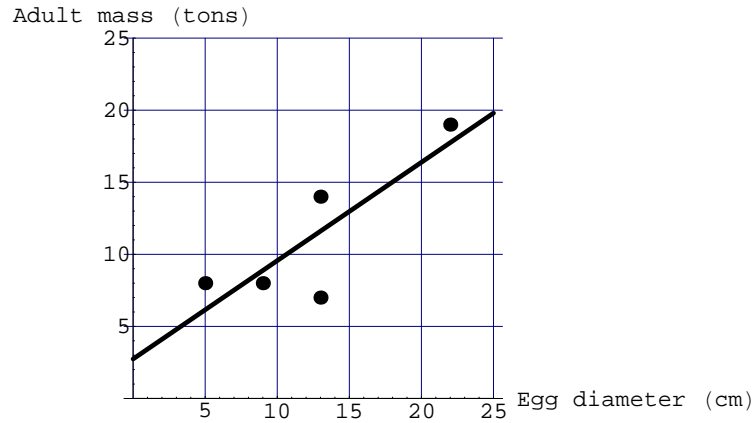
6. (5 points.) In 2002, Jane Smith got a score of 590 on the mathematics part of the SAT, and John Doe got a score of 24 on the ACT Assessment mathematics test. SAT math scores in 2002 were normally distributed with mean 516 and standard deviation 114, and ACT math scores in 2002 were normally distributed with mean 20.6 and standard deviation 5.0. Find the standardized scores for both students. Assuming both tests measure the same kind of ability, who did better?

*Solution:* Jane Smith's  $z$ -score is  $z_1 = (590 - 516)/114 \approx 0.649123$ . John Doe's  $z$ -score is  $z_2 = (24 - 20.6)/5.0 = 0.68$ . Since  $0.649123 < 0.68$ , John Doe is a larger number of standard deviations above the mean, and thus did better.

7. (4 points/part; total 12 points.) A scientist studying the fire-breathing monsters of the planet Yuggxth wants to know if egg size is a good predictor of adult mass for these creatures. At considerable personal risk, he has managed to obtain the following data on five individuals.

Individual	Egg diameter (cm)	Adult mass (tons)
A	22	19
B	9	8
C	13	7
D	5	8
E	13	14

a. Draw a scatterplot on the axes provided. Be sure to label your axes, and make an appropriate choice of which variable to put on the horizontal axis.



b. Find the equation of the least squares regression line and plot it on the graph above.

*Solution:* My computer gave  $\hat{y} \approx 2.74121 + 0.682161x$ . See the graph above for the plot.

c. What is the correlation between egg diameter and adult mass? What percentage of the variation in adult mass is explained by egg diameter?

*Solution:* My computer gave  $r \approx 0.832861$ . So  $r^2 \approx 0.693658$ , which means that about 69.37% of the variation in adult mass is explained by egg diameter.