Chapter 3

Sudoku: A Puzzle

In this book, we consider a puzzle to be a type of game. A puzzle is problem designed to challenge one’s brain and to be entertaining. Many people spend part of almost every day working on crossword puzzles, Bridge or chess puzzles, number or word puzzles, and the other types of puzzles printed in daily newspapers and in a variety of magazines. They enjoy the challenge and the feelings of success as they solve the problem or accomplish the task presented by the puzzle. You can learn about a number of different puzzles at http://en.wikipedia.org/wiki/Puzzle.

Note to Teachers: My belief is that every person is a teacher. Some do it as a profession, while others do it merely as an everyday part of their lives. I am a teacher who writes books. One of my teaching strategies is to try to get the reader to take an active part in their own learning. The previous paragraph provides an example of this. Why should I spend my writing time and effort trying to duplicate the good work that someone has already done and made available free in the Wikipedia? (Perhaps you are not familiar with the Wikipedia. It is a free encyclopedia where all of the entries have been contributed for free use, and readers can edit the entries.) Moreover, suppose you click on the link and begin to read about puzzles. There is a good chance you will find some information that seems particularly interesting to you, and you will follow up on it. Your learning will be driven by intrinsic motivation. You will be learning because you want to learn. Great!

A Game Without an Opponent

Chapter 1 contains a discussion of competition, independence, and cooperation. Most puzzles fall into the middle category; they are neither competitive not cooperative. Of course, if you like to take a competitive view of almost everything, you can think of a puzzle as a game in which you are competing against yourself. You are trying to solve a challenging problem or accomplish a challenging task. Typically, you are doing this for fun—because you want to. You ask yourself question such as:

• Do I have the knowledge, skills, and persistence to solve this specific puzzle? (For example, perhaps you are looking at a crossword puzzle. Some are much more difficult than others.)

• Am I enjoying spending time solving this puzzle? (Perhaps you are looking at a Rubric’s Cube. From previous experience, you know that you get little or no enjoyment in trying to solve such spatial puzzles.)

• Am I getting better at solving this type of puzzle? (If you do jigsaw puzzles or crossword puzzlers frequently, you will get better at doing such puzzles.)

• How good am I (in solving this type of puzzle) relative to other people?
• Am I learning anything by solving this puzzle. (Perhaps you wonder if this brain exercise is good for your brain.)

• Why am I spending so much time “playing” with the puzzle, when I could be doing other, more productive, work. Puzzles, like other types of games, can be addictive. Am I addicted?

Introduction to Sudoku

In the remainder of this chapter, the Sudoku puzzle is used to illustrate various aspects of learning to solve a puzzle and increasing one’s level of expertise in solving a puzzle. Figure 2.1 illustrates the playing board. The coordinate system is similar to that used in chess. It helps us to communicate precisely about the location of each of the 81 spaces on the board. Notice that the board is divided into nine 3x3 regions, numbered 1 through 9.

Figure 2.1. Sudoku board grid and nine regions

Figure 2.2 illustrates an actual puzzle.

Figure 2.2 An example of a Sudoku puzzle.

A specific puzzle is specified by the set of givens entered onto the board, as illustrated in Figure 2.2. The goal (the problem) is to enter a numerical digit from 1 through 9 in each empty space of the 9x9 grid so that:

• Each of the nine regions region contains all of the digits 1 through 9.
• Each horizontal row and each vertical column contains all of the digits 1 through 9.
The rules or goal of this puzzle are very simple. Solving the puzzle does not depend on having knowledge of math or any other subject. Indeed, the puzzle might just as well make use of nine different letters from the alphabet or nine different geometric shapes. Sudoku is not a math or a word puzzle.

**A 4x4 Example and a High-Road Transferable Strategy**

In this chapter, we will explore the 9x9 Sudoku puzzle. However, there are 4x4, 16x16, and other variations on this puzzle.

Just for fun, try solving the two 4x4 Sudoku puzzles given in Figure 2.3. These two puzzles are the same, except that one uses digits and one uses letters. Notice that it is assumed that you can make up a correct goal (an appropriate set of rules) for these puzzles. That is, without any help from your author, you can transfer the rules of this game from a 9x9 board to a 4x4 board.

![Figure 2.3. Two identical 4x4 Sudoku puzzles, one using digits, one using letters.](image)

The chances are that you will decide that the 4x4 Sudoku puzzle is too simple to be much of a challenge for you. However, it might well be a challenge for young children.

In addition, it illustrates a very important aspect in problem solving. If a particular problem seems too difficult for you, try to create a simpler version of the problem or create a closely related problem that is not as difficult. The process of creating and solving a simpler version or a related problem may well give you insights that will help you to solve the more complex problem.

Throughout this chapter we will be looking for general strategies for problem solving that are applicable over a wide range of problems. The goal is to have you add each of these to your repertoire of high-road transferable problem-solving strategies. By the time you finish reading this chapter, you may well have significantly improved your general problem-solving skills. Moreover, you may well have developed some teaching strategies that will be very valuable to your students.

Let’s name our newly discovered strategy the *create a simpler problem strategy*. The strategy has several purposes. It may help you to better understand the original problem. Solving the simpler problem may help you gain insights that will help you solve the more complex problem. If your simpler problem is carefully chosen, solving it will contribute to solving your original problem.

To add *create a simpler problem* to your repertoire of high-road transfer strategies, you must identify and consciously explore a number of examples that are meaningful to you. High-road transfer involves identifying a number of examples that are meaningful to you.
This requires reflective thinking. Here is a personal example. When I write a book—such as this one—I am not able to just sit down and write the whole book in a linear fashion. Indeed, I cannot even produce an outline that stands a decent chance of actually fitting the final product. To get started, I set myself a much simpler problem. I use a word processor to record my ideas as I brainstorm possible goals, audience, and content for the book.

I then set myself the problem of ordering my brainstormed set of ideas into a somewhat logical, coherent order. During this process, I throw out some ideas and add some new ideas.

I then set myself another simple problem—to develop a short summary and a set of references for some of the topics that seem particularly important. I can solve this problem off the top of my head and by use of the Web. In the process of solving it, I get some new ideas to add to my original brainstormed list. I may well rearrange the order of the brainstormed list, and I may well throw out some of the items in the list.

Okay, now it’s up to you. As you explore your own examples, think carefully about how you will help your students to learn this strategy. Make up some examples of the sorts that may be particularly relevant to them. Think about how you will help them to find personal examples. Think about how the sharing of such personal examples in class may help all members of the class find additional personal examples.

Metacognition

The next two sections are diversions, seemingly leading us away from solving the 9 x 9 Sudoku puzzle of Figure 2.2. However, we will return to this puzzle after the diversions.

A puzzle provides a situated learning environment. While some puzzles require considerable knowledge from outside the puzzle environment, others require very little outside knowledge. The Sudoku puzzle requires the player to be able to recognize and distinguish between each of nine different symbols. However, it does not depend on being able to read or to do math.

Even before we begin studying the Sudoku puzzle in some detail, you can do some introspection or metacognition (thinking about your thinking) as you are first faced by this problem-solving puzzle situation. Here are some questions that might help you learn more about yourself:

1. What are your personal feelings and thoughts as you first encounter a puzzle—especially, a puzzle of a type that you have not previously attempted to solve?
2. For you, personally, do you think digits, letters, or geometric shapes would be easiest for you in a Sudoku puzzle? Why?
3. Think about some non-Sudoku puzzle that you have solved or attempted to solve in the past. Was this an enjoyable experience? Did you develop a reasonable level of expertise with this puzzle? How much time and effort did it take you to develop your current level of expertise with this puzzle? Do you feel you are close to your upper limit in how good you can get in solving this type of puzzle?

The metacognitive questions given above are all stated in the context or situation of learning to solve a type of puzzle. However, they are applicable to learning how to solve problems in any
discipline. That is, the questions represent a set of ideas that are applicable as one studies problem solving in any new discipline.

This is a very important idea. For many people, recreational puzzles represent a relatively non-threatening learning environment. Within this environment, you can learn about yourself as a learner. You can see yourself making learning gains, moving from an absolute novice to a person with an appreciable level of skill. In many puzzle-solving situations, you can see appreciable gains in expertise over a relatively short time.

Metacognition is an important aid to learning to solve problems in any discipline. It can be called the metacognition strategy for learning to solve problems. Think about the idea of high-road transfer of metacognition to the study of other types of problems. What is unique about puzzle problems that does not readily transfer to other types of problems? What is there about puzzle problems that transfers to other types of problems?

As you struggle with proving answers to these types of questions, think about your students being faced by the same issues and struggles. What can you do, as a teacher, to help your students learn to routinely use the metacognition strategy?

**Is the Puzzle Problem Solvable?**

Suppose you are now thinking about how to get started in solving the puzzle in Figure 2.2. Perhaps you spend some time looking at the puzzle, checking to see if the givens in any region, row, or column already violate the solution requirement that each row, column, and region must contain the digits 1 to 9. If the givens in a row, column, or region already contain two copies of a digit, then these givens cannot be part of a solution to the puzzle. That is, the puzzle that has these givens has no solution.

This is an important observation (a Big Idea!). For many people, the term problem means a math problem that has exactly one solution. However, a problem may have no solution, one solution, or more than one solution.

Solvability is an important issue in problem solving, and it is usually poorly taught in our precollege educational system. To help illustrate this, it may well be that you believe that every math problem has exactly one solution. Your goal, when faced by a math problem, is to “get the right answer.”

Think about each of the following simple math problem examples:

1. Find a positive integer that, when multiplied by itself, gives the integer 16. This problem has exactly one solution.
2. Here is a slight modification of the problem. Find an integer that, when multiplied by itself, gives the integer 16. This problem has exactly two solutions.
3. Next, consider the similar problem: Find an integer that, when multiplied by itself, gives the integer 15. This problem does not have a solution.
4. Here is a slight change in the unsolvable problem. Find a number that, when multiplied by itself, gives the integer 15. This problem has two solutions, and they are both irrational numbers.
5. Another slight change to the problem opens up the idea of imaginary numbers. Find a number that, when multiplied by itself, gives the integer minus 15 (that is, \(-15\)).

6. Now, here is still another math problem. Find two integers that, when added together, give the integer 12. With a little thought, you should be able to convince yourself that this problem has an infinite number of solutions.

7. Here is a slight modification of this problem. Find two integers that, when added together, give the number 11 \(\frac{1}{2}\). Now the problem has no solution.

I hope that by now you are convinced that even a quite simple problem may be unsolvable, may have exactly one solution, may have more than one—but still a finite number of solutions, or may have an infinite number of solutions.

In summary, this section introduces a problem-solving strategy called the explore solvability strategy. When faced by a challenging problem, think about whether the problem is solvable. Spend some time exploring the idea that the problem might not be solvable, or that it might have one or many solutions. Think about the idea that if the problem has more than one solution, then perhaps one solution is better in some sense than another solution. What are criteria for a “good” solution? Work to understand the problem so that you can tell if you are making progress toward developing a solution.

You should spend some time adding this strategy to your repertoire of high-road transfer problem-solving strategies. Begin by finding some examples that are personally meaningful to you. Then spend some time developing ideas on how you will go about helping your students learn this strategy. One approach is to routinely expose your students to problems that look like the others they are studying, but that are unsolvable or have more than one solution.

**Getting Started in Solving the Puzzle**

Finally, we are now ready to begin solving the Sudoku puzzle given in Figure 2.2. You should now be suspicious that perhaps the puzzle has no solution, or perhaps it has more than one solution. You might want to do a quick check of the givens to see if it is obvious that the puzzle has no solution. However, you should be aware that even if the set of givens do not make a row, column, or region with two copies of one of the digits 1-9, this still does not tell us whether the puzzle is solvable or whether it has more than one solution.

Let’s pretend that I am an absolute novice in solving Sudoku puzzles. I stare at the puzzle for a while. My eyes tend to go to the upper left region, Region 7.

Within this region, for some reason my eye catches on the empty space b8. I think to myself: “This empty space needs to contain one of the digits 1-9. Right now, the digit 1 is not in Region 7. What happens if I place a 1 into the space b8? The result is shown in Figure 2.4
Figure 2.4. Trying a “1” in space b8.

Placing a 1 into space b8 is a step in the direction of having all nine digits in Region 7. However, you can now see that Row 8 in which I have inserted the digit 1 already contains a 1. Thus, the move is a mistake—a move that cannot lead to solving the puzzle. I have just used the guess and check strategy. I made a guess based on the information that currently the digit 1 does not appear in Region 7. I checked the result by looking at the row and column in which I placed the 1.

In many problem-solving situations, the guess and check strategy can be used mentally, without actually making a move. In Figure 2.4, it is easy to make the proposed move in my mind’s eye, and then to do the checking in my mind’s eye. That is, I don’t have to physically write a 1 into space b8 in order to “see” that this will make Row 8 have two 1s. Undoubtedly you have heard the expression: “Look before you leap.” That is an admonition to do a visual/mental check of possible results before taking an action.

In addition, it is a problem-solving strategy. That should be part of your repertoire of high-road transfer problem-solving strategies. This strategy goes by other names, such as engage brain before opening mouth strategy. Please spend some time thinking about how to help your students add this strategy to their repertoire of general problem-solving strategies.

**Persistence and Self-confidence**

We still haven’t made any progress in solving our Sudoku puzzle. Let’s try another approach. We are still examining the space b8. Figure 2.5 shows all possible moves that are not eliminated by a quick consideration the current entries in row 8, column b, and cell 7. That is, Figure 2.5 illustrates a start on an exhaustive search approach to space b8, after making a quick mental elimination of obviously incorrect choices.
Aha! I am beginning to see why a Sudoku puzzle can be a mental challenge. I stare at cell 7, and I mentally contemplate various possibilities. For example, I might mentally contemplate leaving the 7 in space b8, and putting placing the 2 and 4 as shown in Figure 2.6.

Now, if my mind’s eye (mental image) is working well enough, I see that my contemplated sequence of moves is incorrect, since the situation that has emerged is that I will need to place a 1 into space c8, and that will mean that there are two 1s in row 8.

If my working memory (short-term memory) is good enough, I might well make my way through this maze of possibilities. In attempting to do so, I will be exercising my working memory and other parts of my brain. With practice, I will get better at this aspect of attempting to solve a Sudoku puzzle.

An alternative is to step back a little. Think of my first trial as being an exploration of cell 7. After putting quite a bit of effort into this exploration, I did not experience much (if any) success.

I could quit right now—just give up, and claim, “I am too dumb to learn to solve Sudoku puzzles. Probably this puzzle does not have a solution. Anyway, who cares?” Alternatively, I can persist, try a different cell to explore, and perhaps discover another strategy that might be helpful.

Think about this situation from a teaching/learning point of view. Many of our students have become convinced that they cannot learn to solve complex problems. They have learned that it is
much easier to say, “I can’t do it.” than it is to persist, continue to learn, and continue to make incremental progress.

Persistence and self-confidence are two important characteristics of good problem solvers. Think about your own levels of persistence and self-confidence as a learner and as a problem solver. What might you do to improve your levels of these two characteristics? What might you do as a teacher to help your students increase their levels of persistence and self-confidence?

Games provide one possible piece of an answer to the question. As a teacher, parent, older sibling, and so on, you can use games to create challenging learning and problem-solving environments in which a learner gets an opportunity to gain in persistence and in self-confidence. With proper help from you, the learner can transfer these gains in persistence and self-confidence to other learning and problem-solving situations.

The Elimination Strategy

I will not give up! I am ready to select another region to explore in the Sudoku puzzle shown in Figure 2.2. As I explore the board, this time my eye catches on Region 5, and the empty space in the exact center of the board. The combination of Region 4, Row 5, and Column E has a lot of givens. Indeed, mentally or with the aid of pencil and paper I quickly discover that each of the digits 1-9 except the digit 5 is in the set of givens for the combination of Region 4, Row 5, and Column E. Thus, e5 has to be a 5. My first success! See Figure 2.8.

We have just discovered and used the elimination strategy. In exploring the space e5, we eliminated as many possible moves as we could. It turned out to be easy to eliminate all but one possible move. The elimination strategy is a good one to add to your repertoire of high-road transferable problem-solving strategies.

Continuing with my somewhat inane personal examples, in the morning when I get up I am faced by the problem of what to wear. I have a number of long sleeve shirts, and a number of short sleeve shirts. Thus, depending on the day’s expected temperature, I can quickly eliminate about half of my shirts from consideration. I also have a number of dress shirts and a number of non-dress shirts. I can quickly eliminate one of these categories by thinking about whether this is a work or non-work day. These two eliminations greatly simplify my selection problem.
Here is a somewhat more complex example of using the elimination strategy. I am faced by the problem of obtaining some up to date information on a topic that will be in the book I am writing. A little thought eliminates from consideration my own personal knowledge and the books in my personal library that I have read. I also quickly eliminate all of the books and journals in the physical library on my campus, since I am at home and I want a quick solution to my problem.

This line of elimination and thinking leads me to doing a Google search on the Web. Unfortunately, my search produces about a million hits. That is, Google tells me that it may have found as many as a million sources of the information that I seek.

I definitely need to do some more elimination. I can narrow my search—for example, I can increase the number of terms in my search strategy. However, I thought carefully in developing my original search terms, and so it is not easy to narrow the search.

An alternative approach, one that I most often use, is to explore the brief descriptions of the first half dozen hits. This uses a guess and check strategy. If one catches my eye as possibly being relevant (a good guess), I go to the Website and browse it.

If this Website does not meet my needs, I will browse a couple more of the top numbered hits. In this guess and check process, I will be gaining information that will help me to narrow or reformulate my search. If none of the hits I browse meet my needs, I may decide to eliminate all million of the hits found by Google, and formulate a new search.

Finally, let’s go back to our Sudoku puzzle. Notice that there are now only two blank spaces in Column E. Using the elimination strategy, you see that these must contain the digits 3 and 4. By a mental guess and check you easily arrive at Figure 2.9

![Figure 2.9. Two more successful moves in Column e.](image)

Keep working on this puzzle. (Hint: Cell f3 looks like a fruitful cell to explore.) Pay careful attention to the strategies that you use. Each time you use one of the strategies named in this chapter, make note of this fact. This is a good way to solidify strategies in your repertoire of high-road transferable problem-solving strategies. If you use a strategy that has not been discussed earlier in this chapter, explore its for possible inclusion in your repertoire of high-road transfer problem-solving strategies.
Final Remarks

If you had not previously worked with a Sudoku puzzle, you have now learned the rules and practiced a little in solving a puzzle. You can now see that our goal is to use games and puzzles as a vehicle to help students get better at problem solving and to address other important goals in education.

Using a discover-based approach, we discovered some very important things that apply to problem solving in all disciplines. These include some high-road transferable general problem-solving strategies:

- Create a simpler problem
- Explore solvability
- Guess and check (also named look before you leap)
- Elimination

Researchers have found that the typical student has a quite small repertoire of general-purpose strategies that may be applicable when faced by a new, novel problem. In just a few minutes, we “discovered” four such strategies while exploring the Sudoku puzzle. Through appropriate teaching, students can add these to their repertoire of high-road transfer problem-solving strategies.

Strategies and strategic thinking are part of the more general topic, computational thinking. All of the strategies listed above can be carried out by a thinking human being. Two of them are well suited to implementation in computer programs. Thus, both the human and the computer aspects of computational thinking are represented. In subsequent chapters, we will explore computational thinking in more detail.

Many people enjoy learning new puzzles precisely because it provides them an opportunity to discover strategies that are particularly powerful in the puzzle. However, educational researchers tell us that relatively few people automatically transfer such strategies to use in other puzzles and to solving real world problems. Explicit teaching (by a teacher, or by the learner) is a major help in overcoming this difficulty.

Activities for the Reader

1. Many popular puzzles and games are available in handheld, battery powered, electronic format. Use the Web to see the features of competing models for generating and playing Sudoku puzzles, and how much they cost.

2. Go to the Web and find a puzzle that you have not previously “played” or tried to solve. Explore the puzzle using techniques somewhat like those illustrated in this chapter’s exploration of Sudoku. Do metacognition and reflect on the learning experience. If this is a written assignment, keep detailed notes on the overall activity and then use them to support doing the written assignment.

3. Think about how you, personally, deal with novel, challenging problems that you encounter. Do you have any strategies that you tend to use frequently, and that are often effective? (Have you thought about the possibility of sharing this strategy with your students?) Do you have any strategies that you
tend to use frequently, and that are seldom or almost never effective? (Have you thought about the possibility of helping your students to discover some of their personal ineffective strategies?

4. Suppose that you have a textbook that you have used before, and you want to look up something in it that you are fairly sure is in the book. What strategies do you use? Are these strategies applicable to looking up information in other types of books? Are they applicable to looking up something on the Web?

Activities for use with Students

1. Talk to several children to learn whether they can tell you some general-purpose strategies that they use when faced by novel problems. In the process, include a focus on whether the children have vocabulary (such as the word strategy) useful in carrying on the conversation and in thinking about how they approach novel problems. Also, focus on problems from many different disciplines—not just math problems or math exercises.

2. Working with a group of students, such as a whole class, determine how many are familiar with Sudoku. If quite a few are familiar with this puzzle, then have the Sudoku-experienced class members teach the game to the others, working in one-on-one or in very small groups instruction mode. If few are familiar with the puzzle, teach it to the class. Make use of your Sudoku-knowledgeable students as aides to help the other students as they work on a puzzle. Then debrief this learning experience with the whole class. Direct the conversation so you gain increased insight into students helping students, students being helped by students, and the overall student experience in learning and playing with this puzzle.

3. Select one of the general high-road transferable problem-solving strategies discussed in this chapter. Use it to explain the meaning of high-road transfer of learning to your students. Engage them in gaining the knowledge and skills to do high-road transfer of this strategy. Do whole class brainstorming on type of problems in which this strategy might be applicable. For example, when trying to write a sentence that contains a word a student does not know how to spell, guess and check might be a useful approach. The “check” might come from looking at the spelled word ("It seems to look right."), from use of a dictionary, or from use of a spelling checker on a computer. Repeat this activity once a week for a number of weeks, teaching other strategies from this chapter or from your own repertoire of high-road transferable problem-solving strategies.