Math 95 Fall 2006 Name: ______________________

Quiz 7 Solutions

Class time: __________

1. Multiply the following polynomials to derive the *perfect square* and *difference of squares* formulae. You will want to use these formulas for the rest of the questions!

   (a) $(A-B)(A+B) = A^2 - B^2$

   (b) $(A+B)^2 = A^2 + 2AB + B^2$

   (c) $(A-B)^2 = A^2 - 2AB$

2. **Instructions:** Completely factor all of the polynomials below. Show all of your work, including noting any formulas you use. You may use any of the formulas above and the following:

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

   (a) $81x^3 - 3$
   First we factor out the GCF$=3$ to get $3(27x^3-1)$. Now, since $27x^3-1$ is a binomial with a minus sign it is either a difference of squares or a difference of cubes, in this case it is a difference of cubes with $A = 3x$ and $B = 1$. Just apply the formula:

$$\begin{aligned} 27x^3 - 1 &= (3x-1)((3x)^2 + 3x + 1) \\ &= (3x-1)(9x^2 + 3x + 1) \end{aligned}$$

   Hence

$$81x^3 - 3 = 3(3x-2)(9x^2 + 3x + 1)$$

   (b) $4x^9 - 400x$
   The GCF is $4x$ so $4x^9 - 400 = 4x(x^8 - 100)$. Again, $x^8 - 100$ is a binomial and so either the difference of squares of the difference of cubes. In this case it is a difference of squares where $A = x^4$ and $B = 10$ so

$$x^8 - 100 = (x^4 - 10)(x^4 + 10)$$

   Since 10 isn't a perfect square each of the above factors are prime. So finally,

$$4x^9 - 400x = 4x(x^4 + 10)(x^4 - 1)$$

(c) $27x^2 + 36xy + 12y^2$

First take out the GCF, 3 to get $3(9x^2 + 12xy + 4y^2)$. Before trying any of our longer strategies for factoring this notice that the first and last terms are perfect squares, i.e. $(3x)^2$ and $(2y)^2$. A quick check show that the middle term, $12xy$, is the same as $2(3x)(2y)$ and therefore $9x^2 + 12xy + 4y^2$ is of the form $A^2 + 2AB + B^2$ where $A = 3x$ and $B = 2y$. Applying the formula we get

$$\begin{aligned} 27x^2 + 36xy + 12y^2 &= 3(9x^2 + 12xy + 4y^2) \\ &= 3(3x + 2y)^2 \end{aligned}$$

(d) $3y^2 + 22y - 16$

This is a little harder as the GCF is 1 and $3y^2$ isn't a perfect square so we need to use one of our strategies. Let's use the ac method.

$ac = -48$

Now we need to find two numbers that multiply to -48 and add to 22. The two numbers are 24 and -2.

Now we rewrite our middle term, $22y$ as $22y = 24y - 2y$ and then factor by grouping:

$$\begin{aligned} 3y^2 + 22y - 16 &= 3y^2 + 24y - 2y - 16 \\ &= (3y^2 + 24y) - (2y + 16) \\ &= 3y(y + 8) - 2(y + 8) \\ &= (y + 8)(3y - 2) \end{aligned}$$