

Quiz 5 Solutions

Solve the following inequalities. Express all of your solution sets in interval notation.

1. $-4(x + 2) > 3x + 20$

$$\begin{aligned} & -4(x + 2) > 3x + 20 \\ \implies & -4x - 8 > 3x + 20 \\ \implies & -20 - 8 > 3x + 4x \\ \implies & -28 > 7x \\ \implies & -4 > x \\ \implies & x < -4 \end{aligned}$$

In interval notation the solution set is $(-\infty, -4)$.

2. $-6 \leq \frac{x}{2} - 4 < -3$

$$\begin{aligned} -6 & \leq \frac{x}{2} - 4 < -3 && \text{multiply everything by 4 to clear the denominator} \\ \implies -12 & \leq x - 8 < -6 && \text{add 8 to every piece to get } x \text{ by itself} \\ \implies -4 & \leq x < 2 \end{aligned}$$

In interval notation the solution set is $(-4, 2]$.

3. $4x + 3 < -1$ or $2x - 3 \geq -11$

To solve compound inequalities that involve the word “or”, we solve each inequality individually then take the **union** of their solution sets.

$$\begin{aligned} & 4x + 3 < -1 & \Bigg| & 2x - 3 \geq -11 \\ \implies & 4x < -4 & \Bigg| & \implies 2x \geq -8 \\ \implies & x < -1 & \Bigg| & \implies x \geq -4 \end{aligned}$$

So $x < -1$ **or** $x \geq -4$. In interval notation: $(-\infty, -1) \cup (-4, \infty) = \text{ALL REAL NUMBERS}$.

4. $2|y + 6| = 10$

$$2|y + 6| = 10 \quad \text{divide both sides by 2}$$

$$|y + 6| = 5 \quad \text{rewrite without absolute values as an “or” statement}$$

$$y + 6 = 5 \quad \text{or} \quad y + 6 = -5 \quad \text{solve both equations}$$

$$y = -1 \quad \text{or} \quad y = -11 \quad \text{now check each solution with the original equation}$$

$$2|(-1) + 6| = 2|5| = 2 \cdot 5 = 10$$

$$2|(-11) + 6| = 2|-5| = 2 \cdot 5 = 10$$

So both solutions work!

5. $|x - 3| = |5 - x|$

This is saying that for some x , the distance of $x - 3$ from 0 **is the same** as the distance of $5 - x$ from 0. So there are two possibilities: either $x - 3$ and $5 - x$ are the same, i.e. $x - 3 = 5 - x$, or they are opposites, i.e. $x - 3 = -(5 - x) = -5 + x$. So these are the two equations we need to solve:

$$\begin{array}{l} x - 3 = 5 - x \\ 2x = 8 \\ x = 4 \end{array} \left| \begin{array}{l} \implies \\ \implies \\ \implies \end{array} \right. \begin{array}{l} x - 3 = -5 + x \\ -3 = -5 \\ \text{False!} \end{array}$$

So there is only one possible solution, $x = 4$, which we still have to check in the original equation:

Does $|4 - 3| = |5 - 4|$? Yes!

6. $|x + 1| \leq 5$

This is saying that the distance of $x + 1$ to 0 is less than 5, which means that the number $x + 1$ is somewhere between -5 and 5 , i.e. $-5 \leq x + 1 \leq 5$. We will solve this inequality in more or less the same fashion as in exercise 2.

Alternately, we could write this as two inequalities, joined by an “and”, then take the intersection of their solution sets - this is the hard way.

$$\begin{array}{l} -5 \leq x + 1 \leq 5 \\ \implies -5 - 1 \leq x \leq 5 - 1 \\ \implies -6 \leq x \leq 4 \end{array}$$

In interval notation the solution set is $[-6, 4]$.

Now the harder way:
$$\implies \begin{array}{l} x + 1 \geq -5 \\ x \geq -6 \end{array} \left| \begin{array}{l} x + 1 \leq 5 \\ x \leq 4 \end{array} \right.$$

So $x \geq -6$ **and** $x \leq 4$. To find where these two sets intersect I would draw a picture of both intervals and see that they overlap on the interval $[-6, 4]$.