Math 95
October 23rd, 2006
Midterm Exam I Solutions **True/False.** Mark "T" if the statement is **always** true. Mark "F" if the statement is **ever** false. You **do not** need to justify your answers though this is a good way to see if your intuition is correct.

T F The relation $\{(1,2),(2,5),(7,2)\}$ is a function.
True. None of the inputs are repeated so for every input there is only one output.

T F A function can have more than $x-$intercept.
True. It can have infinitely many in fact! (But at most one y-intercept)

T F A line with slope -3 is perpendicular to a line with slope 3.
False. The product of the slopes of two perpendicular lines should be -1, not -9.

T F It is possible for the intersection of two sets to be empty when neither set is empty.
True. Let $A = \{2,3\}$ and $B = \{1,4\}$. Then $A \cap B =$.

T F It is possible for the union of two sets to be empty when neither set is empty.
False. Each set is included in the union so if neither set was empty then all of their elements will be in the union, so it can't be empty!

T F If $|x-4| = 7$ then $|x| = 11$.
False. One solution is $x = 11$, but $x = -11$ isn't another solution because $|-11-4| = 15 \neq 7$.

T F A system of linear equations with infinitely many solutions is called a dependent system.
True. Just a definition.

T F Another way to write "$4 < x$ or $x < 8$" is $4 < x < 8$.
False. $4 < x < 8$ **is shorthand for** $4 < x$ **and** $x < 8$**.**

T F The equation $|3x+1| + 2 = 1$ has no solutions.
True. It is equivalent to the equation $|3x+1| = -1$; the absolute value of a quantity can never be negative!

T F The slope of a horizontal line is zero.
True. If you forget, just sketch one, find two points on it, and use the formula for slope . . .

1. Answer the questions below regarding the following four functions f, g, r, h.

$f(x) = 3x^2 - 2x + 1$ $\qquad g(x) = \dfrac{4}{x+1}$

x	$r(x)$
0	-1
1	10
2	1
3	0

(a) Compute:

$f(2)$ $= 3 \cdot 2^2 - 4 + 1$ $= 12 - 4 + 1 = 9$	$r(0)$ $r(0) = -1$	$h(-1)$ oops! I don't have a copy of the graph.	$(g + r)(1)$ $= g(1) + r(1)$ $= \frac{4}{2} + 10 = 12$
$\left(\frac{r}{h}\right)(2)$ $= \frac{r(2)}{h(2)} = \frac{1}{h(2)}$	$(hf)(1) - h(f(1))$	$f(a + h)$ $= 3(a + h)^2 - 2(a + h) + 1$ $= 3(a^2 + 2ah + h^2) - 2a - 2h + 1$ $= 3a^2 + 6ah + 3h^2 - 2a - 2h + 1$	

(b) Find the domain of

i. f; all real numbers

ii. g; $x \neq -1$

iii. r; $\{0, 1, 2, 3\}$

iv. h; write your answer in interval notation.
I don't have graph to be able to answer this question but to find the domain, look at the x axis.

v. Extra credit: $\frac{f}{r}$; the domain is the intersection of the domain of f and the domain of r, so $\{0, 1, 2, 3\}$ except that we have to take out any values that make $r = 0$, so the domain of $\frac{f}{r}$ is $\{0, 1, 2\}$.

(c) Find the range of h.
To find the range, look at the y-axis.

(d) For what value of x is $h(x) = 0$?
This question is basically asking you to find all of the x-intercepts, so just look for where h crosses the x-axis.

2. Write the equation of the line passing through the points $(-2, 3)$, $(1, -5)$.

   First we need to find the slope

   $$m = \frac{3-(-5)}{-2-1} = -\frac{8}{3}$$

   Now just choose one of the points above for the point-slope formula: $y - y_1 = m(x - x_1)$
   The equation for the line is

   $$y - 1 = -\frac{8}{3}(x+5)$$

3. Write the equation of the line perpendicular to $3y - 2x = 5$ with $y-$intercept 100.
   The first thing we need to do is figure out the slope of the line above by writing it in slope-intercept form:

   $$3y - 2x = 5 \qquad \Longrightarrow \qquad y = \frac{2}{3}x + \frac{5}{3}$$

   So the equation of the line perpendicular to $3y - 2x = 5$ with $y-$intercept 100 is

   $$y = -\frac{3}{2}x + 100$$

4. Solve the system of equations using any method you like:

   $$\begin{aligned} 2x + 3y &= 6 \\ 4x &= -6y + 12 \end{aligned}$$

   I think the addition method will probably work best for this system.

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + 6y &= 12 \end{aligned}$$

Put the second equation into standard form

$$\begin{aligned} -4x - 6y &= -12 \\ 4x + 6y &= 12 \end{aligned}$$

Multiply the first equation by -2 to cancel out the x's

$$\begin{aligned} -4x - 6y &= -12 \\ + \quad 4x + 6y &= 12 \\ \hline 0x + 0y &= 0 \end{aligned}$$

add the equations together

Since both x and y were eliminated and the resulting statement is true, there are infinitely many solutions. The solution set is

$$\{(x, y) | 2x + 3y = 6\}$$

5. Solve for x in the equation $|4x - 7| = 5$.

   Remember that absolute value means **distance**. So this equation is saying find all x such that $4x - 7$ is exactly 5 units away from 0. Well, in order to be 5 units away from 0, either $4x - 7 = 5$ or $4x - 7 = -5$.
   Solving each of the two equations gives us the solutions $x = 3$ or $x = \frac{1}{2}$.
   We need to check each of the solutions with the original equation: $|4(3) - 7| = |5| = 5$ and $|4(1/2) - 7| = |2 - 7| = |-5| = 5$. So they both work.

6. Find the union: $\{a, b, c, d, e, f\} \cup \{d, e, f, g, h\} =$

   Remember that union means "join together", so the union will have all the elements from both sets. Hence $\{a, b, c, d, e, f\} \cup \{d, e, f, g, h\} = \{a, b, c, d, e, f, g, h\}$

7. Find the intersection: $\{x|x \leq 3\} \cap \{x|x > 1\} =$

   The intersection is where both sets overlap. Probably the best way to see where these sets overlap is to use a number line and remember that you put a bracket for $\leq$ or $\geq$ and parentheses for $<$ or $>$. The correct answer is

   $$\{x|x \leq 3\} \cap \{x|x > 1\} = (-1, 3]$$

8. The solution set to the inequality $|3x - 2| \leq 4$ is:

   Once again, think about absolute value as **distance from 0**. So we're interested in the set of all numbers x such that the number $3x - 2$ is within 4 units of 0. That means that

   $$3x - 2 \leq 4 \text{ and } 3x - 2 \geq -4$$

   Solving each inequality we get that

   $$x \leq 2 \text{ and } x \geq -\frac{2}{3}$$

   In other words,

   $$-\frac{2}{3} \leq x \leq 2$$

   which is given by the interval

   $$[-\tfrac{2}{3}, 2]$$

9. The solution set to $2x + 7 < -11$ or $-3x - 2 < 13$ is

   Solving each inequality we see that $x < -9$ or $x > -5$.
   In interval notation, $(-\infty, -9) \cup (-5, \infty)$.

10. For full credit set up and solve, showing all of your work and writing your answer in a complete sentence. I strongly suggest using the addition method. (*Remember, multiplication with decimals works exactly the same as with integers, you just have to move the decimal point over however many digits were behind it in each piece of the product.* For example, $(.2)(.3) = .06$ and $\frac{6}{.3} = 20$)

Let me say first off that I copied this problem incorrectly so the solution doesn't make sense but the method for setting it up and solving it is still okay.

You invested \$7000 in two accounts paying 6% and 7%, respectively. If the total interest earned for the year was \$520, how much was invested at each rate?

Let x be the amount invested at 6%.
Let y be the amount invested at 7%.

So we know that $x + y = 7000$.

The interest earned from investing x dollars was $0.06x$.
The interest earned from investing y dollars was $0.07y$.

So we know that $0.06x + 0.07y = 520$.

Hence we need to solve the system of equations:

$$\begin{aligned} x + y &= 7000 \\ 0.06x + 0.07y &= 520 \end{aligned}$$

Let's solve the inequality using the addition method. I'll multiply the top equation by -0.06 in order to eliminate the x variable:

$$\begin{aligned} x + y &= 7000 \\ 0.06x + 0.07y &= 520 \end{aligned}$$

$$\begin{array}{rrcl} & -.06x - .06y & = & -420 \\ + & 0.06x + 0.07y & = & 520 \\ \hline & 0x + .01y & = & 100 \end{array}$$

The last equation, $0.01y = 100$ which implies that $y = 10000$ and hence $x = -3000$. Note that this solution doesn't make sense because we only had \$7000 to start with and both x and y are assumed to be non-negative.