Assignment 5; Due Friday, October 28

The first midterm will be on Monday, October 31. I will have extensive review sheets next week.

For this assignment, read section seven on compact spaces. Then do the following problems:

- 6.6ad
- (Graduate students only) 6.6l
- 7.13 ab
- (Graduate students only) 7.13cg
- 7.13h
- Find an open cover of R which does not contain a finite subcover. Repeat this problem for $[0, 1)$.
- Consider the three examples which follow. Which of these spaces are compact? If the space *is not* compact, then find an open cover which does not contain a finite subcover. If the space *is* compact, prove it.
  1. Q, the set of rational numbers, with the topology induced from $Q \subseteq R$
  2. $S^2 - \{\text{north pole}\}$
  3. the Klein bottle, obtained as a quotient space of $[0, 1]^2$ by the usual gluing operation along the boundary
- (Graduate students only): A topological space X is a Hausdorff space if whenever $x \neq y$ in X, we can find open neighborhoods $\mathcal{U}$ and $\mathcal{V}$ of x and y such that $\mathcal{U} \cap \mathcal{V} = \emptyset$. If A and B are nonintersecting closed subsets of a compact Hausdorff space X, prove that there exist nonintersecting open sets $\mathcal{U}$ and $\mathcal{V}$ with $A \subseteq \mathcal{U}$ and $B \subseteq \mathcal{V}$.
- Prove that the open ball $\{ \ x \ : \ ||x|| < 1\}$ and the closed ball $\{ \ x \ : \ ||x|| \leq 1\}$ in R^n are not homeomorphic.