

Here is an outline of the topics we covered this course.

- Real numbers
 - Know what real numbers are and how we can represent subsets of the real numbers using (1) the number line, (2) inequalities, (3) interval notation.
- Definition of a function
 - Know what the domain, range, and rule of a function are. Also, be sure that you understand the phrase “every input has a unique output” in the definition of a function.
- Function notation
 - Make sure you understand the difference between $f(a + b)$ and $f(a) + f(b)$, as well as the difference between $f(x + h)$ and $f(x) + h$.
 - You should be able to do look at the rule of a function f and compute things like $2f(3x - 6) + 12$.
 - Be sure to understand how to compute the difference quotient given the rule of a function.
- Inverse functions
 - You need to know what inverse functions are. For example, if I tell you that $f(17) = 23$, then you should be able to tell me $f^{-1}(23) = 17$ and vice versa.
 - You need to know what it means to be a one-to-one function and what this has to do with inverses. . . in other words know that a function has an inverse if and only if the function is one-to-one.
 - Know the relationship between the domain and range of f with the domain and range of f^{-1} .
 - Given $f(x)$ (where f is one-to-one) you should be able to find $f^{-1}(x)$.
 - You should know how to sketch the graph of f^{-1} given the graph of f [reflect about the line $y = x$].
- Graphs of functions
 - Understand the definition of a graph. . . i.e. the point (a, b) is on the graph of f if and only if $f(a) = b$.
 - You should be able to look at a graph of f and read off the domain of f , the range of f , the x and y intercepts, as well as things like $f(-3)$ or $f(12)$.
- Graph transformations
 - Understand vertical and horizontal shifts, vertical stretching, and reflections about the x and y axes and how each changes the rule of a function.

- Be careful to use orders of operations when listing graph transformations.
 - You should be able to look at the graph of a function f and sketch a graph of something like $-3f(x+2) - 6$. I would first write down the transformations (in order) and then transform the graph step by step being sure to label each graph.
- Even and odd functions
 - Know that f is even exactly when $f(-x) = f(x)$, and f is odd exactly when $f(-x) = -f(x)$.
 - Know that the graphs of even functions are symmetric about the y -axis, and the graphs of odd functions are symmetric about the origin (that is 180° rotation about $(0,0)$ fixes the graphs of odd functions).
 - You should be able to look at $f(x)$ and determine whether f is even or odd by computing $f(-x)$ and comparing it to $f(x)$ and $-f(x)$.
- Operations on functions
 - Given two functions f and g , you should know what the functions $f + g$, $f - g$, fg , and f/g are.
 - Know how to compose functions, i.e. if you are given $f(x)$ and $g(x)$ you should be able to find $(f \circ g)(x)$ as well as $(g \circ f)(x)$.
 - Remember that $f \circ g \neq g \circ f$ in general.
 - Know the round trip theorem and how to use it to determine if two functions are inverse to one another.
- Linear functions
 - Know slope intercept form $f(x) = mx + b$ and what both m and b tell you about the graph of f .
 - You should be able to find the rule of a linear function through two points, through one point given it's slope, through one point parallel to another line, or through one point perpendicular to another line.
 - Know what graphs of linear functions look like [lines!!].
 - be able to solve linear equations like $2(x - 3) = -\frac{1}{2}x + 2$.
- Quadratic functions
 - Know the “vertex form” $f(x) = a(x - h)^2 + k$ and what each of a, h, k tell you about the graph of f .
 - You should be able to put the rule of a function $f(x) = ax^2 + bx + c$ into vertex form by completing the square.
 - You should be able to find the rule of a quadratic function given its vertex and another point on its graph.
 - Know what graphs of quadratic functions look like [parabolas!!].

- You should be able to solve quadratic equations by (1) factoring, (2) completing the square, (3) using the quadratic formula.
 - You should know what the graph of $f(x) = \sqrt{x}$ looks like and how to use graph transformations to sketch the graph of something like $f(x) = 2\sqrt{-x} + 7$.
- Circles
 - Know the following equation for a circle $(x - a)^2 + (y - b)^2 = r^2$ and what each of a, b, r represent.
 - You should be able to put an equation like $x^2 + 8x = 3y - y^2$ into the form above by completing the square, and use this form to read off the center and radius of a circle.
- Square roots.
 - Know what the graph of $s(x) = \sqrt{x}$ looks like and using this graph (along with graph transformations) you should be able to sketch the graph of any function with rule $f(x) = a\sqrt{x - b} + c$.
- Polynomial functions
 - Know how to do long division.
 - Know the relationship between factors and remainders.
 - Know that the remainder when $f(x)$ is divided by $x - c$ is $f(c)$.
 - Know that c is a root of a f exactly when $x - c$ is a factor of $f(x)$.
 - Know how to use long division to solve polynomial equations.
 - Know what graphs of polynomial functions look like (smooth and continuous).
 - Know what the degree, leading coefficient, and constant term of a polynomial tell you about a graph. In fact you should be able to look at a graph of a polynomial function and say whether the degree is even or odd, whether the leading coefficient is positive or negative, the value of the constant term, and the minimal degree.
- Absolute values
 - you should know how to write the rule of $f(x) = |x|$ as a piecewise function.
 - You should know what the graph of the function $A(x) = |x|$ looks like and using this graph (along with graph transformations) you should be able to sketch the graph of any function with rule $f(x) = a|x - b| + c$.
 - Be able to solve absolute value equalities as well as inequalities.
- Piecewise functions
 - Understand the notation for the rule of a piecewise function.
 - You should be able to look at the rule of a piecewise function f and
 - * list values like $f(3)$ and $f(0)$.

- * give the domain of f .
- * sketch its graph and label (with x and y coordinates) all
 - x -intercepts,
 - y -intercepts,
 - holes,
 - endpoints.
- * use the graph to state the range of f

- Rational functions.

- You should be able to find the rule of f^{-1} given the rule of f where $f(x)$ is something like $\frac{2x-1}{4+7x}$ or $\frac{1+5x^7}{12x^7}$.
- You should be able to use the round trip theorem to determine whether or not two rational functions are inverse to one another.
- You should be able to factor the numerator and denominator of many rational functions.
- Once the numerator and denominator of the rule of a rational function have been factored, you should be able to determine each of the following:
 - * The domain
 - * Any vertical asymptotes (you should write these as $x = a$ where a is some constant)
 - * Any holes (both x and y coordinates).
 - * The horizontal asymptote (written $y = b$ for some constant b .)
 - * Any x -intercepts
 - * Any y -intercepts

- Exponential Functions

- You need to understand what negative and rational exponents mean. Also, you need to know that $a^0 = 1$ for all positive real numbers a .
- Be able to simplify expressions with exponents using the exponent laws on page 329 in the book.
- Know what the graph of $f(x) = a^x$ looks like (it will depend on a). Using this information you should be able to sketch the graph of functions with rules like $f(x) = -12e^{-x} + 4$.
- Know the formulas for exponential growth and decay. In particular, you should know the interest formulas we have discussed in class paying attention to what each variable represents.

- Logarithmic Functions

- Know the definition of a logarithm is. In particular, understand how logarithmic statements are inverse to exponent statements.

- You need to know what the graph of a logarithmic function looks like. Also, be sure to know $\log_b(1) = 0$ for any $b > 1$.
- Know that $\ln(x) = \log_e(x)$ and $\log(x) = \log_{10}(x)$.
- Know the properties of logarithms on page 378 as well as the logarithm laws on page 379 of the book.
- Know how to use logarithms to solve exponential equations. Conversely, know how to use exponentials to solve logarithmic equations.
- Know how to find $f^{-1}(x)$ where f has a rule similar to the following:
 - * $f(x) = -3 \log_7(4x - 5) + 23$
 - * $23 \cdot 5^{2x+6} - 12$

- Story Problems

- Be sure to know how to find the maximum or minimum of a quadratic function (i.e. find the vertex). Understand how this relates to max/min story problems we have discussed in this course.