Math 251 Review for Final

Here are some practice problems for the final.

1. Definitions and Theorems:
   (a) State the ε, δ definition of a limit.
   (b) State the squeeze theorem.
   (c) State the difference quotient definition of a derivative.
   (d) State the Extreme Value Theorem.
   (e) State Fermat's Theorem.
   (f) State the Mean Value Theorem.
   (g) State L'Hospital's Rule.

2. Use the definition of a derivative (i.e. the difference quotient) to find $f'(x)$ [You should not use L'Hospital's rule here]
   (a) $f(x) = \sqrt{2x-1}$
   (b) $f(x) = x^2 - 3x + 3$
   (c) $f(x) = \frac{4x}{x-5}$

3. Find all numbers c which satisfies the mean value theorem for
   (a) $f(x) = x^2 + 3x$ on the interval $[-1, 2]$
   (b) $f(x) = 3x^3 + 2x$ on the interval $[0, 1]$
   (c) $f(x) = \ln(x^3)$ on the interval $[1, e]$
   (d) $f(x) = e^{6-x}$ on the interval $[5, 6]$

4. Find the derivative of g where
   (a) $g(x) = (x^5 - 3x)\ln(2x)$
   (b) $g(x) = e^{\tan(x)}$
   (c) $g(x) = \frac{3x-5}{x^2+3}$
   (d) $g(x) = 3\sqrt{x} + \frac{4}{x^3}$
   (e) $g(x) = \cos(e^{1-4x})$

5. Prove the following
   (a) $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
   (b) $\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$
   (c) $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
   (d) $\frac{d}{dx}\ln(x) = \frac{1}{x}$

6. Find the equation of the tangent line to the curve $x^3y^2 - 3xy = y + x$ at the point $(0, 0)$.

7. Find the equation of the tangent line to the curve $y^2 = 4 + x^2y$ at the point $(0, -2)$.

8. Compute $\lim x \to 0 x^4 \sin(x)$.

9. Prove that there exists a number c in the interval $[1, e]$ with $f(c) = 0$ where

$$f(x) = \ln(x^4) - x$$

10. Evaluate the following limits:

    (a) $\lim_{x\to 0} \frac{\sin^2 x}{3x^2}$

    (b) $\lim_{x\to\infty} \frac{x^{5/3}+2x}{x+2}$

    (c) $\lim_{x\to -3} \frac{\sin(x+3)}{(x+3)^3}$

11. Suppose f is a function with

$$\frac{2x^4 + 1}{-2} \leq f(x) \leq \frac{\cos(x) - 1}{x^2}$$

    for all nonzero x in the interval $(-2, 2)$. Find $\lim_{x\to 0} f(x)$.

12. Suppose the function given by $f(t) = \ln(t) + 2t^2$ gives the position of a particle at time t.

    (a) What is the domain of f?

    (b) Find the velocity when $t = 1$.

    (c) Find the rule of a function which gives the acceleration at time t.

    (d) When is the acceleration zero?
    [keep in mind part (a)]

13. Work through story problems in sections 3.10 and 4.7 from homework assignments 8 and 10.

14. Prove $\lim_{x\to -4}(3 - 2x) = 11$ using the ε, δ definition of a limit.

15. Let $f(x) = \frac{x}{x^2+1}$. Find the intervals of increase and decrease and all local max and mins.

16. Find the intervals of concavity and all inflection points of the function

$$f(x) = \frac{1}{8}x^8 - \frac{14}{15}x^6$$

17. Use the first derivative test to show that the function f has a local max at $x = 0$ where
$$f(x) = \frac{1}{x^2 + 1}$$

18. Use the second derivative test to show that the function f has a local max at $x = 0$ where
$$f(x) = \frac{1}{x^2 + 1}$$

19. Sketch the graph of f' below the given graph of f: