Math 251
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Assignment #6
Partial Solutions

Additional Exercises:

1. Prove $\frac{d}{dx}(\cos x) = -\sin x$ using the definition of the derivative.
   [Hint: Look at how we proved $\frac{d}{dx}(\sin x) = \cos x$ in class.]

   Solution: Remember that we know the two limits

$$(L1) \quad \lim_{h\to 0} \frac{\sin h}{h} = 1 \qquad \text{and} \qquad (L2) \quad \lim_{h\to 0} \frac{\cos(h) - 1}{h} = 0$$

Also we will need the addition identity for cosine which states

$$\cos(x+h) \overset{(*)}{=} \cos(x)\cos(h) - \sin(x)\sin(h)$$

Now

$$\begin{aligned}
\frac{d}{dx}(\cos x) &= \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} \\
&\overset{(*)}{=} \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \\
&= \lim_{h\to 0} \frac{\cos(x)\cos(h) - \cos(x)}{h} - \lim_{h\to 0} \frac{\sin(x)\sin(h)}{h} \\
&= \lim_{h\to 0} \frac{\cos(x)(\cos(h) - 1)}{h} - \lim_{h\to 0} \frac{\sin(x)\sin(h)}{h} \\
&= \cos(x) \lim_{h\to 0} \frac{\cos(h) - 1}{h} - \sin(x) \lim_{h\to 0} \frac{\sin(h)}{h} \\
&\overset{(L1)\&(L2)}{=} \cos(x)\cdot 0 - \sin(x)\cdot 1 = -\sin x.
\end{aligned}$$