Math 251
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Assignment #5
Partial Solutions

From the Textbook:

§ 3.1 #45 Find the points on the curve $y = 2x^3+3x^2-12x+1$ where the tangent is horizontal.

Solution: The tangent is horizontal when the derivative is zero, so first we find $\frac{dy}{dx}$:

$$y' = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x + 2)(x - 1)$$

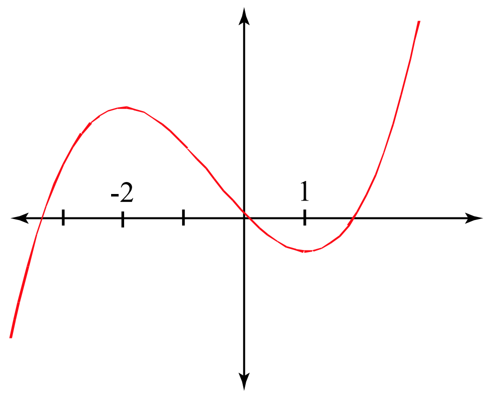
Setting $y' = 0$ and solving for x shows us the tangent is horizontal when $x = -2$ and when $x = 1$. Now when $x = -2$ we have

$$y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 1 = -16 + 12 + 24 + 1 = 21,$$

and when $x = 1$ we have

$$y = 2(1)^3 + 3(1)^2 - 12(1) + 1 = 2 + 3 - 12 + 1 = -6.$$

Therefore the points where the tangent line is zero are $(-2, 21)$ and $(1, -6)$. Here's a picture:



§ 3.1 #46 For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent?

Solution: The tangent is horizontal when the derivative is zero, so first we find $f'(x)$:

$$f'(x) = 3x^2 + 6x + 1$$

Now we set $f'(x) = 0$ and solving for x. $f'(x)$ does not factor "nicely" so I'm going to complete the square to find its roots (you can also use the quadratic formula).

$$3x^2 + 6x + 1 = 0 \quad \Leftrightarrow \quad 3x^2 + 6x = -1 \quad \Leftrightarrow \quad x^2 + 2x = -\frac{1}{3}$$

$$\Leftrightarrow \quad x^2 + 2x + 1 = -\frac{1}{3} + 1 \quad \Leftrightarrow \quad (x+1)^2 = \frac{2}{3} \quad \Leftrightarrow \quad x + 1 = \pm\sqrt{\frac{2}{3}}$$

$$\Leftrightarrow \quad x = -1 \pm \sqrt{\frac{2}{3}} = -\frac{3}{3} \pm \frac{\sqrt{6}}{3} = \frac{-3 \pm \sqrt{6}}{3}.$$

§ 3.2 #40 A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

(a) What does it mean to say $f(20) = 10,000$ and $f'(20) = -350$?

Solution: Because $f(p)$ is the quantity of fabric sold when the price is p, saying $f(20) = 10,000$ is the same as saying "when the selling price is 20 dollars per yard, the manufacturer sells 10,000 yards of fabric." The derivative $f'(p)$ refers to the rate of the change in quantity of fabric sold as the price changes. So $f'(20) = -350$ means "when the selling price is 20 dollars per yard, the rate of the change in quantity of fabric sold as the price changes is -350 yards per dollar." That is to say if the manufacturer increases the price a very small amount, say d dollars, then they will sell about $350d$ fewer yards of fabric.

(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

Solution: Using the product rule we see

$$R'(p) = \left(\frac{d}{dp}p\right) f(p) + p\left(\frac{d}{dp}f(p)\right) = f(p) + pf'(p)$$

so that

$$R'(20) = f(20) + 20f'(20) = 10,000 + 20(-350) = 3,000.$$

This means the instantaneous change in revenue as the price changes is $\$3,000$ per dollar. That is to say if the manufacturer increases the price a very small amount, say d dollars, then the revenue will increase by about $\$3,000d$.