Name: 3/3/06

Math 251 Exam 2

1. (2pts each) Write the rules for each of the following making sure to use correct notation. Assume f and g are differentiable and c is a constant.

   (a) $\frac{d}{dx}(f(x)+g(x))$

   (b) $\frac{d}{dx}(cf(x))$

   (c) $\frac{d}{dx}(f(x)g(x))$

   (d) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$

   (e) $\frac{d}{dx}(f(g(x)))$

2. (10pts) Prove the following equality. Show all work, and justify steps when appropriate.

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

3. (10pts) Find the rule of $f'(x)$ where

$$f(x) = (1+x^2)\arctan(x)$$

   [Hint: you can use the result from problem 2 even if you did not finish problem 2]

4. (10pts) Find the global (absolute) maximum and minimum of the function f on the interval $[1, 3]$ where

$$f(x) = \frac{x}{x^2 + 4}$$

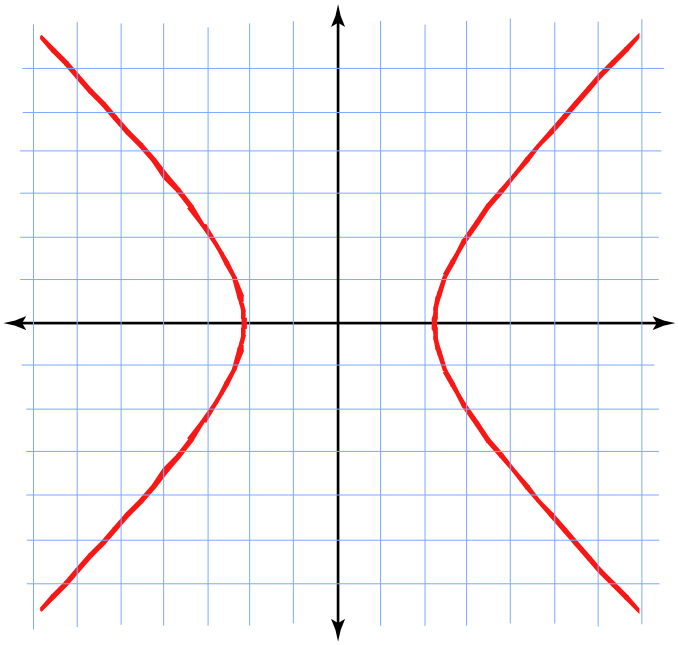
5. (10pts each) Find the derivative of the following functions.

   (a) $f(x) = \cos(2e^x)$

   (b) $f(x) = \frac{4x+7}{3\ln(x)}$

   (c) $f(x) = (8\sqrt{x})^{2-x}$

6. (a) (2pts) Here's a picture of the curve $x^2 - y^2 = 5$. As accurately as you can, sketch the tangent line to the curve at the point $(3, -2)$.



(b) (10pts) Find the equation of the tangent line to the point $(3, -2)$ on the curve

$$x^2 - y^2 = 5.$$

[You should be able to check your answer with part (a)]

7. (10pts) The volume V of a cylindrical can with radius r and height h is given by $V = \pi r^2 h$. Suppose the radius of the can is increasing at a rate of 2 inches per second, and the height of the can is decreasing at a rate of 3 inches per second. How fast is the volume changing when both the radius and the height are 1 inch?

   [Be sure to include units.]

8. (10pts) Evaluate the following limit. Be sure to show all your work using correct notation.

$$\lim_{x \to 0} \left(\frac{(2x)^3}{\sin^2(x) \sin(4x)} \right)$$

Bonus: (10pts) Suppose f is a function with inverse function f^{-1} and both f and f^{-1} are differentiable everywhere. Prove

$$\frac{d}{dx} \left(f^{-1}(x) \right) = \frac{1}{f'(f^{-1}(x))}$$