

Name:

1/31/06

Math 251 Exam 1

1. (5pts) State the squeeze theorem.

2. (5pts) What is the definition of a derivative?

[all I'm looking for here is the rule of  $f'(x)$ ]

3. (10pts) State where the function  $f$  is discontinuous, given that

$$f(x) = \begin{cases} x - 3 & \text{if } x < 1, \\ x^2 - 2x - 1 & \text{if } 1 \leq x < 3, \\ 0 & \text{if } x = 3, \\ 2^x - 6 & \text{if } x > 3. \end{cases}$$

4. Let  $f(x) = \sqrt{4x - 3}$

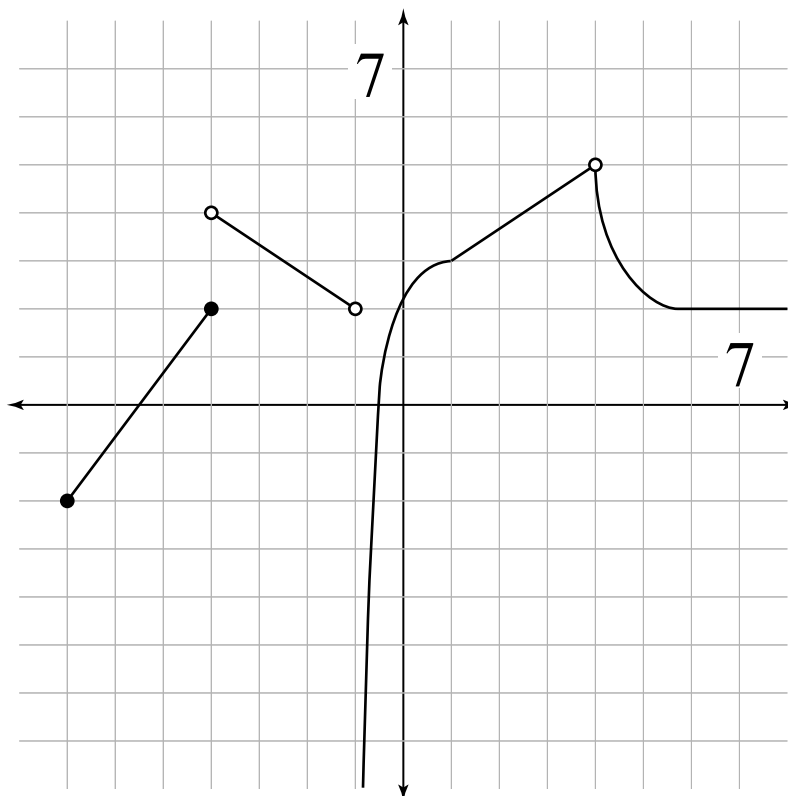
(a) (10pts) Find the rule of  $f'(x)$ .

(b) (10pts) Give the equation of the tangent line to  $f$  at the point  $(7, 5)$ .

5. (10pts) Evaluate the following limit, if it exists.

$$\lim_{x \rightarrow \infty} \frac{13x^2 + 6x - 112}{x^3 - 6x}$$

6. (10pts) Consider the following graph of the function  $f$



State the following values, if they exist.

$$\lim_{x \rightarrow \infty} f(x)$$

$$\lim_{x \rightarrow -1^+} f(x)$$

$$\lim_{x \rightarrow -1} f(x)$$

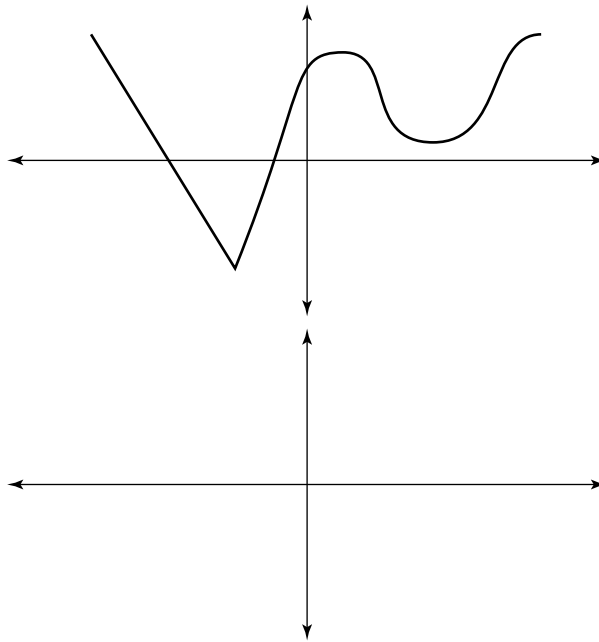
$$\lim_{x \rightarrow 4} f(x)$$

$$\lim_{x \rightarrow -4^+} f(x)$$

7. (10pts) Show that  $f(c) = 0$  for some number  $c$  between 0 and 2 where

$$f(x) = 3^x + \sqrt[3]{4x} - 7.$$

8. (10pts) Sketch a graph of  $\frac{df}{dx}$  below the given graph of  $f$ .



9. (10pts) Prove that  $\lim_{x \rightarrow -2} (4x + 1) = -7$  using the  $\varepsilon, \delta$  definition of a limit.

10. (10pts) Suppose  $f$  is a function which satisfies

$$-x \leq f(x) \leq \frac{x+1}{x^2+3x+2}$$

whenever  $-2 < x < 0$ . Use the squeeze theorem to find  $\lim_{x \rightarrow -1} f(x)$ .

Extra Credit (10pts) A fixed point of a function  $f$  is a number  $a$  such that  $f(a) = a$ . Show that the function with rule  $f(x) = x^6 - 2^x + \sqrt{x} + 3x$  has a fixed point.