

Name:

1/31/06

Math 251 Exam 1

1. (5pts) What is the definition of a derivative?

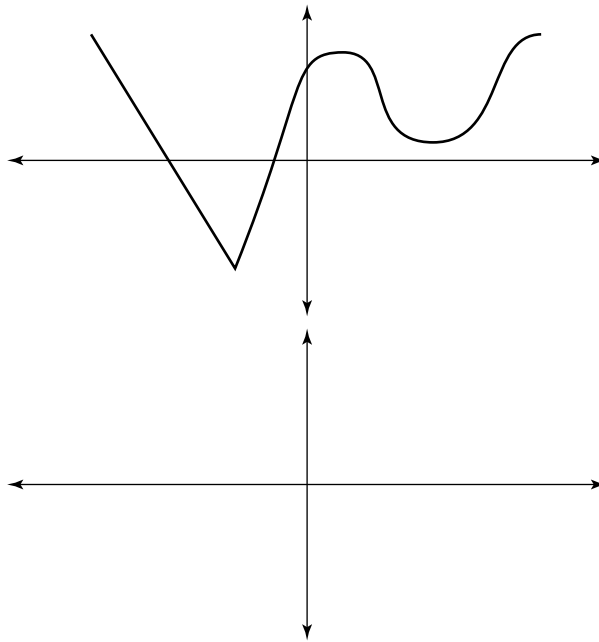
[all I'm looking for here is the rule of $f'(x)$]

2. (5pts) State the squeeze theorem.

3. (10pts) State where the function f is discontinuous, given that

$$f(x) = \begin{cases} x - 3 & \text{if } x < 1, \\ x^2 - 2x - 1 & \text{if } 1 \leq x < 3, \\ 0 & \text{if } x = 3, \\ 2^x - 6 & \text{if } x > 3. \end{cases}$$

4. (10pts) Sketch a graph of $\frac{df}{dx}$ below the given graph of f .



5. (10pts) Prove that $\lim_{x \rightarrow -3} (4x + 1) = -11$ using the ε, δ definition of a limit.

6. Let $f(x) = \sqrt{3x - 2}$

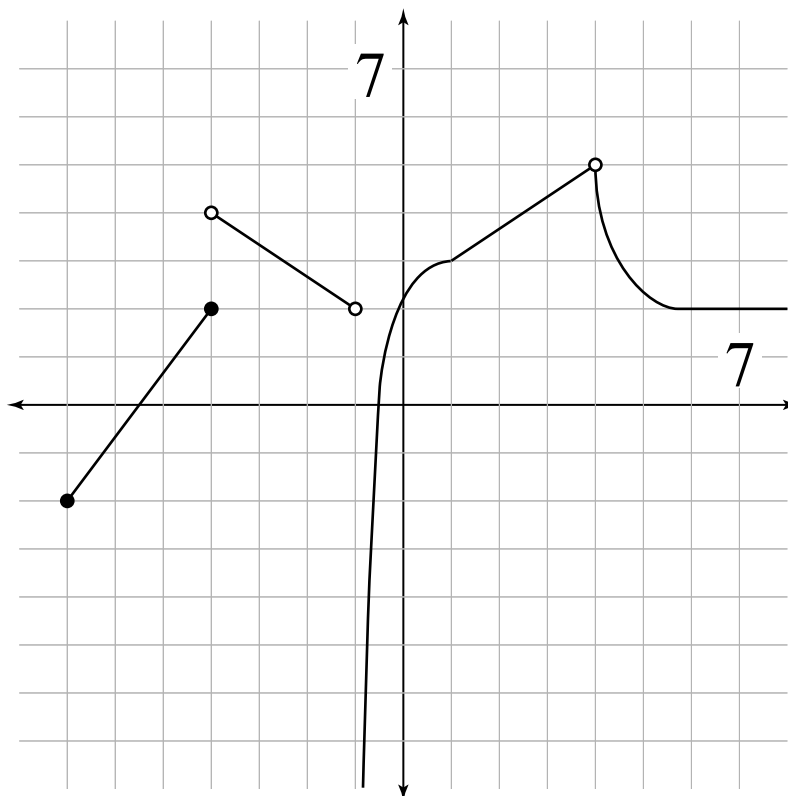
(a) (10pts) Find the rule of $f'(x)$.

(b) (10pts) Give the equation of the tangent line to f at the point $(9, 5)$.

7. (10pts) Evaluate the following limit, if it exists.

$$\lim_{x \rightarrow \infty} \frac{12x^2 - 5x + 108}{x^3 + 3x}$$

8. (10pts) Consider the following graph of the function f



State the following values, if they exist.

$$\lim_{x \rightarrow \infty} f(x)$$

$$\lim_{x \rightarrow -1} f(x)$$

$$\lim_{x \rightarrow -1^+} f(x)$$

$$\lim_{x \rightarrow 4} f(x)$$

$$\lim_{x \rightarrow -4^-} f(x)$$

9. (10pts) Show that $f(c) = 0$ for some number c between 0 and 2 where

$$f(x) = 3^x + \sqrt[3]{4x} - 7.$$

10. (10pts) Suppose f is a function which satisfies

$$-x \leq f(x) \leq \frac{x+1}{x^2+3x+2}$$

whenever $-2 < x < 0$. Use the squeeze theorem to find $\lim_{x \rightarrow -1} f(x)$.

Extra Credit (10pts) A fixed point of a function f is a number a such that $f(a) = a$. Show that the function with rule $f(x) = x^6 - 2^x + \sqrt{x} + 3x$ has a fixed point.