Math 251 Quiz 4 Solutions

1. (10pts each) Find the derivatives of the following functions.

   (a) $f(x) = 7x^4 + 3x^2 - 101$

   Solution:

   $$f'(x) = \frac{d}{dx}(7x^4+3x^2-101) = 7\frac{d}{dx}x^4+3\frac{d}{dx}x^2-\frac{d}{dx}101 = 7(4x^3)+3(2x^1)+0 = 28x^3+6x$$

   (b) $g(x) = 12e^x - \sqrt[3]{x}$

   Solution:

   $$f'(x) = \frac{d}{dx}(12e^x - \sqrt[3]{x}) = 12\frac{d}{dx}e^x - \frac{d}{dx}x^{1/3} = 12e^x - \frac{1}{3}x^{-2/3}$$

   (c) $k(t) = \frac{t^3}{\sqrt{t}} + \frac{5}{t^2}$

   Solution:

   $$f'(t) = \frac{d}{dt}\left(\frac{t^3}{\sqrt{t}} + \frac{5}{t^2}\right) = \frac{d}{dt}\left(t^{5/2} + 5t^{-2}\right) = \frac{5}{2}t^{3/2} - 10t^{-3}$$

2. (15pts) Find the equation of the tangent line to the curve $y = 3x^4 + 12x - 7e^x$ at the point $(0, -7)$.

Solution: We know

$$\frac{dy}{dx} = \frac{d}{dx}(3x^4 + 12x - 7e^x) = 3\frac{d}{dx}x^4 + 12\frac{d}{dx}x - 7\frac{d}{dx}e^x = 12x^3 + 12 - 7e^x$$

So at the point $(0, -7)$

$$\frac{dy}{dx} = 12(0)^3 + 12 - 7e^0 = 5$$

So the slope of the tangent line at the point $(0, -7)$ is 5. Using the point slope form $y - y_1 = m(x - x_1)$ for the equation of a line with slope m going through the point (x_1, y_1) we see the equation of the tangent line is $y - (-7) = 5(x - 0)$ or

$$y = 5x - 7$$

3. (15pts) Find all points on the curve $y = x^3 + 3x^2 - 12$ where the tangent line is horizontal.

Solution: The tangent line is horizontal when $y' = 0$. So we need to find an expression for $y' \ldots$

$$y' = \frac{d}{dx}(x^3 + 3x^2 - 12) = \frac{d}{dx}(x^3) + 3\frac{d}{dx}(x^2) - \frac{d}{dx}(12) = 3x^2 + 6x$$

Now we set $y' = 0$ and solve for x

$$3x^2 + 6x = 0 \quad \Rightarrow \quad 3x(x + 2) = 0 \quad \Rightarrow \quad x = 0 \text{ or } x = -2$$

So we have found the x-coordinates of the points where the tangent line is horizontal. To find the y-coordinate we plug these back into the original equation for y.

$$\text{when } x = 0 \qquad y = (0)^3 + 3(0)^2 - 12 = -12$$

$$\text{when } x = -2 \qquad y = (-2)^3 + 3(-2)^2 - 12 = -8 + 12 - 12 = -8$$

So the points where the tangent line is horizontal are $(0, -12)$ and $(-2, -8)$.