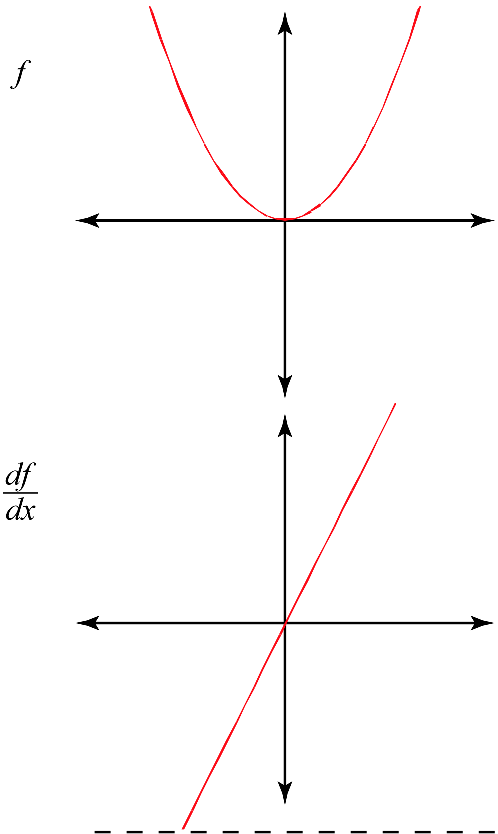
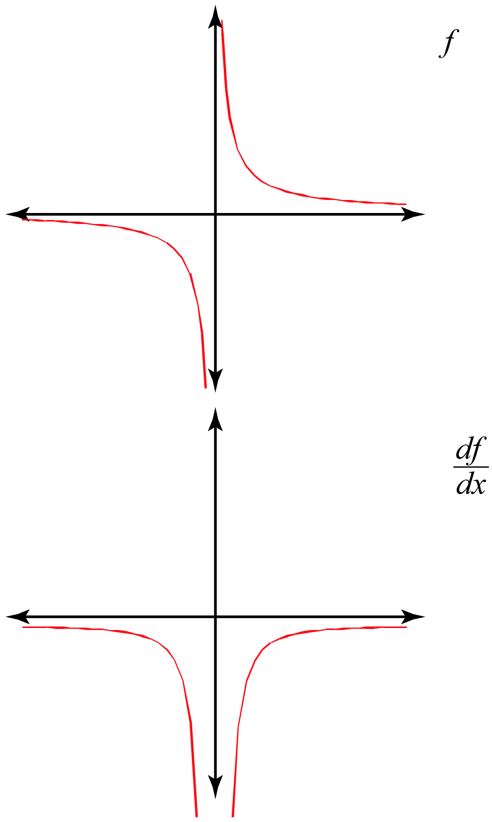
1. First sketch a graph of the function f. Next sketch a graph of the derivative $\frac{df}{dx}$ without actually finding the rule of the derivative.

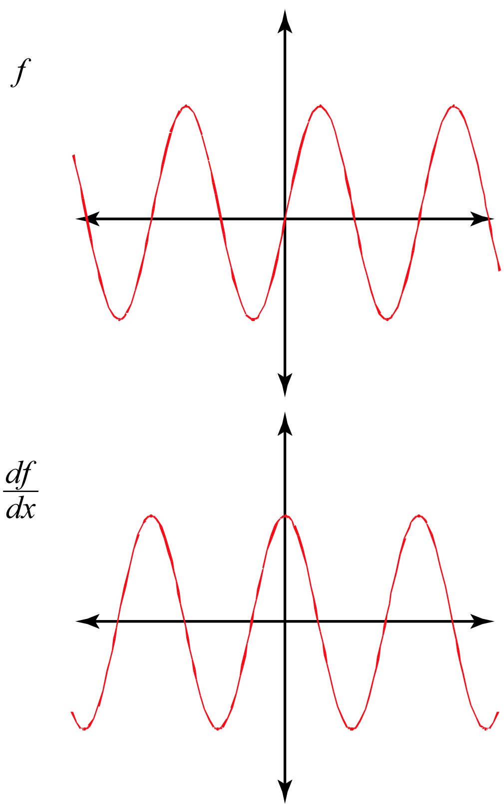
(a) $f(x) = x^2$



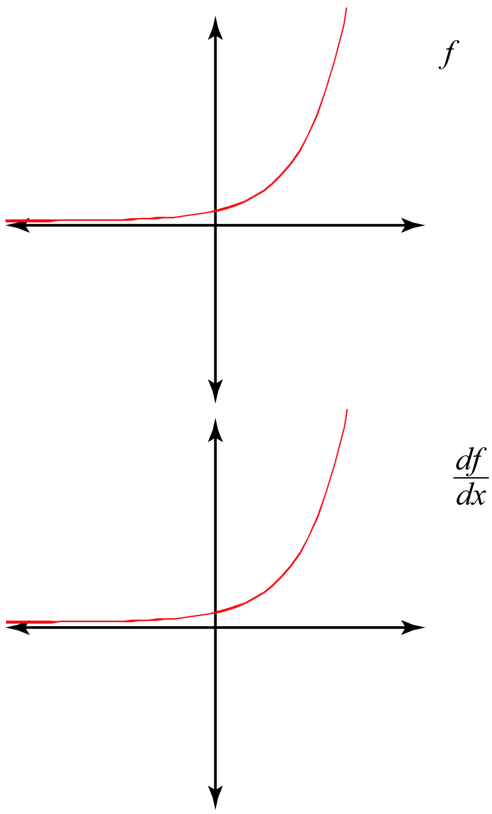
(b) $f(x) = \frac{1}{x}$



(c) $f(x) = \sin(x)$

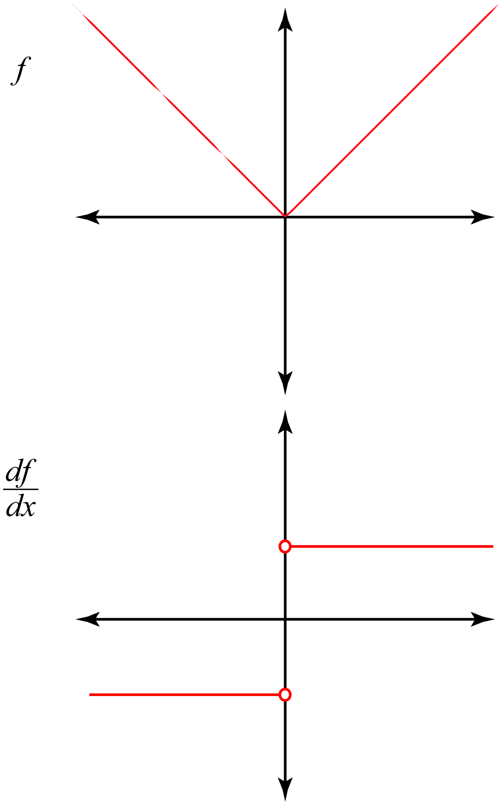


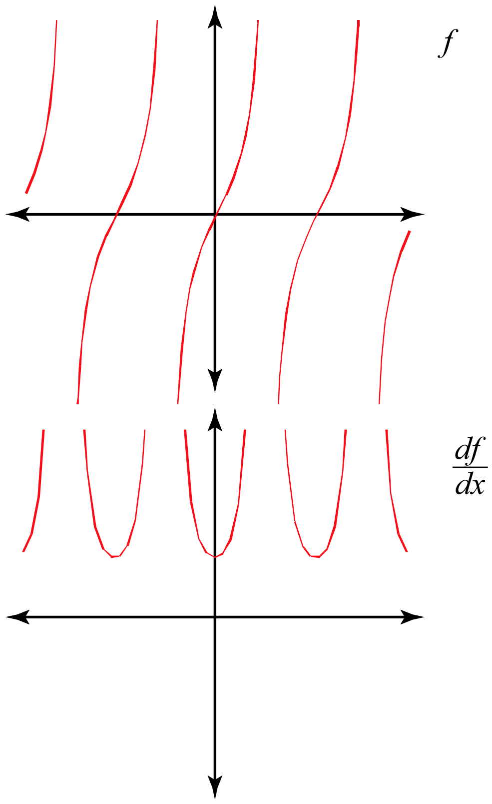
(d) $f(x) = e^x$



(e) $f(x) = |x|$

(f) $f(x) = \tan(x)$





2. (a) Let $f(x) = x^2$ and compute $\frac{d}{dx}f(x)$ using the difference quotient. Compare your answer with 1(a).

Solution:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$= \lim_{h\to 0}\frac{2xh+h^2}{h} = \lim_{h\to 0}\frac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h) = 2x+0 = 2x.$$

Thus $\boxed{f'(x) = 2x}$ Notice this agrees with 1(a).

(b) Let $f(x) = \frac{1}{x}$ and compute $\frac{d}{dx}f(x)$ using the difference quotient. Compare your answer with 1(b).

Solution:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}$$

$$= \lim_{h\to 0}\frac{\left(\frac{x-(x+h)}{x(x+h)}\right)}{h} = \lim_{h\to 0}\frac{\left(\frac{-h}{x(x+h)}\right)}{h} = \lim_{h\to 0}\left(\frac{-h}{x(x+h)}\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{-1}{x^2+hx}\right) = \frac{-1}{x^2+(0)x} = -\frac{1}{x^2}.$$

Thus $\boxed{f'(x) = -\frac{1}{x^2}}$ Notice this agrees with 1(b).

(c) Let $f(x) = \sin(x)$. Using your answer for 1(c), make a guess at the rule of f'.

Solution: It looks like $\boxed{f'(x) = \cos(x).}$

(d) Let $f(x) = e^x$. Using your answer for 1(d), make a guess at the rule of f'.

Solution: It looks like $\boxed{f'(x) = e^x.}$

3. Let $y = 3x^2 - 5x + 2$. Compute y' and give the equation of the tangent line to the graph of $y = 3x^2 - 5x + 2$ at the point $(3, 14)$.

Solution:

$$y' = \lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) + 2 - (3x^2 - 5x + 2)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5x - 5h + 2 - 3x^2 + 5x - 2}{h} = \lim_{h \to 0} \frac{6xh + 3h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h - 5)}{h} = \lim_{h \to 0}(6x + 3h - 5) = 6x + 3(0) - 5 = 6x - 5.$$

When $x = 3$ we have $y' = 6(3) - 5 = 13$. Thus the slope of the tangent line through the point $(3, 14)$ is 13. So the equation of that tangent line is

$$y - 14 = 13(x - 3)$$

or

$$\boxed{y = 13x - 25}$$

4. Let $f(x) = \sqrt[3]{x}$.

(a) Use the difference quotient to calculate $f'(0)$, if it exists.
[Hint: $h = (\sqrt[3]{h})^3$.]
Solution:

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\sqrt[3]{0+h} - \sqrt[3]{0}}{h} = \lim_{h\to 0} \frac{\sqrt[3]{h}}{h}$$

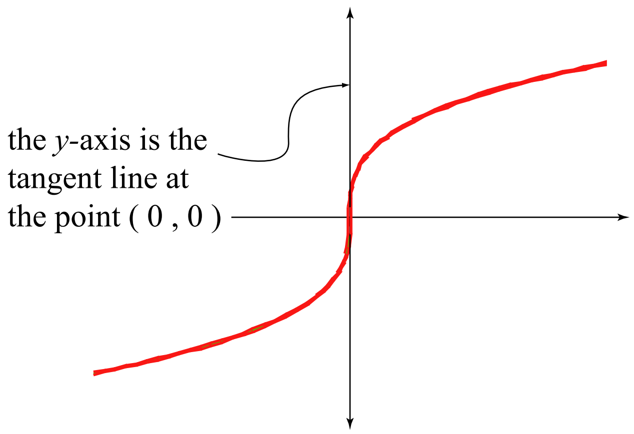
$$= \lim_{h\to 0} \frac{\sqrt[3]{h}}{(\sqrt[3]{h})^3} = \lim_{h\to 0} \frac{1}{(\sqrt[3]{h})^2}.$$

Since $(\sqrt[3]{h})^2$ is positive for any non-zero h, and $\lim_{h\to 0}(\sqrt[3]{h})^2 = 0$, we see that

$$\lim_{h\to 0} \frac{1}{(\sqrt[3]{h})^2} = \infty.$$

However, $f'(x)$ must be a real number for any x in it's domain (i.e. $f'(x) \neq \infty$ for any x) thus $\boxed{f'(x) \text{ is undefined}}$

(b) Sketch a graph of $f(x)$ along with the line tangent to f at the point $(0, 0)$.
[Hint: The tangent line will cross through the graph of f.]
Solution:



(c) Explain how your answers in parts (a) and (b) are related.

Solution: In part (a) we found that $f'(0)$ is undefined. In part (b) we see this graphically by noticing the tangent line at the point $(0, 0)$ is vertical, and therefore has undefined slope.