Read This!!!

So here is a review sheet from a DIFFERENT 112 class. Because this is from a different instructor, things may seem a bit different. It may be helpful for you to see things in a slightly different way...It may not. **If I were you, I would make sure I understood everything we've done in our class BEFORE you looked at this!** In particular: Do you know how to do all the practice problems? Do you understand all the True/False questions? Have you gone over all the old quizzes and exams?...what about old homework assignments?

Final Exam Review Sheet Math 112

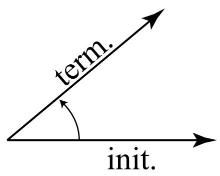
Note: Some of the problems on this review sheet are designed to be ***very*** challenging. It is my belief that if you can get to the point where you can do these problems quickly and with little assistance from the book then you will know the concepts and will be well prepared for the exam.

Question 1 : ***(The Basics)***

Recall that in Section 6.1 *we started by defining angles (both positive and negative), angles in standard position, and coterminal angles. Use those ideas to complete the following:*

(*a*) *Define what it means for two angles to be* **coterminal**. *Then for each of the following angles find two other angles that are coterminal to the given one. (either by drawing or by angle measure)*

(*i*)



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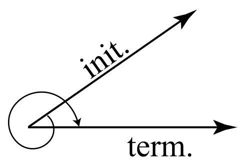

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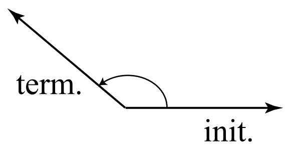

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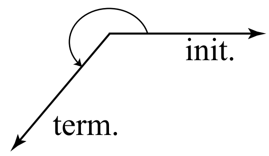


(*v*) 30° (*vi*) $\frac{\pi}{2}$ *Radians* (*vii*) 1° (*viii*) $\frac{3\pi}{7}$ *Radians*

(*b*) *If we place our angles in the coordinate plane, what does it mean for an angle to be in* **standard position**? *Once you've defined that, how do we define* **positive angles** *versus* **negative angles**? *(i.e. What is the difference between the two?)*

(*c*) *Recall that we defined (and use!) Radians rather than degrees because they have many nice properties when applied in calculus and physics. The formal definition was that* the radian measure of an angle is equal to the distance traveled along the unit circle by the point P as it moves from its starting position on the initial side to its final position on the terminal side of the angle. *However, what we are really concerned with in this class is how to* **use** *radians. So how many radians are there in one full revolution around the circle? And more specifically, how can we convert from any angle measure in degrees into the same angle measure in radians??*

(*d*) *Now convert the following angles measures from degrees to radians (or vice versa):*

(*i*) $\frac{240}{8\pi}^{\circ}$ (*ii*) $\frac{5\pi}{6}$ *Radians* (*iii*) 1° (*iv*) $\frac{\pi}{180}$ *Radians*

(*v*) 30° (*vi*) $\frac{\pi}{2}$ *Radians* (*vii*) 36° (*viii*) 125°

Question 2 : ***(Arc Length & Angular Speed)***

(*a*) *Recall that one of the nice things about measuring angles in radians was that this gave us a quick way of finding the arc length given any angle that subtends it and the radius of our circle. What was the formula for arc length given and angle in radians? How did we come up with this formula??*

(*b*) *Calculate the arc lengths subtended by the following angles in radians, θ, with given radii, R (i-iii). Then find the following central angles given the radii, R, and arc lengths, S (iv-vi):*

(*i*) $\theta = \frac{240\pi}{8}$ *and* $R = \frac{1}{10}$ *ft* (*ii*) $\theta = \frac{5\pi}{6}$ *and* $R = 6$ *cm* (*iii*) $\theta = \frac{9\pi}{7}$ *and* $R = 63$ *miles*

(*iv*) $S = 12$ *ft and* $R = \frac{3}{4}$ *ft* (*v*) $S = 54\pi$ *cm and* $R = 9$ *cm* (*vi*) $S = \frac{19}{4}$ *ft and* $R = 24$ *inches*

(*c*) *Another nice thing about measuring angles in radians was that we had a very nice formula for converting angular speed of a circle to the linear speed of a point moving on the edge of a circle. First, what is the formula for* **angular speed***? Next, what is the equation for converting from angular speed to linear speed?? Use this information to do the following problems. (Note: Pictures may be helpful to you!!)*

(*i*) *What is the angular speed of the second hand on a clock in radians per* **minute***? (Hint: Use the fact that you know exactly what angle the second hand passes through in one minute!)*

(*ii*) *What is the angular speed of the minute hand on a clock in radians per* **minute***? What about in radians per* **hour***?? If the minute hand is* 6 *inches long, then what is the linear speed of a point at the end of the minute hand?*

(*iii*) *Suppose a Merry-Go-Round is rotating at 3 revolutions every 2 minutes. What is the angular speed of the Merry-Go-Round? What is the linear speed of a horse on the Merry-Go-Round that is* 6 *feet from the center? If a second horse is* 9 *feet from the center, what is the difference in linear speed between the two horses? Who is going faster??*

(*iv*) *Suppose you get selected to be on Wheel of Fortune. If the wheel has a diameter of* 10 *feet and you spin it at 18 revolutions every 30 seconds, what is the linear speed of the point at the end in* **feet per minute***??*

Question 3 : ***(Trig Functions and Special Values)***

(*a*) *How did we define the sine and cosine functions on any real number θ (using the unit circle)? Be explicit here (see pg. 423 if you forget). How did we define the tangent function once we had the sine and cosine functions? What are the domains and ranges for each of these functions?*

(*b*) *Recall that we had an equivalent definition when we only knew a point on the terminal side of the angle rather than the specific point where it crosses the unit circle. How did we define our basic trig functions using the point in the plane?*

(*c*) *Recall that there were some special triangles which allowed us to find the exact values of sine, cosine, and tangent of some specific inputs (and all coterminal inputs!). Find* $sin(\theta)$, $cos(\theta)$, $tan(\theta)$,

$csc(\theta)$, $sec(\theta)$, and $cot(\theta)$ for each of the following special values:

$(i)\ \theta = 0$	$(ii)\ \theta = \frac{\pi}{6}$	$(iii)\ \theta = \frac{\pi}{4}$	$(iv)\ \theta = \frac{\pi}{3}$
$(v)\ \theta = \frac{\pi}{2}$	$(vi)\ \theta = \frac{2\pi}{3}$	$(vii)\ \theta = \frac{3\pi}{4}$	$(viii)\ \theta = \frac{5\pi}{6}$
$(ix)\ \theta = \pi$	$(x)\ \theta = \frac{7\pi}{6}$	$(xi)\ \theta = \frac{5\pi}{4}$	$(xii)\ \theta = \frac{4\pi}{3}$
$(xiii)\ \theta = \frac{3\pi}{2}$	$(xiv)\ \theta = \frac{5\pi}{3}$	$(xv)\ \theta = \frac{7\pi}{4}$	$(viii)\ \theta = \frac{11\pi}{6}$

(d) Given a point on the terminal side of the angle θ find $sin(\theta)$, $cos(\theta)$, $tan(\theta)$, $csc(\theta)$, $sec(\theta)$, and $cot(\theta)$:

$(i)\ (3, 4)$	$(ii)\ (-6, 8)$	$(iii)\ (12, -5)$	$(iv)\ (-15, -20)$
$(v)\ (\sqrt{5}, -\sqrt{11})$	$(vi)\ (-\sqrt{6}, \sqrt{3})$	$(vii)\ (-\sqrt{3}, -1)$	$(viii)\ (\sqrt{17}, 0)$

Question 4 : ***(Algebra & Identities)***

(a) State **and prove** *the following identities: (You should* **know** *these for the test!)*

- *(i) Pythagorean Identity (with sine and cosine)*
- *(ii) Periodicity Identities (for sine, cosine, tangent, cosecant, secant, and cotangent)*
- *(iii) Negative Angle Identities (for all six trig functions)*
- *(iv) Reciprocal Identities (for all six trig functions)*
- *(v) New Pythagorean Identities (with tangent & secant, or cotangent & cosecant)*
- *(vi) Addition and Subtraction Identities (for sine, cosine, and tangent)*
- *(vii) Cofunction Identities (for all six trig functions)*

(b) Now use those identities to simplify the following expressions:

$(i)\ sin(2\pi - x)tan(3\pi + x)$	$(ii)\ \frac{sin(x)[cos^2(4\pi+x)-sin^2(2\pi-x)]}{cos(x-2\pi)}$	$(iii)\ 2cos(x - 2\pi) - cos(x + 2\pi) - cos(4\pi - x)$
$(iv)\ sin^2(-x)$	$(v)\ (\frac{4cos^2t}{sin^2t})(\frac{sin^2t}{2sin^2t+2cos^2t})^2$	$(vi)\ sin(-x) + sin(x) + cos(x) + cos(-x)$

(c) Prove the following identities: (I highly recommend actually picking a strategy each time before you do any work. But remember that there is no wrong *strategy so just try things!!)*

$(i)\ \frac{tanx+cotx}{cscx} = secx$ $(ii)\ \frac{cos^4\theta - 2cos^2\theta + 1}{sin^2\theta cos^2\theta} = tan^2\theta$ $(iii)\ sec^4x - tan^4x = 1 + 2tan^2x$

$(iv)\ \frac{cosx-siny}{cosy-sinx} = \frac{cosy+sinx}{cosx+siny}$ $(v)\ cos^4x - sin^4x = cos^2x - sin^2x$ $(vi)\ sin^2x(cotx+1)^2 = cos^2x(tanx+1)^2$

(d) Prove the following identities:

$(i)\ \frac{cos(x-y)}{cos(x+y)} = \frac{coty+tanx}{coty-tanx}$ $(ii)\ sec(x+y) = \frac{1}{cosxcosy - sinxsiny}$

$(iii)\ \frac{cos(x-y-\pi)}{sinxcos(y+\pi)} = tan(\frac{\pi}{2}-x) + cot(x+2\pi)$ $(iv)\ sinxsiny = \frac{1}{2}[cos(x-y) - cos(x+y)]$

Question 5 : ***(Basic Graphs)***

(a) Sketch the basic sine, cosine, and tangent graphs. Specifically, how did we graph the tangent function for the first time (this is an important characterization of the tangent function!!)??

Question 6 : ***(Periodic Graphs and Simple Harmonic Motion)***

(a) Given a function of the form $f(t) = Asin(bt+c)$ *or* $g(t) = Acos(bt+c)$ *what do each of the constant terms represent? (i.e. What do they tell us about the important features of our function?)*

(b) Define in words what the **period** *of a function is. Also define in words what the* **amplitude** *is. If we know the* **phase shift** *of our function, what specifically does this tell us about the graph of the function?*

(c) Graph the following functions: (Note: (vi) is very challenging and will not be on the exam, but it is a good problem for testing whether you really understand why our graphs work the way they do!)

$(i)\ f(x) = 3sin(2x-\pi)$ $(ii)\ g(x) = \frac{1}{2}cos(\frac{1}{3}x + \frac{\pi}{2})$ $(iii)\ h(t) = -2sin(3t + \frac{\pi}{2})$

$(iv)\ f(t) = -\frac{3}{2}cos(x+7)$ $(v)\ g(t) = \pi sin(\pi t - 3\pi)$ $(vi)\ p(x) = 2tan(2x-4)$

(d) Write a function with the following characteristics: (Note: there are many correct answers for each one!)

$(i)\ Amp = 3,\ Period = \pi,\ and\ Phase\ Shift = -\frac{\pi}{3}$ $(ii)\ Amp = \frac{1}{2},\ Per = \frac{\pi}{3},\ P.S. = -\frac{\pi}{6}$

$(iii)\ Amp = 4,\ Per = \frac{4}{3},\ P.S. = -\frac{7}{6}$ $(iv)\ Amp = \frac{4}{3},\ Per = -6,\ P.S. = 1$

Question 7 *: (Miscellaneous Questions)*

(a) Recall that given two pieces of pertinent information we can often evaluate all of our trig functions for unknown angles. For example, when given a point on the terminal side (so the two pieces of information are x and y) we can find all of our trig functions. Likewise, if given just one coordinate and the radius of the line to that point we can find everything. Continuing in this thread we can be given sinθ and the quadrant our angle is in and this is also enough information to solve for all of our trig functions. Using this idea take the given pieces of information in each case and find sinθ, cosθ, tanθ, cscθ, secθ, and cotθ:

(i) $sin(\theta) = .6$ *and* $tan(\theta) = -.75$

(ii) $cos(\theta) = \frac{\sqrt{3}}{2}$ *and* $\frac{3\pi}{2} < \theta < 2\pi$

(iii) $sin(\theta) = -\frac{8}{10}$ *and* $cos(\theta) > 0$

(iv) *pt. on terminal side of* θ *is* $(\sqrt{4}, -\sqrt{5})$

(v) $sin(x) = \frac{1}{2}$, $cos(y) = \frac{4}{5}$, $\theta = x + y$, *and* x *and* y *are in quadrant* 1

(vi) *pt. on terminal side of* θ *is* $(-\sqrt{11}, \sqrt{5})$

(vii) $sin(x) = -\frac{\sqrt{7}}{4}$, $cos(y) = -\frac{\sqrt{5}}{3}$, $\theta = x - y$, *and* x *and* y *are in quadrant* 3

(viii) $tan(x) = -\frac{12}{5}$, $\theta = x - \frac{\pi}{2}$, *and* x *is in quadrant 2*

(b) Use the Addition and Subtraction Identities to evaluate the following trigonometric expressions **exactly***:*

(i) $sin(\frac{\pi}{2} + \frac{\pi}{3})$

(ii) $cos(\frac{\pi}{4} + \frac{\pi}{6})$

(iii) $tan(\frac{\pi}{4} - \frac{\pi}{3})$

(iv) $sec(\frac{3\pi}{4} - \frac{5\pi}{6})$

(v) $csc(-\frac{4\pi}{3} + \frac{\pi}{2})$

(vi) $sin(\frac{5\pi}{12})$

(vii) $csc(-\frac{\pi}{12})$

(viii) $cot(\frac{7\pi}{12})$

$sec(\frac{5\pi}{12}) + csc(\frac{11\pi}{12})$

Question 8 *:* **(More Algebra & Identities)**

(a) Use the Addition Identities to prove the Double-Angle Identities for Sine, Cosine, and Tangent (i.e. what is $sin(2\theta)$*,* $cos(2\theta)$*,* $tan(2\theta)$*?)*

(b) Now that you have the Double-Angle Identities written down, use them to solve the following problems **exactly** *without a calculator:*

(i) *Given* $sin(\theta) = .6$ *and* $\frac{\pi}{2} \leq \theta \leq \pi$ *Find* $sin(2\theta)$, $cos(2\theta)$, *and* $tan(2\theta)$

(ii) *Given* $cos(\theta) = \frac{\sqrt{12}}{4}$ *and* $\frac{3\pi}{2} \leq \theta \leq 2\pi$ *Find* $sin(2\theta)$, $cos(2\theta)$, *and* $tan(2\theta)$

(iii) *Given* $tan(\theta - \pi) = \frac{-\sqrt{5}}{2}$ *and* $\frac{3\pi}{2} \leq \theta \leq 2\pi$ *Find* $sin(2\theta)$, $cos(2\theta)$, *and* $tan(2\theta)$

*(iv) *Given* $arctan(\frac{6}{\sqrt{13}}) = \alpha$ *Find* $sin(2\alpha)$, $cos(2\alpha)$, *and* $tan(2\alpha)$

*(v) *Given* $cos^{-1}(-\frac{2}{\sqrt{5}}) = \theta$ *Find* $sin(2\theta)$, $cos(2\theta)$, *and* $tan(2\theta)$

(c) *Recall the Half-Angle Identities:* $sin(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{2}}$, $cos(\frac{x}{2}) = \pm\sqrt{\frac{1+cosx}{2}}$, $tan(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{1+cosx}}$. *Use these identities to evaluate the following trig expressions* **exactly***:*

(i) $cos(\frac{\pi}{8})$ (ii) $sin(\frac{3\pi}{8})$ (iii) $tan(\frac{x}{2})$ *given that* $tan^{-1}(3) = x$

(iv) $cos(-\frac{5\pi}{12})$ (v) $tan(-\frac{\pi}{12})$ (vi) $sec(\frac{\theta}{2})$ *given that* $cos^{-1}(\frac{4}{\sqrt{20}}) = \theta$

Question 9 *:* **(Proofs and Simplifications Using Identities)**

(a) *Prove the following identities:*

(i) $\frac{sin(2x)-1}{(sin(\frac{\pi}{2}-x)-cos(\frac{\pi}{2}-x))^2-1} = -1$ (ii) $\frac{sinx(secx+1)}{tan(x+3\pi)} = \frac{tan(x-\pi)}{cscx(secx-1)}$ (iii) $2cosx - 2cos^3x = sin(x)sin(2x)$

(iv) $\frac{cos^4x-sin^4x}{1-cot^4(\frac{\pi}{2}-x)} = cos^4x$ (v) $tanx + cotx = sec(x)csc(x)$ (vi) $\frac{cos(x-y)}{cos(x)cos(y)} = 1 + tan(x)tan(y)$

(b) *Recall the power reducing identities* $sin^2x = \frac{1-cos2x}{2}$ *and* $cos^2x = \frac{1+cos2x}{2}$. *Use these identities to write the following expressions with no exponents!*

(i) sin^4x (ii) $sin^3x + cos^3x$ (iii) $cos^4(3\theta)$

Question 10 *:* **(A Little Review of Inverses)**

(a) *Recall that we needed a special condition on our functions to be able to find an inverse at all. This condition was being* **one-to-one**. *Define what it means for a function to be one-to-one.*

(b) *Why is it that we needed our functions to be one-to-one for us to be able to define inverses??*

(c) *For each of the following one-to-one functions, find the inverse function.*

(i) $f(x) = 7x - \frac{14}{3}$ (ii) $g(x) = -x$ (iii) $h(x) = (x-4)^3 + 6$ (iv) $k(x) = -\frac{1}{8}x^3$

(v) $p(x) = \frac{3}{8}e^x$ (vi) $q(x) = ln(x) + 4$ (vii) $n(x) = \frac{1}{2}log(e^{3x-2})$ $(viii)$ $s(x) = \frac{x+2}{x-4}$

(d) *State the Round-Trip Theorem for inverse functions* $f(x)$ *and* $g(x)$ *with ALL details! (If you don't remember what these details are, refer to section* 3.7*)*

Question 11 *:* **(Review of** e^x **and** $ln(x)$**)**

(a) *Recall that we* **defined** *the Natural Logarithm function* $ln(x)$ *to be the inverse of the Natural Exponential function* e^x. *How exactly did we state that definition? This special kind of definition gave us a direct correspondence between exponential statements and logarithmic ones. Use that idea to complete the following statements:*

(i) *"If* $e^a = b$ *then* $ln(b) = ...$*"* (ii) *" If* $ln(u) = v$ *then...*"

(b) *Now use those statements to take each statement below and write an equivalent one. (i.e. change from logarithmic expressions to exponential expression or vice versa):*

(i) $ln(29) = 3.3673$ (ii) $e^{5.0626} = 158$ (iii) $ln(14) = 2.6391$

(iv) $e^{5.5} = 244.692$ (v) $ln(186) = 5.2257$ (vi) $ln(k + t) = d$

(c) *State the complete form of the Round-Trip Theorem for* e^x *and* $ln(x)$. *(See your definition from above!)*

Question 12 *:* **(Inverse Trig Functions)**

(a) *Recall that we defined inverse trig functions in a similar way (i.e. we* **created** *the functions to be inverses of our restricted trig functions). State the definitions of the inverse trig functions* $sin^{-1}(x)$, $cos^{-1}(x)$, *and* $tan^{-1}(x)$ *with all possible details! (If you don't remember all of the details then go back and look it up in section* 7.4*)*

(b) *State the Round-Trip Theorem properly for each pair of a restricted trig function and its inverse (i.e.* $cos(x)$ *and* $arccos(x)$*). (Again look back to your proper definition above!)*

(c) *Evaluate each of the following expressions: (Hint: For many of them you can use the Round-Trip Theorem, but what must you check first??):*

(i) $sin(sin^{-1}(\frac{1}{2}))$ (ii) $cos^{-1}(cos(\frac{3\pi}{2}))$ (iii) $tan(tan^{-1}(47.68))$

(iv) $cos(cos^{-1}(-\frac{\pi}{4}))$ (v) $sin(sin^{-1}(1.375))$ (vi) $cot(tan^{-1}(-4))$

(d) *Evaluate the following expressions* **exactly***:*

(i) $cos[sin^{-1}(\frac{1}{2})]$ (ii) $sin[cos^{-1}(-\frac{\sqrt{3}}{2})]$ (iii) $cos[tan^{-1}(-\frac{4}{3})]$

(iv) $cos[sin^{-1}(\frac{\sqrt{7}}{4})]$ (v) $csc[tan^{-1}(\frac{\sqrt{5}}{\sqrt{20}})]$ (vi) $3sec[sin^{-1}(\frac{2\sqrt{2}}{3})]$

(e) *For good examples of word problems with pictures already drawn for you, refer to problems 52, 53, and 54 in section* 7.4

Question 13 *:* **(Solving Trig Equations)**

(a) *Recall that we came up with an algorithm for how to solve all three of the basic forms of trig equations (i.e.* $sinx = c$, $cosx = c$, *and* $tanx = c$*). Write down this Algorithm for finding all possible solutions to a basic trig equation. How did we come up with these solutions?*

(b) *Solve the following trig equations: (i.e. Find* **ALL** *solutions!)*

$(i)\ sin(\theta) = \frac{\sqrt{2}}{2}$ $(ii)\ cos(\theta) = \frac{\sqrt{3}}{2}$

$(iii)\ sin(3x) = -\frac{4}{5}$ $(iv)\ -4cos^2(x) - 4sin(x) = -5$

$(v)\ cot(2x) = -\frac{1}{\sqrt{3}}$ $(vi)\ 2sinxcosx - 2cosx - \sqrt{3}sinx + \sqrt{3} = 0$

$(vii)\ tan(\frac{x}{2}) = -1$ $(viii)\ 10sin(x)cos(x) - sin(2x) = -2\sqrt{2}$

Question 14 *: (Trig Functions with Angles. a.k.a. "Triangle Trig")*

(*a*) *Recall that we used the Point-in-the-Plane definition of our Trig functions to come up with definitions for use in Right Triangles (this is an important condition!!) by placing those Right Triangles into the coordinate plane. Try to reproduce our proof of the following definitions:*

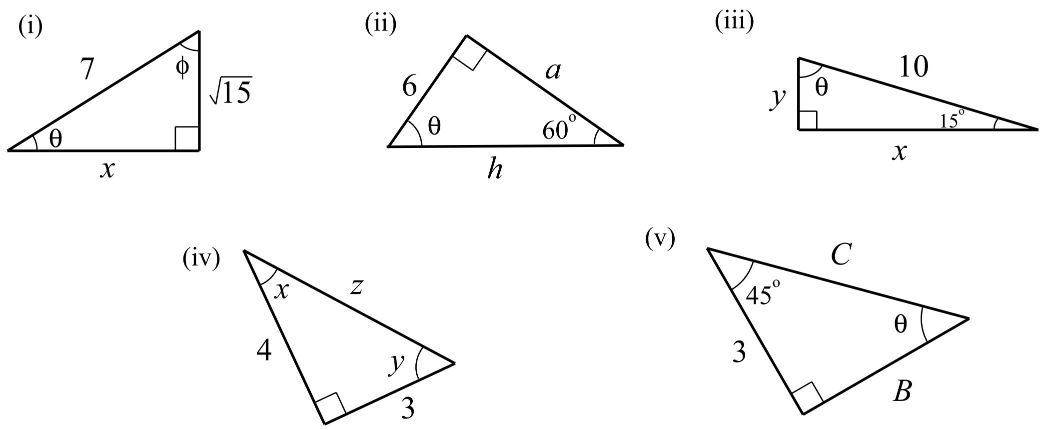
$(i)\ sin(\theta) = \frac{opposite}{hypotenuse}$ $(ii)\ cos(\theta) = \frac{adjacent}{hypotenuse}$ $(iii)\ tan(\theta) = \frac{opposite}{adjacent}$

$(iv)\ csc(\theta) = \frac{hypotenuse}{opposite}$ $(v)\ sec(\theta) = \frac{hypotenuse}{adjacent}$ $(vi)\ cot(\theta) = \frac{adajcent}{opposite}$

(*b*) *Just to remind you if you've forgotten, there is a helpful mnemonic tool for remembering the Triangle Trig definitions: "SOH CAH TOA" which corresponds to the first row of definitions above. We can then use the reciprocal identities to get the others.*

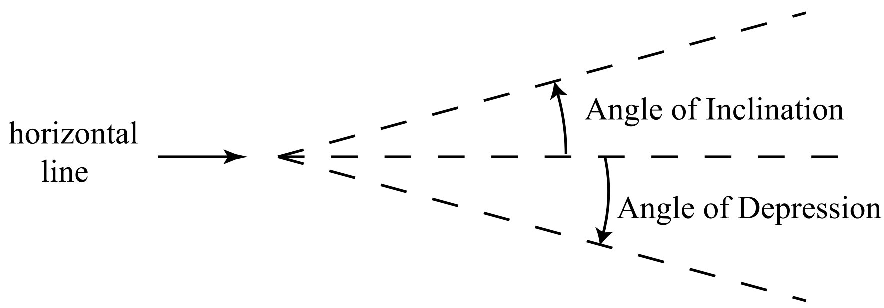
(*c*) *Recall that "Solving a Right Triangle" means finding the measures of all angles and lengths of all sides of the triangle. The trig definitions above combined with the Pythagorean Theorem* ($a^2 + b^2 = c^2$) *allows us to take 2 pieces of information (not including the fact that we have a right angle) and solve a triangle completely (provided of course that at least one of the pieces of information is a side length). Why do we need one of the pieces of information to be a side length??*

(*d*) *Use the strategy above to solve the following triangles completely:*



Question 15 *: (Triangle Trig Word Problems)*

(*a*) *Recall that we defined the* **Angle of Inclination** *as the positive angle rotated "up" from a horizontal line and we defined the* **Angle of Depression** *as the positive angle rotated "down"* *from a horizontal line. (See diagram below)*



(b) *Solve the following word problems using Triangle Trig: (DRAW PICTURES!!!)*

(i) *Lisa Simpson goes for a walk along the river one sunny afternoon and decides that she'd like to know how wide the river is. Being the smart girl she is, she comes up with a plan. There is a tree directly across the river from her (on the edge of the bank). She then walks 120 feet parallel to the river and notices that the angle between her path and the tree is now* $60°$. *How wide is the river? What if the angle created was* $45°$*?* $30°$*??*

(ii) *As Bart Simpson prepares to attempt a jump over Springfield Gorge on his skateboard he walks up a steep path to the top of a hill overlooking the Gorge (he needs a place to start from so he can pick up enough speed). Suppose the Angle of Elevation of the path on the hill is* $45°$ *and Bart walks up the hill for 200 ft. How high above the edge of the Gorge is Bart before he gets on his skateboard? (Vertical height!) What if he had walked 150 ft and the angle of elevation had been* $60°$*? Or 300 ft and* $30°$*??*

(iii) *As Yoda gets ready to fight Count Dooku he takes a moment to really size up his opponent and the situation that he's now in. His eye-level is at 30 inches from the ground. If the angle of depression between his eye-level and Dooku's feet is* $15°$ *and the angle of elevation between his eye-level and the top of Dooku's head is* $30°$ *then how far apart are they? And how tall is Dooku? (Hint: You should use the Half-Angle Identity when working with* $15°$*)*

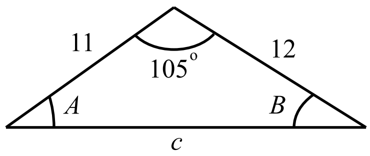
(iv) *In one of his crazy schemes, Peter Griffin decides that he's going to ride a zipline from the top of a nearby building through the window into his favorite bar The Drunken Clam. The height of the building he's on is 10 meters and the height of the window is 1 meter. If the angle of depression from the top of the building to the window is* $22.5°$ *then how much wire does he need to make the zipline? (Hint:* $22.5°$ *is* **half** *of* $45°$*!)*

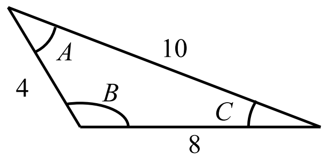
Question 16 *:* **(Law of Cosines)**

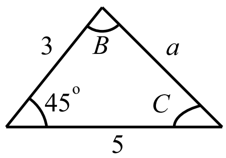
(a) *State and prove the Law of Cosines (all three cases!)*

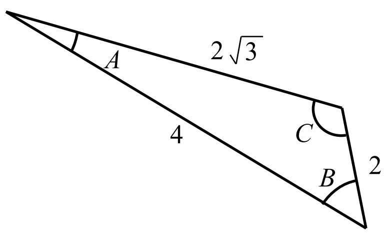
(b) *Now use the original Law of Cosines to prove the Alternate Form of the Law of Cosines (all three cases!)*

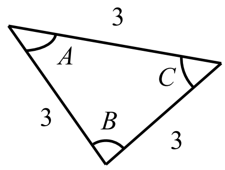
(c) *Recall that the Law of Cosines (and its Alternate Form) give us the ability to solve two types of oblique triangles (that is, triangles without a right angle), namely Side-Angle-Side Triangles (SAS)* *and Side-Side-Side Triangles (SSS). First of all, what does each of these designations mean (with respect to what kind of information we're given)?? Now, solve the following oblique triangles:*

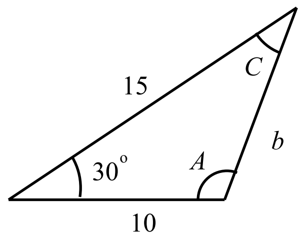








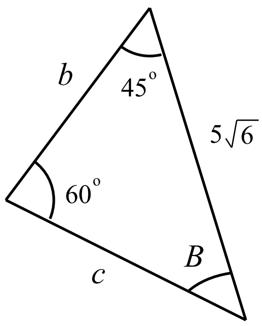


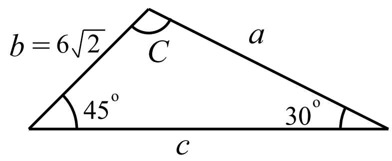


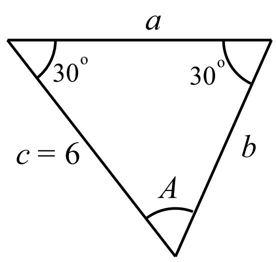
Question 17 *:* **(Law of Sines)**

(*a*) *State and prove the Law of Sines (all three pieces!)*

(*b*) *Recall that the Law of Sines gives us a way to solve Angle-Angle-Side triangles (AAS) and guarantees that in this case we will always get one solution. Solve the following AAS triangles using the Law of Sines:*

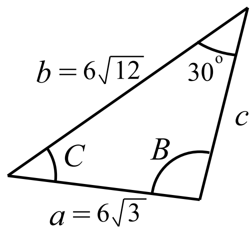






(*c*) *Recall also that the Law of Sines gives us a way to work with Side-Side-Angle triangles (SSA), however this is where we run into what is known as the* **ambiguous case**. *Explain exactly why we have an ambiguous case here (pictures are helpful!!).*

(*d*) *Now solve the following SSA triangles using the Law of Sines:*



$C = 45°$	$B = 135°$
$a = 6\sqrt{2}$	$c = 3\sqrt{6}$
$c = \frac{12}{\sqrt{3}}$	$b = 6\sqrt{3}$

Question 18 : **(Complex Numbers)**

(*a*) *Recall that we created the complex numbering system with the goal of solving polynomial equations that we weren't able to solve using the real numbers (i.e.* $x^2 + 1 = 0$*). In creating this system we came up with a new number* $i = \sqrt{-1}$ *Write down the* 5 *properties of complex numbers (if you don't remember these, check pg. 306)*

(*b*) *Evaluate the following expressions and write your answer in the standard form for complex numbers* $a + bi$ *(i.e. do the addition, subtraction, multiplication or division)*

(*i*) $(3 + i) + (6 - 4i)$ (*ii*) $(-2 + 3i) - (14 + \frac{1}{2}i)$ (*iii*) $(9i - 6) + 3(-2 - 3i)$

(*iv*) $(6 + i)(3 - 2i)$ (*v*) $(-3 + 4i)(4 + 3i)$ (*vi*) $4i(2 - 6i)(-1 + \frac{2}{3}i)$

(*vii*) $(a + bi)(a - bi)$ (*viii*) $\frac{4-2i}{3+i}$ (*ix*) $\frac{3(2i-7)}{-i(4+3i)}$

(*x*) i^{57} (*xi*) $(-3i + 5)^2$ (*xii*) $(-i)^{15}$

(*c*) *Define the* **conjugate** *of any complex number. (Recall that the product of any complex number with its conjugate is guaranteed to be a* **non-negative, real number**.

(*d*) *Find the conjugate of each of your answers from part* (*b*) *above.*

(*e*) *Compute the following square roots by using complex numbers:*

(*i*) $\sqrt{-2}$ (*ii*) $\sqrt{-7}$ (*iii*) $-\sqrt{4}$ (*iv*) $-3\sqrt{\frac{1}{9}} + \sqrt{-144}$

(*v*) $\sqrt{-16}$ (*vi*) $3\sqrt{-5}$ (*iii*) $\sqrt{-\frac{1}{4}}$ (*iv*) $3\sqrt{16} + \sqrt{-25}$

Question 19 : **(Theory of Equations)**

(*a*) *State the Fundamental Theorem of Algebra.*

(*b*) *Recall that having Complex Numbers allows us to factor any polynomial into linear factors. i.e. Given any polynomial* $f(x)$ *of degree* $n > 0$ *with leading coefficient* d *we can find* n *complex numbers (as many as the degree of the polynomial!!)* $c_1, c_2, \ldots, c_n$ *such that we can factor* $f(x)$ *to get* $f(x) = d(x - c_1)(x - c_2)...(x - c_n)$*. Moreover, we know that the complex numbers* $c_1, c_2, \ldots, c_n$ *are the* **only** *roots of* $f(x)$*. Use this information when you approach the following problems:*

(i) Find a polynomial $f(x)$ *of degree* 4 *that has roots 1 (with mult. 1),* -2 *(with mult. 2), and 0 AND such that* $f(2) = 16$.

(ii) Find a polynomial $g(x)$ *of degree* 3 *that has roots* -1 *and* i *AND such that* $f(3) = 80$.

(iii) Find a polynomial $h(x)$ *of degree* 5 *that has roots* $3+i$, $2i$, *and* $-\frac{1}{2}$ *with leading coefficient* $d = -2$.

(iv) Find a polynomial $h(x)$ *of degree* 4 *that has* $6-2i$ *as a root (with mult. 2) and with leading coefficient* $d = 3$.

(c) When doing the problems above, did you remember the rule that for any polynomial function $f(x)$, *if a complex number is a root of* $f(x)$ *then its conjugate is a root also? Go back and make sure that you used this fact while doing the problems above.*

(d) Find **all** *of the roots of the following polynomials (remember that if it is a degree* n *polynomial then there should be exactly* n *roots, although they may not all be different): (Hints: You may need to use division, substitution, factoring, etc. etc.)*

(i) $f(x) = x^3 - 8$ *(ii)* $g(x) = x^4 + 3x^2 + 2$ *(iii)* $h(x) = x^4 - x^2 - 1$

(v) $f(x) = x^2 - 2x + 5$ *(vi)* $g(x) = \frac{1}{2}x^2 + 3x + 5$ *(iii)* $h(x) = x^5 + 2x^3 - 8x$ *Given that* $\sqrt{2}$ *is a root*

Question 20 *:* **(The Complex Plane and Polar Form)**

(a) Recall how we defined the Complex Plane (or Complex Coordinate Plane). Plot the following complex numbers in the complex plane:

(i) $3+i$ *(ii)* $-2+3i$ *(iii)* $3(-2-3i)$ *(iv)* $\frac{3}{2} - 4i$

(b) We then used the Complex Plane to define the absolute value of any complex number. How exactly did we define the absolute value of a complex number?? i.e. $|a+bi| = \ldots$

(c) This gave us a way of expressing any complex number in **Polar Form**. *Explain how we came up with the formula* $a + bi = r[cos\theta + isin\theta]$ *with* $a = rcos\theta$ *and* $b = rsin\theta$.

(d) Express the following complex numbers in Polar Form: (check your answers!!)

(i) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ *(ii)* $-i$ *(iii)* $-2\sqrt{3} + 2i$ *(iv)* $-6 - 6i$

(v) $\frac{1}{4} - i\frac{\sqrt{3}}{4}$ *(vi)* -1 *(vii)* $-2\sqrt{2} + 2i\sqrt{2}$ *(viii)* $i - 1$

(e) Recall that the whole purpose of introducing Polar Form was to simplify other calculations with complex numbers. Towards this end we developed rules for multiplication and division of complex numbers in Polar Form. State AND **prove** *the Multiplication and Division Rules for Complex Numbers written in Polar Form.*

(f) Now use those rules to evaluate the following expressions (i.e. write the complex numbers in

polar form if necessary and then compute) For the left four... check your work by comparing with the calculations that you did above in Standard Form (you'll need to convert back to Standard Form to compare them!):

(i) $(6+i)(3-2i)$

(ii) $(2[cos(\frac{5\pi}{6})+isin(\frac{5\pi}{6})])(3[cos(\frac{7\pi}{4})+isin(\frac{7\pi}{4})])$

(iii) $\frac{4-2i}{3+i}$

(iv) $(-3[cos(\frac{3\pi}{2})+isin(\frac{3\pi}{2})])(\frac{1}{2}[cos(\frac{\pi}{6})+isin(\frac{\pi}{6})])$

(v) $4i(2-6i)(-1+\frac{2}{3}i)$

(vi) $(-1[cos(\frac{2\pi}{11})+isin(\frac{2\pi}{11})])(\frac{4}{3}[cos(\frac{3\pi}{5})+isin(3\frac{\pi}{5})])$

(vii) $\frac{3(2i-7)}{-i(4+3i)}$

$(viii)$ $(-6[cos(\frac{\pi}{8})+isin(\frac{\pi}{8})])(\frac{5}{2}[cos(\frac{3\pi}{8})+isin(\frac{3\pi}{8})])$

Question 21 *:* **(Roots of Complex Numbers)**

(a) *State and* **explain** *DeMoivre's Theorem for taking powers of complex numbers written in Polar Form.*

(b) *Use DeMoivre's Theorem to compute the following:*

(i) $(\frac{1}{2}+i\frac{\sqrt{3}}{2})^5$

(ii) $(-sqrt3+i)^6$

(iii) $(-2\sqrt{3}+2i)^4$

(iv) $(-6-6i)^{10}$

(v) $(\frac{1}{4}-i\frac{\sqrt{3}}{4})^{12}$

(vi) $(1+i)^{11}$

(vii) $(-2\sqrt{2}+2i\sqrt{2})^4$

$(viii)$ $(i-1)^5$

(c) *Define what an n^{th} Root of Unity is.*

(d) *Find the 3rd, 4th, 5th, 6th, 8th, 10th, and 12th roots of unity.*

Study hard and I'm sure everything will go fine. As before, if you know how to do everything on this review sheet from memory (and quickly!) then you will definitely be prepared for this exam and there will be no surprises! Good Luck!