Examples for Assignment #5

Here are a few examples for finding the inverses of logarithmic and exponential functions. Remember that $\ln(x)$ means $\log_e(x)$ and $\log(x)$ means $\log_{10}(x)$. Here are the main two properties of logs

$$\text{I. } \log_b(b^k) = k$$
$$\text{II. } b^{\log_b(k)} = k$$

Also recall the 3 steps for finding the rule of f^{-1}

Step 1. Set $f(y) = x$.
Step 2. Solve for y.
Step 3. Set $f^{-1}(x) = y$

Examples:

1. Find the rule of f^{-1} for all of the following.

   (a) $f(x) = 4\ln(x + 17)$

   Solution:

   Step 1. $f(y) = 4\ln(y + 17)$, so we set

   $$x = 4\ln(y + 17)$$

   Step 2. Now we solve for y:

   $$x = 4\ln(y + 17)$$
   $$\Rightarrow \frac{x}{4} = \ln(y + 17)$$
   $$\Rightarrow e^{x/4} = e^{\ln(y+17)}$$
   $$\Rightarrow e^{x/4} = y + 17 \qquad \text{(using prop. II)}$$
   $$\Rightarrow e^{x/4} - 17 = y$$

   Step 3. Setting $f^{-1}(x) = y$ we see

   $$f^{-1}(x) = e^{x/4} - 17.$$

(b) $f(x) = 12\log_4(2x+9)$

Solution:

Step 1. $f(y) = 12\log_4(2y+9)$, so we set

$$x = 12\log_4(2y+9)$$

Step 2. Now we solve for y:

$$\begin{aligned}
x &= 12\log_4(2y+9) \\
\Rightarrow \frac{x}{12} &= \log_4(2y+9) \\
\Rightarrow 4^{x/12} &= 4^{\log_4(2y+9)} \\
\Rightarrow 4^{x/12} &= 2y+9 \qquad \text{(using prop. II)} \\
\Rightarrow 4^{x/12} - 9 &= 2y \\
\Rightarrow \frac{4^{x/12}-9}{2} &= y
\end{aligned}$$

Step 3. Setting $f^{-1}(x) = y$ we see

$$f^{-1}(x) = \frac{4^{x/12}-9}{2}.$$

(c) $f(x) = 18 \cdot 19^{x+6}$

Solution:

Step 1. $f(y) = 18 \cdot 19^{y+6}$, so we set

$$x = 18 \cdot 19^{y+6}$$

Step 2. Now we solve for y:

$$\begin{aligned}
x &= 18 \cdot 19^{y+6} \\
\Rightarrow \frac{x}{18} &= 19^{y+6} \\
\Rightarrow \log_{19}\left(\frac{x}{18}\right) &= \log_{19}\left(19^{y+6}\right) \\
\Rightarrow \log_{19}\left(\frac{x}{18}\right) &= y+6 \qquad \text{(using prop. I)} \\
\Rightarrow \log_{19}\left(\frac{x}{18}\right) - 6 &= y
\end{aligned}$$

Step 3. Setting $f^{-1}(x) = y$ we see

$$f^{-1}(x) = \log_{19}\left(\frac{x}{18}\right) - 6.$$

(d) $f(x) = 2 \cdot e^{5x+7}$

Solution:

Step 1. $f(y) = 2 \cdot e^{5y+7}$, so we set

$$x = 2 \cdot e^{5y+7}$$

Step 2. Now we solve for y:

$$\begin{aligned} x &= 2 \cdot e^{5y+7} \\ \Rightarrow \frac{x}{2} &= e^{5y+7} \\ \Rightarrow \ln\left(\frac{x}{2}\right) &= \ln\left(e^{5y+7}\right) \\ \Rightarrow \ln\left(\frac{x}{2}\right) &= 5y + 7 \qquad \text{(using prop. I)} \\ \Rightarrow \ln\left(\frac{x}{2}\right) - 7 &= 5y \\ \Rightarrow \frac{\ln\left(\frac{x}{2}\right) - 7}{5} &= y \end{aligned}$$

Step 3. Setting $f^{-1}(x) = y$ we see

$$f^{-1}(x) = \frac{\ln\left(\frac{x}{2}\right) - 7}{5}.$$

2. Use the round trip theorem to check you answers for problems 1(a) and 1(d).

Solution

1(a) We need to show $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Well,

$$f(f^{-1}(x)) = f\left(e^{x/4} - 17\right) = 4\ln\left(e^{x/4} - 17 + 17\right) = 4\ln\left(e^{x/4}\right) \overset{\text{by I}}{=} 4 \cdot \frac{x}{4} = x,$$

and

$$f^{-1}(f(x)) = f^{-1}\left(4\ln(x+17)\right) = e^{\frac{4\ln(x+17)}{4}} - 17 = e^{\ln(x+17)} - 17 \overset{\text{by II}}{=} x + 17 - 17 = x.$$

1(d) Again, we need to show $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Well,

$$f(f^{-1}(x)) = f\left(\frac{\ln\left(\frac{x}{2}\right) - 7}{5}\right) = 2 \cdot e^{5 \cdot \frac{\ln\left(\frac{x}{2}\right)-7}{5}} + 7$$

$$= 2 \cdot e^{\ln\left(\frac{x}{2}\right)-7} + 7 \overset{\text{by II}}{=} 2 \cdot \frac{x}{2} - 7 + 7 = 2 \cdot \frac{x}{2} = x,$$

and

$$f^{-1}(f(x)) = f^{-1}\left(2 \cdot e^{5x+7}\right) = \frac{\ln\left(\frac{2 \cdot e^{5x+7}}{2}\right) - 7}{5}$$

$$= \frac{\ln\left(e^{5x+7}\right) - 7}{5} \overset{\text{by I}}{=} \frac{5x + 7 - 7}{5} = \frac{5x}{5} = x.$$