

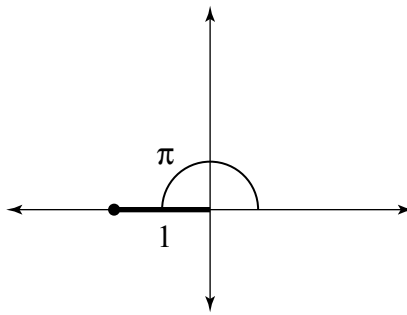
Math 112
Jonny Comes
Fall 2005
Assignment #9
Partial Solutions

Additional Exercises: (Be sure to justify all your answers)

1. Plot each of the following complex numbers in the complex plane, and write each of the complex numbers in standard form ($a + bi$).

(a) $e^{\pi i}$

Solution: Here's the picture:

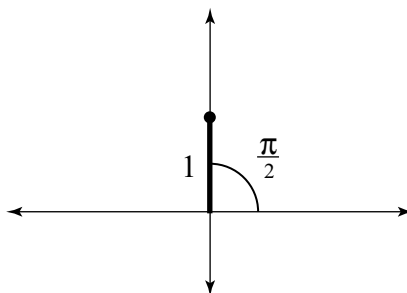


Now to put $e^{\pi i}$ into standard form:

$$e^{\pi i} = r \cos \theta + ir \sin \theta = 1 \cos \pi + i \cdot 1 \sin \pi = -1$$

(b) $e^{\frac{\pi}{2}i}$

Solution: Here's the picture:

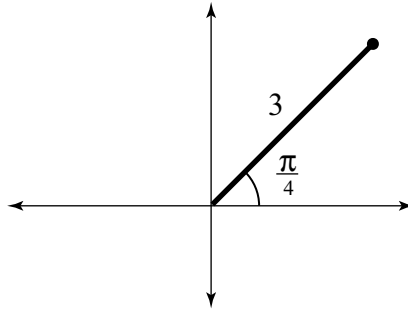


Now to put $e^{\frac{\pi}{2}i}$ into standard form:

$$e^{\frac{\pi}{2}i} = r \cos \theta + ir \sin \theta = 1 \cos \frac{\pi}{2} + i \cdot 1 \sin \frac{\pi}{2} = i$$

(c) $3e^{\frac{\pi}{4}i}$

Solution: Here's the picture:

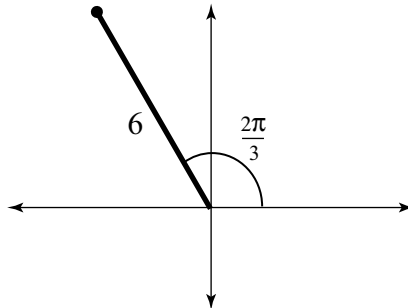


Now to put $3e^{\frac{\pi}{4}i}$ into standard form:

$$\begin{aligned} 3e^{\frac{\pi}{4}i} &= r \cos \theta + ir \sin \theta = 3 \cos \frac{\pi}{4} + i \cdot 3 \sin \frac{\pi}{4} \\ &= 3 \left(\frac{\sqrt{2}}{2} \right) + 3i \left(\frac{\sqrt{2}}{2} \right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i \end{aligned}$$

(d) $6e^{\frac{2\pi}{3}i}$

Solution: Here's the picture:

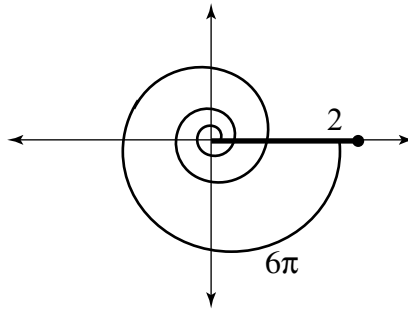


Now to put $6e^{\frac{2\pi}{3}i}$ into standard form:

$$\begin{aligned} 6e^{\frac{2\pi}{3}i} &= r \cos \theta + ir \sin \theta = 6 \cos \frac{2\pi}{3} + i \cdot 6 \sin \frac{2\pi}{3} \\ &= 6 \left(-\frac{1}{2} \right) + 6i \left(\frac{\sqrt{3}}{2} \right) = -3 + 3\sqrt{3}i \end{aligned}$$

(e) $2e^{6\pi i}$

Solution: Here's the picture:

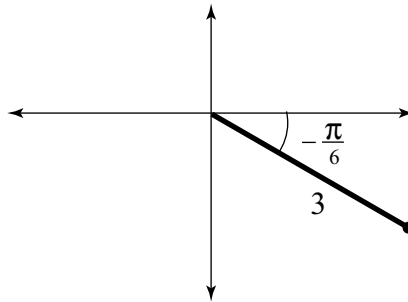


Now to put $2e^{6\pi i}$ into standard form:

$$2e^{6\pi i} = r \cos \theta + ir \sin \theta = 2 \cos 6\pi + i \cdot 2 \sin 6\pi = 2$$

(f) $3e^{-\frac{\pi}{6}i}$

Solution: Here's the picture:



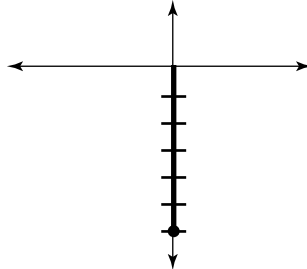
Now to put $3e^{-\frac{\pi}{6}i}$ into standard form:

$$\begin{aligned} 3e^{-\frac{\pi}{6}i} &= r \cos \theta + ir \sin \theta = 3 \cos \left(-\frac{\pi}{6}\right) + i \cdot 3 \sin \left(-\frac{\pi}{6}\right) \\ &= 3 \left(\frac{\sqrt{3}}{2}\right) + 3i \left(-\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} - \frac{3}{2}i \end{aligned}$$

2. Plot each of the following complex numbers in the complex plane, and write each of the complex numbers in polar form using Euler e -notation ($re^{i\theta}$).

(a) $-6i$

Solution: Here's the picture:

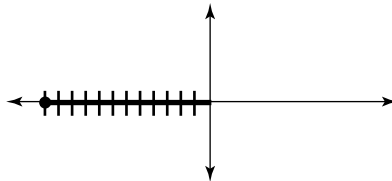


Now to put $-6i$ into polar form we first notice $r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-6)^2} = \sqrt{36} = 6$ [or you can just see the radius is 6 from the picture]. Now from the picture we see $\theta = -\frac{\pi}{2}$ so that

$$-6i = 6e^{-\frac{\pi}{2}i}$$

(b) -12

Solution: Here's the picture:

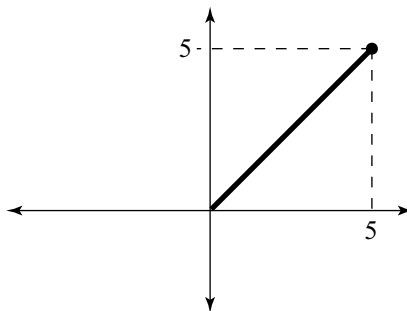


Now to put -12 into polar form we first notice $r = \sqrt{a^2 + b^2} = \sqrt{(-12)^2 + 0^2} = \sqrt{144} = 12$ [or you can just see the radius is 12 from the picture]. Now from the picture we see $\theta = \pi$ so that

$$-12 = 12e^{\pi i}$$

(c) $5 + 5i$

Solution: Here's the picture:

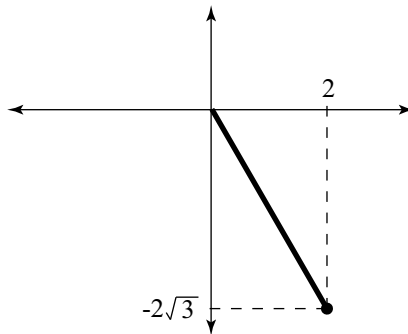


Now to put $5 + 5i$ into polar form we first notice $r = \sqrt{a^2 + b^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$. Now since $\arctan(b/a) = \arctan(5/5) = \arctan(1) = \frac{\pi}{4}$ and this agrees with the picture we see $\theta = \frac{\pi}{4}$ [or you can just see from the picture that $\theta = \frac{\pi}{4}$]. So

$$5 + 5i = 5\sqrt{2}e^{\frac{\pi}{4}i}$$

(d) $2 - 2\sqrt{3}i$

Solution: Here's the picture:

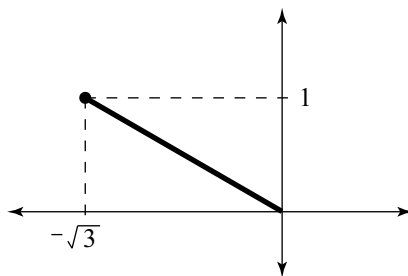


Now to put $2 - 2\sqrt{3}i$ into polar form we first notice $r = \sqrt{a^2 + b^2} = \sqrt{2^2 + (-2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$. Now since $\arctan(b/a) = \arctan(-2\sqrt{3}/2) = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$ and this agrees with the picture we see $\theta = -\frac{\pi}{3}$. So

$$2 - 2\sqrt{3}i = 4e^{-\frac{\pi}{3}i}$$

(e) $-\sqrt{3} + i$

Solution: Here's the picture:



Now to put $-\sqrt{3} + i$ into polar form we first notice $r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$. Now $\arctan(b/a) = \arctan(-1/\sqrt{3}) =$

$-\frac{\pi}{6}$, but this does not agree with the picture so $\theta = \arctan(b/a) + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$. So

$$-\sqrt{3} + i = 2e^{\frac{5\pi}{6}i}$$

3. Write the following in Euler e -notation ($re^{i\theta}$).

(a)

$$(2e^{\pi i}) (5e^{-\frac{\pi}{2}i})$$

Solution:

$$(2e^{\pi i}) (5e^{-\frac{\pi}{2}i}) = 2 \cdot 5e^{\pi i - \frac{\pi}{2}i} = 10e^{\frac{\pi}{2}i}$$

(b)

$$(\sqrt{2}e^{\frac{\pi}{3}i}) (\sqrt{2}e^{\frac{\pi}{6}i})$$

Solution:

$$(\sqrt{2}e^{\frac{\pi}{3}i}) (\sqrt{2}e^{\frac{\pi}{6}i}) = \sqrt{2} \cdot \sqrt{2}e^{\frac{\pi}{3}i + \frac{\pi}{6}i} = 2e^{\frac{3\pi}{6}i} = 2e^{\frac{\pi}{2}i}$$

(c)

$$\frac{6e^{\frac{3\pi}{2}i}}{2e^{-\frac{\pi}{2}i}}$$

Solution:

$$\frac{6e^{\frac{3\pi}{2}i}}{2e^{-\frac{\pi}{2}i}} = \frac{6}{2}e^{\frac{3\pi}{2}i - (-\frac{\pi}{2}i)} = 3e^{\frac{4\pi}{2}i} = 3e^{2\pi i} = 3$$

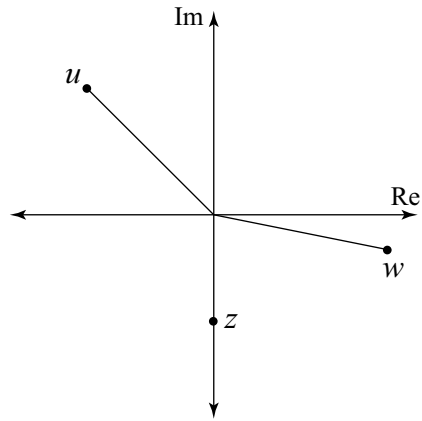
(d)

$$\frac{7e^{\frac{\pi}{6}i}}{21e^{\frac{2\pi}{3}i}}$$

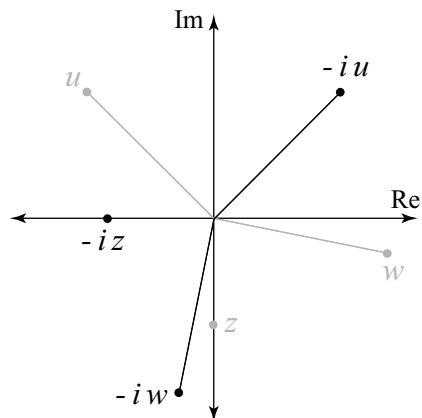
Solution:

$$\frac{7e^{\frac{\pi}{6}i}}{21e^{\frac{2\pi}{3}i}} = \frac{7}{21}e^{\frac{\pi}{6}i - \frac{2\pi}{3}i} = \frac{1}{3}e^{-\frac{3\pi}{6}i} = \frac{1}{3}e^{-\frac{\pi}{2}i}$$

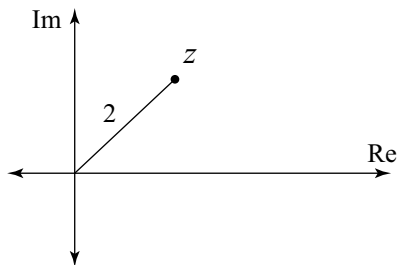
4. Plot the points $-iz$, $-iw$ and $-iu$ where z, w , and u are the complex numbers pictured below



Solution: Because $-i = e^{-\frac{\pi}{2}i}$, multiplication by $-i$ is just a rotation by $-\pi/2$ radians (or 90° clockwise) so here is the picture:



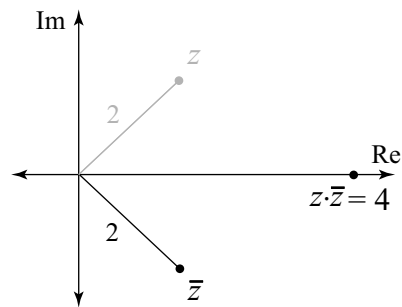
5. Plot the two points \bar{z} and $z \cdot \bar{z}$ where z is the complex number pictured below.



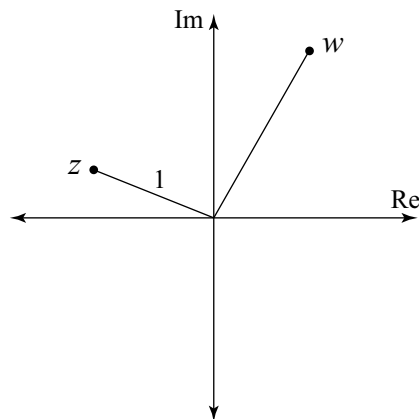
Solution: We know $z = 2e^{i\theta}$ for some θ , thus $\bar{z} = 2e^{-i\theta}$, and

$$z \cdot \bar{z} = (2e^{i\theta})(2e^{-i\theta}) = 4e^{i\theta-i\theta} = 4e^0 = 4$$

we have the following picture:

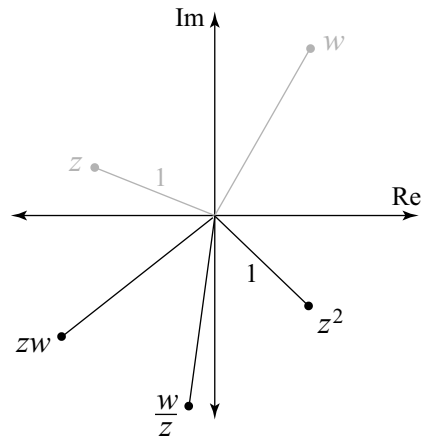


6. Plot the three points z^2 , zw , and $\frac{w}{z}$ where z and w are the complex numbers pictured below.



Solution: Since $|z| = 1$ we know z^2 has the same modulus (radius) as z but its argument (angle) is twice the argument of z . As for zw , the modulus will be the same as the modulus of w because $|z| = 1$. The argument of zw will be the sum of the arguments of z and w . Finally, the modulus of $\frac{w}{z}$ will be the same as the modulus of w because $|z| = 1$. The argument of $\frac{w}{z}$ will be equal to the argument of w subtract the

argument of z . So, here's the picture:

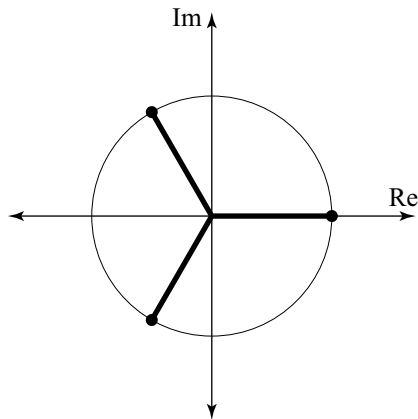


7. (a) Plot all the 3rd roots of unity.

Solution: The 3rd roots of unity are

$$1, \quad e^{\frac{2\pi}{3}i}, \quad \text{and} \quad e^{\frac{4\pi}{3}i}$$

Here's the picture:

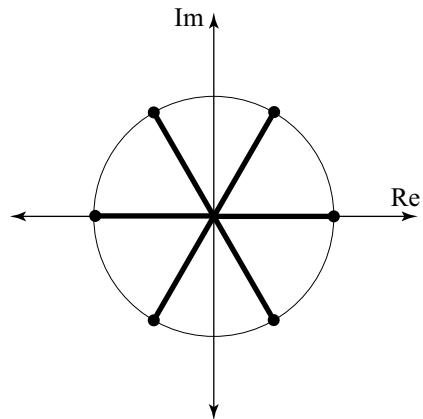


(b) Plot all the 6th roots of unity.

Solution: The 6th roots of unity are

$$1, e^{\frac{\pi}{3}i}, e^{\frac{2\pi}{3}i}, e^{\pi i}, e^{\frac{4\pi}{3}i}, \text{ and } e^{\frac{5\pi}{3}i}$$

Here's the picture:



(c) Plot all the 8th roots of unity.

Solution: The 8th roots of unity are

$$1, e^{\frac{\pi}{4}i}, i, e^{\frac{3\pi}{4}i}, -1, e^{\frac{5\pi}{4}i}, -i, \text{ and } e^{\frac{7\pi}{4}i}$$

Here's the picture:

