Math 112
Jonny Comes
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Assignment #6
Partial Solutions

From the Textbook:

- Section 7.5:

45. Find all solutions in the interval $[0, 2\pi)$.

$$\cot x \cos x = \cos x.$$

Solution:

$$\cot x \cos x = \cos x \quad \Rightarrow \quad \cot x \cos x - \cos x = 0$$

$$\Rightarrow \quad \cos x(\cot x - 1) = 0$$

$$\Rightarrow \quad \textbf{Case 1}: \quad \cos x = 0 \qquad \text{or} \qquad \textbf{Case 2}: \quad \cot x - 1 = 0$$

Case 1: $\cos x = 0$ implies $x = \pm \arccos(0) + 2\pi k$ for any integer k. Since $\arccos(0) = \frac{\pi}{2}$ this gives $x = \pm\frac{\pi}{2} + 2\pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{2} \qquad \text{and} \qquad x = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}.$$

Case 2:

$$\cot x - 1 = 0 \quad \Rightarrow \quad \cot x = 1 \quad \Rightarrow \quad \tan x = \frac{1}{\cot x} = \frac{1}{1} = 1.$$

Now $\tan x = 1$ implies $x = \arctan(1) + \pi k$ for any integer k. Since $\arctan(1) = \frac{\pi}{4}$ this gives $x = \frac{\pi}{4} + \pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{4} \qquad \text{and} \qquad x = \frac{\pi}{4} + \pi = \frac{5\pi}{4}.$$

So the final solution is

$$x = \frac{\pi}{2} \quad \text{or} \quad x = \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}.$$

46. Find all solutions in the interval $[0, 2\pi)$.

$$\tan x \cos x = \cos x.$$

Solution:

$$\tan x \cos x = \cos x \quad \Rightarrow \quad \tan x \cos x - \cos x = 0$$

$$\Rightarrow \quad \cos x(\tan x - 1) = 0$$

$$\Rightarrow \quad \textbf{Case 1}: \quad \cos x = 0 \qquad \text{or} \qquad \textbf{Case 2}: \quad \tan x - 1 = 0$$

Case 1: $\cos x = 0$ implies $x = \pm \arccos(0) + 2\pi k$ for any integer k. Since $\arccos(0) = \frac{\pi}{2}$ this gives $x = \pm\frac{\pi}{2} + 2\pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{2} \qquad \text{and} \qquad x = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}.$$

Case 2:

$$\tan x - 1 = 0 \quad \Rightarrow \quad \tan x = 1.$$

Now $\tan x = 1$ implies $x = \arctan(1) + \pi k$ for any integer k. Since $\arctan(1) = \frac{\pi}{4}$ this gives $x = \frac{\pi}{4} + \pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{4} \qquad \text{and} \qquad x = \frac{\pi}{4} + \pi = \frac{5\pi}{4}.$$

So the final solution is

$$x = \frac{\pi}{2} \quad \text{or} \quad x = \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}.$$

47. Find all solutions in the interval $[0, 2\pi)$.

$$\cos x \csc x = 2\cos x.$$

Solution:

$$\cos x \csc x = 2\cos x \quad \Rightarrow \quad \cos x \csc x - 2\cos x = 0$$

$$\Rightarrow \quad \cos x(\csc x - 2) = 0$$

$$\Rightarrow \quad \textbf{Case 1}: \quad \cos x = 0 \qquad \text{or} \qquad \textbf{Case 2}: \quad \csc x - 2 = 0$$

Case 1: $\cos x = 0$ implies $x = \pm\arccos(0) + 2\pi k$ for any integer k. Since $\arccos(0) = \frac{\pi}{2}$ this gives $x = \pm\frac{\pi}{2} + 2\pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{2} \qquad \text{and} \qquad x = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}.$$

Case 2:

$$\csc x - 2 = 0 \quad \Rightarrow \quad \csc x = 2 \quad \Rightarrow \quad \sin x = \frac{1}{\csc x} = \frac{1}{2}.$$

Now $\sin x = \frac{1}{2}$ implies $x = \arcsin\left(\frac{1}{2}\right) + 2\pi k$ or $x = \pi - \arcsin\left(\frac{1}{2}\right) + 2\pi k$ for any integer k. Since $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ this gives $x = \frac{\pi}{6} + 2\pi k$ or $x = \pi - \frac{\pi}{6} + 2\pi k = \frac{5\pi}{6} + 2\pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \frac{\pi}{6} \qquad \text{and} \qquad x = \frac{5\pi}{6}.$$

So the final solution is

$$x = \frac{\pi}{2} \quad \text{or} \quad x = \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{6} \quad \text{or} \quad x = \frac{5\pi}{6}.$$

48. Find all solutions in the interval $[0, 2\pi)$.

$$\tan x \sec x + 3 \tan x = 0.$$

Solution:

$$\tan x \sec x + 3 \tan x = 0 \quad \Rightarrow \quad \tan x(\sec x + 3) = 0$$

$$\Rightarrow \quad \textbf{Case 1}: \quad \tan x = 0 \qquad \text{or} \qquad \textbf{Case 2}: \quad \sec x + 3 = 0$$

Case 1: $\tan x = 0$ implies $x = \arctan(0) + \pi k$ for any integer k. Since $\arctan(0) = 0$ this gives $x = \pi k$ for any integer k. Of these solutions the ones with $0 \le x < 2\pi$ are

$$x = 0 \qquad \text{and} \qquad x = \pi.$$

Case 2:

$$\sec x + 3 = 0 \quad \Rightarrow \quad \sec x = -3 \quad \Rightarrow \quad \cos x = \frac{1}{\csc x} = -\frac{1}{3}.$$

Now $\cos x = -\frac{1}{3}$ implies $x = \pm \arccos\left(-\frac{1}{3}\right) + 2\pi k$ for any integer k. Of these solutions the ones between 0 and 2π are

$$x = \arccos\left(-\frac{1}{3}\right) \qquad \text{and} \qquad x = -\arccos\left(-\frac{1}{3}\right) + 2\pi.$$

So the final solution is

$$x = 0 \quad \text{or} \quad x = \pi \quad \text{or} \quad x = \arccos\left(-\frac{1}{3}\right) \quad \text{or} \quad x = -\arccos\left(-\frac{1}{3}\right) + 2\pi.$$

Additional Exercises: (Be sure to justify all your answers)

1. True or False:

   (a) If c is some real number, then we know the equation $\tan x = c$ has infinitely many solutions.
   Solution: TRUE. For ANY real number c we know

   $$\tan x = c \quad \Rightarrow \quad x = \arctan(c) + \pi k$$

   for any integer k. Thus there are infinitely many solutions, one for each integer.

   (b) If c is some real number, then we know the equation $\sin x = c$ has infinitely many solutions.
   Solution: FALSE. For example, set $c = 2$. We know $\sin x = 2$ has zero solutions.
   [Any c-value greater then 1 or less than -1 will give a counterexample.]

   (c) If c is some real number, then we know the equation $\sin x = c$ has either zero or infinitely many solutions.
   Solution: TRUE. For any real number c between -1 and 1 we know

   $$\sin x = c \quad \Longrightarrow \begin{cases} x = \arcsin(c) + 2\pi k \\ \text{or} \\ x = \pi - \arcsin(c) + 2\pi k \end{cases}$$

   for any integer k. Thus there are infinitely many solutions when $-1 \leq c \leq 1$, one for each integer. On the other hand, when c is greater than 1 or less than -1 the equation $\sin x = c$ has no solution. So there is either zero or infinitely many solutions.

(d) If $\sec x = c$ where $c > 1$, then we know

$$x = \frac{1}{\arccos(c)} + 2\pi k \qquad \text{or} \qquad x = -\frac{1}{\arccos(c)} + 2\pi k$$

Solution: FALSE. For example if $c = 2$, then $\sec x = 2$ implies $\cos x = 1/2$, thus

$$x = \pm\frac{\pi}{3} + 2\pi k$$

for any integer k. On the other hand, $\arccos(2)$ is undefined, so $\pm\frac{1}{\arccos(c)} + 2\pi k$ is undefined for all integers k.

(e) If $\sec x = c$ where $c > 1$, then we know

$$x = \arccos\left(\frac{1}{c}\right) + 2\pi k \qquad \text{or} \qquad x = -\arccos\left(\frac{1}{c}\right) + 2\pi k$$

Solution: TRUE. $\sec x = c$ implies $\cos x = \frac{1}{c}$ which implies

$$x = \frac{1}{\arccos(c)} + 2\pi k \qquad \text{or} \qquad x = -\frac{1}{\arccos(c)} + 2\pi k$$

for any integer k.