Math 112
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Fall 2005
Assignment #4
Due Friday Oct. 21

From the Textbook:

- Section 7.1: 10-48 even, 58, 64
- Section 7.2: 2, 4, 14, 16, 22-34 even, 38, 40, 46-56 even
- Section 7.3: 4, 6, 14, 24, 30, 34, 42, 44a, 48, 66

Additional Exercises: (Be sure to justify all your answers)

1. Use the half angle identity to show:

   (a) $\sin(\pi/8) = \frac{\sqrt{2-\sqrt{2}}}{2}$

   (b) $\sin(\pi/16) = \frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2}$

   Do you see a pattern forming? What do you think $\sin(\pi/32)$ is?

2. (a) Use the addition & subtraction identities to calculate the exact value of $\sin(\pi/12)$.
   [Hint: $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12}$]

   (b) Now use a half-angle identity to calculate $\sin(\pi/12)$.

   Bonus: If you want to, ON A SEPARATE SHEET OF PAPER, show that your answers in parts (a) and (b) are in fact equal.
   [this is extra credit for a reason, do not be discouraged if you cannot do it]

3. Given that $\tan t = 2/5$ and $\pi < t < 3\pi/2$ find the exact values of $\sin(t)$ and $\cos(t)$.

   [Hint: On the back of this sheet there is an example of a similar problem]

Example: Given that $\cot t = 7/4$ and $0 < t < \pi/2$ find the exact values of $\sin(t)$ and $\cos(t)$.

[This is very similar to question 3 on the second quiz, as well as other homework questions you have had before. The key is to use the pythagorean identity]

Solution: We use the Pythagorean identity, just not the one we usually use. Since the problem gives us $\cot t$ we use the identity

$$1 + \cot^2 t = \csc^2 t.$$

[Which identity should you use for problem 3?]

Now substituting $7/4$ in for $\cot t$ we have

$$\begin{aligned} 1 + \left(\frac{7}{4}\right)^2 &= \csc^2 t \\ \Rightarrow 1 + \frac{49}{16} &= \csc^2 t \\ \Rightarrow \frac{16}{16} + \frac{49}{16} &= \csc^2 t \\ \Rightarrow \frac{65}{16} &= \csc^2 t \\ \Rightarrow \frac{65}{16} &= \csc^2 t \\ \Rightarrow \pm\frac{\sqrt{65}}{4} &= \csc t. \end{aligned}$$

Since $0 < t < \pi/2$ we know $\csc t > 0$ so we have

$$\csc t = \frac{\sqrt{65}}{4}.$$

Now we know $\sin t = \frac{1}{\csc t}$ (because $\csc t = \frac{1}{\sin t}$) so

$$\sin t = \frac{1}{\csc t} = \frac{1}{\left(\frac{\sqrt{65}}{4}\right)} = \frac{4}{\sqrt{65}}.$$

To find $\cos t$ we use our knowledge of $\cot t$ and $\sin t$ as follows:

$$\begin{aligned} \cot t &= \frac{\cos t}{\sin t} \\ \Rightarrow \cos t = \cot t \sin t &= \left(\frac{7}{4}\right)\left(\frac{4}{\sqrt{65}}\right) = \frac{7}{\sqrt{65}}. \end{aligned}$$