Math 112
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Fall 2005
Assignment #3
Partial Solutions

Additional Exercises: (Be sure to justify all your answers)

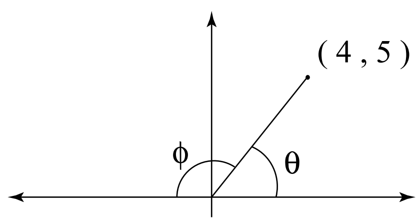
1. Let $f(t) = \cos(t)$ and $g(t) = \sec(t)$. If the point $(a, 7)$ is on the graph of g, what is $f(a)$?

   Solution: Saying the point $(a, 7)$ is on the graph of g is the same as saying $g(a) = 7$. So we have

   $$f(a) = \cos(a) = \frac{1}{\sec(a)} = \frac{1}{g(a)} = \frac{1}{7}.$$

2. Use the following diagram to find $\sin(\theta)$, $\sec(\theta)$, $\cos(\phi)$, and $\cot(\phi)$.

   [Be careful! ϕ is not pictured in standard position.]



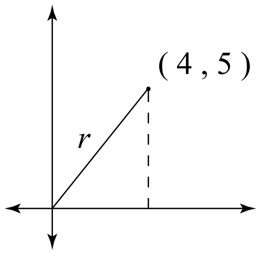
   Solution: For this problem we use the "point in the plane" rules:

   $$\sin(\theta) = \frac{y}{r}, \qquad \cos(\theta) = \frac{x}{r}, \qquad \tan(\theta) = \frac{y}{x}.$$

   These rules along with the definitions of secant cosecant and cotangent give us

   $$\csc(\theta) = \frac{r}{y}, \qquad \sec(\theta) = \frac{r}{x}, \qquad \cot(\theta) = \frac{x}{y}.$$

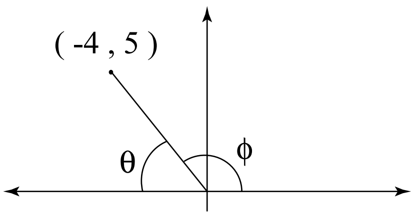
Now to solve the problem. First we find the distance from the origin to the point $(4, 5)$ labelled r in the following diagram.



By the Pyth. theorem we have $r^2 = 4^2 + 5^2 = 16 + 25 = 41$ so that $r = \sqrt{41}$. Using this we have

$$\sin(\theta) = \frac{y}{r} = \frac{5}{\sqrt{41}}, \qquad \sec(\theta) = \frac{r}{x} = \frac{\sqrt{41}}{4}.$$

In order to apply the same method to calculate $\cos(\phi)$ and $\cot(\phi)$ we first need to put ϕ in standard position as pictured below.



Now we see

$$\cos(\phi) = \frac{x}{r} = \frac{-4}{\sqrt{41}}, \qquad \cot(\phi) = \frac{x}{y} = \frac{-4}{5}.$$

3. True/False. The graph of $f(t) = 12\cos(2t - 4)$ passes through the t-axis when $t = \pi/4 + 2$. [Just to clarify, the t-axis is what you might normally think of as the x-axis]

   Solution: TRUE. Saying the graph of f passing through the t-axis when $t = \pi/4 + 2$ is the same as saying $f(\pi/4 + 2) = 0$. So we just need to show $f(\pi/4 + 2) = 0$. Well

$$f\left(\frac{\pi}{4} + 2\right) = 12\cos\left(2\left(\frac{\pi}{4} + 2\right) - 4\right) = 12\cos\left(\frac{\pi}{2} + 4 - 4\right)$$
$$= 12\cos\left(\frac{\pi}{2}\right) = 12 \cdot 0 = 0.$$

4. True/False. For any real number t, $\sin(\csc(t)) = t$.

Solution: FALSE. There are many possible values of t such that $\sin(\csc(t)) \neq t$. [in fact, I don't know how to find a t where the equality does hold] Here are a couple such counterexamples with some justification.

- Let $t = 1000$. Because $\sin(\theta) \leq 1$ for all real numbers θ and $\csc(1000)$ is just some real number, we see $\sin(\csc(1000)) \leq 1$ and therefore $\sin(\csc(1000))$ cannot equal 1000.

  [The same argument would work for any value of t greater than $1\ldots$ A similar argument will work for any value of t less than -1.]
- Let $t = 0$. Because $\csc(0) = \frac{1}{\sin(0)} = \frac{1}{0}$ is undefined, we see $\sin(\csc(0))$ is undefined and is thus not equal to 0.

  [The same argument would work for any number which is not in the domain of $\csc(t)$.]