

Math 112  
Jonny Comes  
Fall 2005  
Assignment #2  
Partial Solutions

**From the Textbook:**

- Section 6.3:

58.

$$\begin{aligned}\frac{1 - \tan^2 t}{1 + \tan^2 t} + 2 \sin^2 t &= \frac{1 - \frac{\sin^2 t}{\cos^2 t}}{1 + \frac{\sin^2 t}{\cos^2 t}} + 2 \sin^2 t \\ &= \left( \frac{\cos^2 t}{\cos^2 t} \right) \left( \frac{1 - \frac{\sin^2 t}{\cos^2 t}}{1 + \frac{\sin^2 t}{\cos^2 t}} \right) + 2 \sin^2 t \\ &= \frac{\cos^2 t - \sin^2 t}{\cos^2 t + \sin^2 t} + 2 \sin^2 t = \frac{\cos^2 t - \sin^2 t}{1} + 2 \sin^2 t \\ &= \cos^2 t - \sin^2 t + 2 \sin^2 t = \cos^2 t + \sin^2 t \\ &= 1.\end{aligned}$$

**Additional Exercises:**

1. Suppose that  $\sin t = 3/5$  and  $\tan t < 0$ . What is the exact value of  $\cos t$ ?

**Solution:** Using the Pythagorean identity we have

$$\begin{aligned}\sin^2 t + \cos^2 t &= 1 \\ \Rightarrow \left( \frac{3}{5} \right)^2 + \cos^2 t &= 1 \\ \Rightarrow \cos^2 t &= 1 - \frac{9}{25}\end{aligned}$$

$$\Rightarrow \cos^2 t = \frac{16}{25}$$

$$\Rightarrow \cos t = \pm \frac{4}{5}.$$

Since  $\sin t$  is positive and  $\tan t$  is negative,  $\cos t$  must be negative. Thus  $\cos t = -4/5$ .

2. Is it possible for  $\sin \theta = 5/13$  and  $\tan \theta = -5/12$  ?

**Solution:** As in the last problem we use the Pythagorean identity to get

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \Rightarrow \left(\frac{5}{13}\right)^2 + \cos^2 \theta &= 1 \\ \Rightarrow \cos^2 \theta &= 1 - \frac{25}{169} \\ \Rightarrow \cos^2 \theta &= \frac{144}{169} \\ \Rightarrow \cos \theta &= \pm \frac{12}{13}.\end{aligned}$$

Thus  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \pm \frac{-5/13}{12/13} = \pm \frac{5}{12}$ . So it is possible for  $\tan \theta = -\frac{5}{12}$ .

3. We know that  $\sin^2(t) + \cos^2(t) = 1$  for any real number  $t$ . What is the expression  $\sin^2(-t) + \cos^2(-t)$  equal to?

**Solution 1:** Well, one way to do this is to notice

$$\begin{aligned}\sin^2(-t) + \cos^2(-t) &= (\sin(-t))^2 + (\cos(-t))^2 = (-\sin(t))^2 + (\cos(t))^2 \\ &= (-1)^2 \sin^2 t + \cos^2 t = \sin^2 t + \cos^2 t = 1.\end{aligned}$$

**Solution 2:** Also you could just note that the point  $(\cos(-t), \sin(-t))$  is on the unit circle, and every point  $(x, y)$  on the unit circle satisfies  $x^2 + y^2 = 1$ , so

$$\cos^2(-t) + \sin^2(-t) = 1.$$

4. Later on in the term we will show  $\sin(\pi/8) = \frac{\sqrt{2-\sqrt{2}}}{2}$ . Use this fact to find  $\cos(\pi/8)$ .

**Solution:** Again, we use the Pythagorean identity.

$$\begin{aligned}\sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right) &= 1 \\ \Rightarrow \left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^2 + \cos^2\left(\frac{\pi}{8}\right) &= 1 \\ \Rightarrow \cos^2\left(\frac{\pi}{8}\right) &= 1 - \frac{2-\sqrt{2}}{4} \\ \Rightarrow \cos^2\left(\frac{\pi}{8}\right) &= \frac{4-(2-\sqrt{2})}{4} \\ \Rightarrow \cos^2\left(\frac{\pi}{8}\right) &= \frac{2+\sqrt{2}}{4} \\ \Rightarrow \cos\left(\frac{\pi}{8}\right) &= \pm \frac{\sqrt{2+\sqrt{2}}}{2}.\end{aligned}$$

Since  $\pi/8$  represents an angle in the first quadrant, and cosine is positive in the first quadrant, we see

$$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}.$$

5. A function  $f$  is called even if  $f(-t) = f(t)$  for all  $t$ .  $f$  is called odd if  $f(-t) = -f(t)$  for all  $t$ .

- (a) Is  $h(t) = \sin(t)$  odd, even, or neither?

**Solution:** Since  $h(-t) = \sin(-t) = -\sin t = -h(t)$  we see that  $h$  is an odd function.

- (b) Is  $k(t) = \cos(t)$  odd, even, or neither?

**Solution:** Since  $k(-t) = \cos(-t) = \cos t = k(t)$  we see that  $k$  is an even function.

- (c) Is  $g(t) = \tan(t)$  odd, even, or neither?

**Solution:** Since  $g(-t) = \tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\tan t = -g(t)$  we see that  $g$  is an odd function.

(d) Is  $f(t) = \tan^2(t) \cos(t) + t \sin(t)$  odd, even, or neither? Since

$$\begin{aligned} f(-t) &= \tan^2(-t) \cos(-t) + (-t) \sin(-t) \\ &= (-\tan t)^2 \cos(t) + (-t)(-\sin(t)) \\ &= \tan^2(t) \cos(t) + t \sin(t) = f(t) \end{aligned}$$

we see that  $f$  is an even function.

6. Is it true that the domain of the function  $f$  whose rule is  $f(t) = \tan(t)$  is all real numbers?

**Solution:** No, it is certainly not true. For example  $\tan(\pi/2)$  is undefined (because  $\cos(\pi/2) = 0$  and we cannot divide by zero). Thus  $\pi/2$  is a real number which is not in the domain of  $f$ .

In fact, any number which represents an angle coterminal to either  $\pi/2$  or  $-\pi/2$  is not in the domain of  $f$ .

7. Is the point  $(\pi/3, 2)$  on the graph of the function  $h$  whose rule is  $h(t) = (\sin(t) + \tan(t) \cos^2(t))^2$ ?

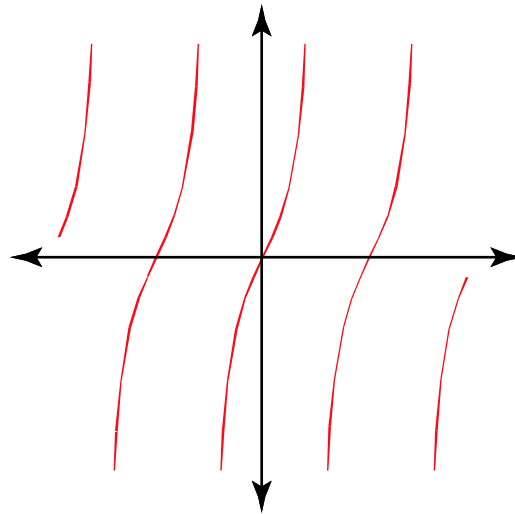
**Solution:** To say that  $(\pi/3, 2)$  is on the graph of  $h$  is equivalent to saying  $h(\pi/3) = 2$ , so all we need to do is find  $h(\pi/3)$ . Well,

$$\begin{aligned} h(\pi/3) &= (\sin(\pi/3) + \tan(\pi/3) \cos^2(\pi/3))^2 \\ &= \left( \frac{\sqrt{3}}{2} + \left( \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right) \cdot \left( \frac{1}{2} \right)^2 \right)^2 \\ &= \left( \frac{\sqrt{3}}{2} + (\sqrt{3}) \cdot \left( \frac{1}{4} \right) \right)^2 = \left( \frac{2\sqrt{3}}{4} + \left( \frac{\sqrt{3}}{4} \right) \right)^2 \\ &= \left( \frac{3\sqrt{3}}{4} \right)^2 = \frac{27}{16}. \end{aligned}$$

However,  $\frac{27}{16} \neq 2$ , so the point  $(\pi/3, 2)$  is NOT on the graph of  $h$ .

8. (a) Is it true that  $\tan \theta$  is always less than 1?  
(b) Is it true that  $\tan \theta$  is always less than 100?  
(c) Is there any numbers  $M$  such that  $\tan \theta$  is always less than  $M$  ?

**Solution:** There is no such number  $M$  such that  $\tan \theta$  is always less than  $M$ . One way of convincing yourself of this is to look at the graph of  $f(t) = \tan t$



Notice that the graph is not bounded above.

Another way to see this, is given any number  $M$ , there is a line with slope  $2M$  through the origin. If we let  $\theta$  be the angle between the  $x$ -axis and this line (in standard position), then  $\tan \theta = 2M > M$ . So I have found an angle  $\theta$  such that  $\tan \theta \not\leq M$ .