Math 112
Jonny Comes
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Assignment #1
Partial Solutions

From the Textbook:

- Section 6.1:

46. The second hand makes on revolution every minute, so in 2 minutes and 15 seconds the second hand will have made $2 + \frac{1}{4} = \frac{9}{4}$ revolutions. To find the radian measure we use the fact that there are 2π radians in one revolution, so we have

$$\frac{9}{4} \text{ revolutions} = \left(\frac{9 \text{ revolutions}}{4}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ revolution}}\right)$$
$$= \frac{9 \cdot 2\pi}{4} \text{ radians} = \frac{9\pi}{2} \text{ radians.}$$

50. Using the answer from problem 46 and the arc length formula $s = r\theta$ we have that the tip of the second hand travels

$$5 \cdot \frac{9\pi}{2} = \frac{45\pi}{2} \text{ centimeters.}$$

[Notice that we used the fact that our answer in problem 46 was given in radians in order to use the formula $s = r\theta$]

74. (a) We use the fact that there are 2π radians in 1 revolution to get

$$15000\frac{\text{rad}}{\text{min}} = \left(15000\frac{\text{rad}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \frac{7500}{\pi}\frac{\text{rev}}{\text{min}}$$

(b) If we let M be the number of minutes it takes for the saw to make 6000 revolutions we have

$$(M \text{ min})\left(\frac{7500}{\pi}\frac{\text{rev}}{\text{min}}\right) = 6000 \text{ rev.}$$

Dropping all the units and solving for M we get

$$M\left(\frac{7500}{\pi}\right) = 6000$$

$$\Rightarrow M\left(\frac{7500}{\pi}\right)\left(\frac{\pi}{7500}\right) = 6000\left(\frac{\pi}{7500}\right)$$

$$\Rightarrow M = \frac{4\pi}{5} \text{ minutes.}$$

Additional Exercises:

1. Show it is NOT true in general that $\sin(a) + \sin(b) = \sin(a + b)$ by finding two numbers a and b with the property

$$\sin(a) + \sin(b) \neq \sin(a + b).$$

Solution: There are many possible correct solutions for this problem, here's one: Let $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$. Then we have

$$\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}.$$

On the other hand

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6} + \frac{2\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1.$$

Since $\frac{1+\sqrt{3}}{2} \neq 1$ we are done.

2. Is it true that $\cos(a - b) = \cos(a) - \cos(b)$? Justify your answer.

Solution: In general it is not true that $\cos(a - b) = \cos(a) - \cos(b)$. To justify your answer you should find numbers a and b such that the equality does not hold, for example if we let $a = \pi$ and $b = 0$ then we have

$$\cos(a - b) = \cos(\pi - 0) = \cos(\pi) = -1$$

whereas

$$\cos(a) - \cos(b) = \cos(\pi) - \cos(0) = -1 - 1 = -2.$$

3. Is it true that $\tan(a) = \tan(b)$ implies $a = b$? Justify your answer.

   Solution: It is not true. There are many counter examples, here's one: $\tan(0) = 0 = \tan(\pi)$ but $0 \neq \pi$.

4. Are there any angles θ and ϕ such that $\sin(\theta) = \sin(\phi)$ but θ and ϕ are not coterminal? Justify your answer.

   Solution: Yes. Here is one such example: $\sin(\pi) = 0 = \sin(0)$ but π and 0 are not coterminal.

5. The following table shows when $\sin(\theta)$ is positive and when it is negative depending on the value of θ.

$\pi/2 < \theta < \pi$ $\sin(\theta) > 0$	$0 < \theta < \pi/2$ $\sin(\theta) > 0$
$\pi < \theta < 3\pi/2$ $\sin(\theta) < 0$	$3\pi/2 < \theta < 2\pi$ $\sin(\theta) < 0$

   Make a table like this one for both cosine and tangent.

   Solution:

$\pi/2 < \theta < \pi$ $\cos(\theta) < 0$	$0 < \theta < \pi/2$ $\cos(\theta) > 0$
$\pi < \theta < 3\pi/2$ $\cos(\theta) < 0$	$3\pi/2 < \theta < 2\pi$ $\cos(\theta) > 0$

$\pi/2 < \theta < \pi$ $\tan(\theta) < 0$	$0 < \theta < \pi/2$ $\tan(\theta) > 0$
$\pi < \theta < 3\pi/2$ $\tan(\theta) > 0$	$3\pi/2 < \theta < 2\pi$ $\tan(\theta) < 0$

6. An ant jumps onto the very end of the second hand of a clock and stays there until he has travelled exactly 20 feet, then he jumps off. If the second hand is 2 feet long, how long did the ant stay on the end of the second hand?

   Solution: Using the formula $s = r\theta$ we have $20 = 2\theta$ where θ is the angle (in radians) which the ant has travelled. Solving for θ we have $\theta = 10$ rad. Now we use the fact that there are 2π radians in 1 revolution, and 1 revolution takes 1 minute to get

$$10 \text{ rad} = (10 \text{ rad}) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) \left(\frac{1 \text{ min}}{1 \text{ rev}}\right) = \frac{5}{\pi} \text{ minutes.}$$