Name: 10/24/05

Math 112 Exam 1

1. (5 pts each) True/False (Justify your answers)

   (a) If $\sin\theta = \frac{\sqrt{2}}{2}$ and $\cos\theta = \frac{\sqrt{2}}{2}$, then we know θ must be $\frac{\pi}{4}$.

   (b) If $\tan(x) = 4$, then $\tan(2x) = 8$.

   (c) If $f(x) = \sin^2(x)\sec x - \cos x$, then $f(-x) = f(x)$.

   (d) For any real number t in the domain of the tangent function, the following is an identity.

$$\tan t = \frac{\sin(t+\pi)}{\cos(t+2\pi)}$$

2. Consider the function f whose rule is

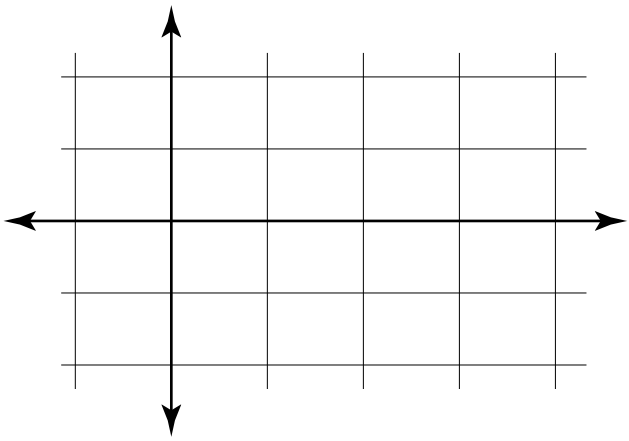
$$f(t) = -2\sin(4t + \pi)$$

(a) (2 pts) What is the amplitude of the graph of f ?

(b) (2 pts) What is the period of the graph of f ?

(c) (2 pts) What is the phase shift of the graph of f ?

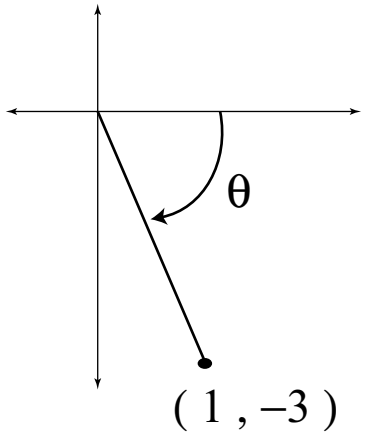
(d) (4 pts) Sketch a complete graph of f on the given axes. Make sure to include the units on the vertical and horizontal axes.



3. (10 pts) Given that $\sin x = \frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$, find the exact value of $\cos\left(\frac{x}{2}\right)$.

4. (10 pts) Find the exact value of $\cos\left(\frac{13\pi}{12}\right)$.
   [Hint: $\frac{13\pi}{12} = \frac{3\pi}{12} + \frac{10\pi}{12}$]

5. (10 pts) Using the following picture, calculate the exact value of $\sec\theta$.



6. (10 pts) Suppose the second hand of a clock is 10 inches. How far (in inches) does the tip of the second hand travel in 80 seconds ?

7. (10 pts) Evaluate the following expression.

$$\frac{\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{2\pi}{3}\right)}{\sin^2(13)+\cos^2(13)}+\tan\left(\frac{-3\pi}{4}\right)$$

8. (10 pts) Prove the following identity

$$\frac{\sec^2 x-\tan^2 x}{\csc x}=\frac{\sin(2x)}{2\cos x}$$

Extra Credit: (5pts) Suppose that $\sin(2x)$ is negative and $\cos(\pi - x)$ is positive. In what quadrant is the terminal side of the angle in standard position with radian measure x ?

[Make sure to justify your answer, and don't be afraid to write a sentence or two.]

Addition and Subtraction Identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Cofunction Identities

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right) \qquad \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec x = \csc\left(\frac{\pi}{2} - x\right) \qquad \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Power-Reducing Identities

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$

$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

Product Identities

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

Factoring Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$