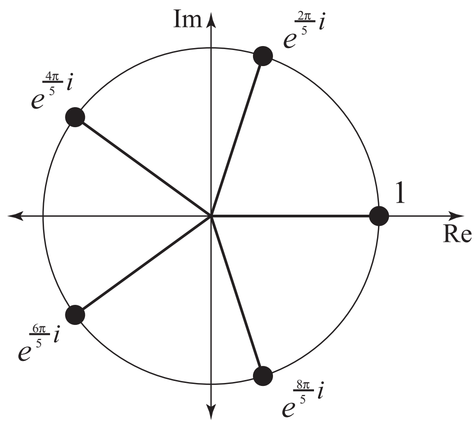
Examples for section 9.2

Roots of Unity: The solutions to the equation $x^n = 1$ are

$$1, \quad e^{\frac{2\pi}{n}i}, \quad e^{\frac{4\pi}{n}i}, \quad e^{\frac{6\pi}{n}i}, \quad \ldots \quad , e^{\frac{2(n-1)\pi}{n}i}$$

1. Plot all the 5th roots of unity.

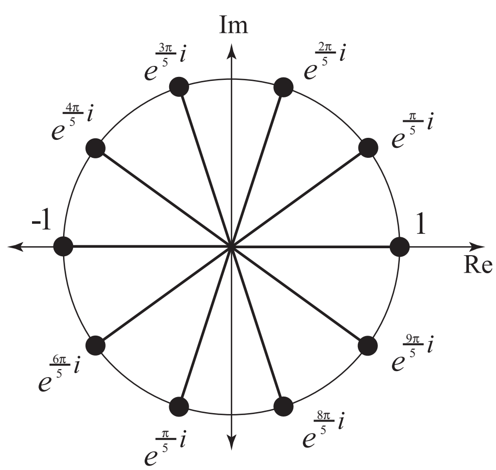
   Solution: The 5th roots of unity are 1, $e^{\frac{2\pi}{5}i}$, $e^{\frac{4\pi}{5}i}$, $e^{\frac{6\pi}{5}i}$, and $e^{\frac{8\pi}{5}i}$. Here's the picture



2. Plot all the 10th roots of unity.

   Solution: Since $\frac{2\pi}{10} = \frac{\pi}{5}$ we see the 10th roots of unity are

   1, $e^{\frac{\pi}{5}i}$, $e^{\frac{2\pi}{5}i}$, $e^{\frac{3\pi}{5}i}$, $e^{\frac{4\pi}{5}i}$, -1, $e^{\frac{6\pi}{5}i}$, $e^{\frac{7\pi}{5}i}$, $e^{\frac{8\pi}{5}i}$, and $e^{\frac{9\pi}{5}i}$. Here's the picture



[Notice that because 10 is a multiple of 5, all 5th roots of unity are also 10th roots of unity... this will always be true. For example all 7th roots of unity are also 77th roots of unity.]

3. Solve the equation $x^4 + x^3 + x^2 + x + 1 = 0$.

   Solution: The key step is to notice that

$$
\begin{array}{rrrrrrr}
 & & x^4 & +x^3 & +x^2 & +x & +1 \\ \cline{2-7}
x-1 \big) & x^5 & & & & & -1 \\
 & -x^5 & +x^4 & & & & \\ \cline{2-3}
 & & x^4 & & & & \\
 & & -x^4 & +x^3 & & & \\ \cline{3-4}
 & & & x^3 & & & \\
 & & & -x^3 & +x^2 & & \\ \cline{4-5}
 & & & & x^2 & & \\
 & & & & -x^2 & +x & \\ \cline{5-6}
 & & & & & x & -1 \\
 & & & & & -x & +1 \\ \cline{6-7}
 & & & & & & 0
\end{array}
$$

   thus $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$. This tells us that the roots of $x^4 + x^3 + x^2 + x + 1$ are all the 5th roots of unity except 1, namely

$$e^{\frac{2\pi}{5}i}, \quad e^{\frac{4\pi}{5}i}, \quad e^{\frac{6\pi}{5}i}, \text{ and } e^{\frac{8\pi}{5}i}$$