

Math 111 Last Quiz Solutions

1. True or False

(a) $f \circ g = g \circ f$ for any functions f and g .

False.

(b) If f and g are one-to-one functions then $f \circ g \circ f^{-1} = g$.

False.

(c) If f and g are one-to-one functions and we know $f(3) = 5$ and $g(7) = 3$ then $(f + g^{-1})(3) = 12$.

True.

2. Let $f(x) = \frac{1}{x+3}$. Find $f^{-1}(x)$ and verify your answer is correct using the round trip theorem.

Solution: To find $f^{-1}(x)$ we first set $f(x) = y$ and solve for x as follows.

$$y = \frac{1}{x+3} \Rightarrow y(x+3) = 1 \Rightarrow yx+3y = 1 \Rightarrow yx = 1-3y \Rightarrow x = \frac{1-3y}{y}$$

Now we set $f^{-1}(y) = x$ and replace all y 's with x 's to get

$$f^{-1}(x) = \frac{1-3x}{x}.$$

Now to check that our answer is correct we need to show both $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$. Well,

$$\begin{aligned} (f \circ f^{-1})(x) &= f(f^{-1}(x)) = f\left(\frac{1-3x}{x}\right) = \frac{1}{\frac{1-3x}{x} + 3} = \frac{1}{\frac{1}{x} - \frac{3x}{x} + 3} \\ &= \frac{1}{\frac{1}{x} - 3 + 3} = \frac{1}{\left(\frac{1}{x}\right)} = x, \end{aligned}$$

and

$$\begin{aligned} (f^{-1} \circ f)(x) &= f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x+3}\right) = \frac{1-3\left(\frac{1}{x+3}\right)}{\left(\frac{1}{x+3}\right)} = \left(1-3\left(\frac{1}{x+3}\right)\right)\left(\frac{x+3}{1}\right) \\ &= \left(\frac{x+3}{1}\right) - 3\left(\frac{1}{x+3}\right)\left(\frac{x+3}{1}\right) = (x+3) - 3 \cdot 1 = x. \end{aligned}$$

3. Let $f(x) = \log(x)$, $g(x) = \frac{1}{x}$, $h(x) = x + 2$. Write k as a composition of f, g , and h where

$$k(x) = \frac{1}{\log(x+2)}$$

Solution:

$$k(x) = \frac{1}{\log(x+2)} = \frac{1}{\log(h(x))} = \frac{1}{f(h(x))} = g(f(h(x)))$$

so

$$\boxed{k = g \circ f \circ h}$$

4. Let $f(x) = x^4$, $g(x) = \ln(6x)$, $h(x) = 3 + x$.

(a) Find $(f \circ g)(x)$.

Solution: $(f \circ g)(x) = f(g(x)) = f(\ln(6x)) = (\ln(6x))^4$.

(b) Find $(g/h)(x)$ and give the domain of g/h .

Solution: $(g/h)(x) = \frac{g(x)}{h(x)} = \frac{\ln(6x)}{3+x}$. Now let's find the domain. Since we can only take logs of positive numbers we need $6x > 0$ or equivalently $x > 0$. Also, we can't divide by zero so we need $3 + x \neq 0$ or equivalently $x \neq -3$. Thus the domain of g/h is $(-\infty, -3) \cup (-3, 0)$.

(c) Compute $(h \circ f \circ h)(-4)$.

Solution:

$$h(f(h(-4))) = h(f(3 + (-4))) = h(f(-1)) = h((-1)^4) = h(1) = 3 + 1 = 4.$$

5. Let $f(x) = \log_3(x)$ and $g(x) = x - 7$. Compute

$$(f \circ g \circ f \circ f \circ g)(x)$$

Solution:

$$\begin{aligned} (f \circ g \circ f \circ f \circ g)(x) &= f(g(f(f(g(x)))))) = f(g(f(f(x - 7)))) = f(g(f(\log_3(x - 7)))) \\ &= f(g(\log_3(\log_3(x - 7)))) = f(\log_3(\log_3(x - 7)) - 7) = \log_3(\log_3(\log_3(x - 7)) - 7) \end{aligned}$$