

Name:

Math 111–Final Exam

Be sure to show all your work in a step by step fashion and clearly indicate your answers. The point value for each problem is labelled next to the problem number.

1. [5pts each] Fill in the blank so that the statement is always true.

(a) The circle with equation  $y^2 + x^2 - 6y = 13$  has center \_\_\_\_\_ and radius \_\_\_\_\_.

(b) The graph of  $e^{x-1} + 3$  is the graph of  $e^x$  shifted \_\_\_\_\_ then \_\_\_\_\_.

(c) The line through the points  $(-2, 3)$  and  $(1, -6)$  is perpendicular to the line

$$y = (\text{_____})x + 327.$$

(d) The function  $f(x) = 3(x + 2)^2 - 103$  has an inverse on the domain \_\_\_\_\_.

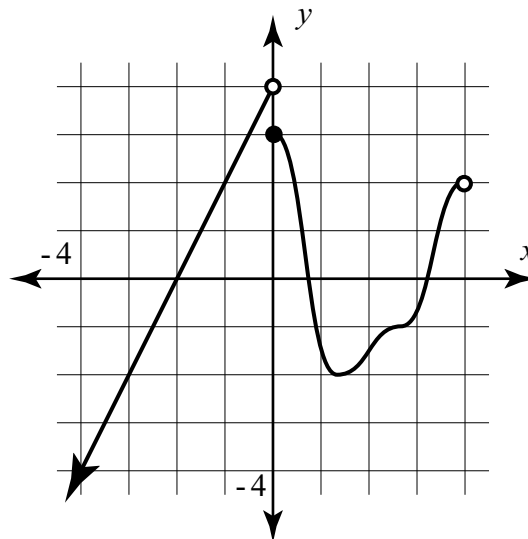
(e) If  $p(x)$  is a polynomial and when  $p(x)$  is divided by  $x - 4$  the remainder is zero, then

$$p(4) = \text{_____}.$$

2. [10pts] Find the inverse function to

$$f(x) = \frac{3x}{2-x}$$

3. [5pts each] Let  $f(x)$  be the function whose graph is the following:



(a) What is  $f(0)$ ?

(b) For what approximate value(s) of  $x$  does  $f(x) = 2$ ?

(c) Use interval notation to write the range of  $f(x)$ .

4. [10pts] Recall the difference quotient for a function  $f$  is

$$\frac{f(x+h) - f(x)}{h} \quad (h \neq 0)$$

Compute and simplify the difference quotient for

$$f(x) = 3 - 2x^2.$$

5. [10pts] Suppose you invest  $P$  dollars into an account. If the interest is 2% and is compounded continuously, how long will it take for your investment to double?

6. [10pts] Let

$$g(x) = 2x^3 + 4x^2 - 26x + 20.$$

Use the fact that  $g(1) = 0$  to find all the roots of  $g(x)$ .

7. [3pts each] Use the rational function

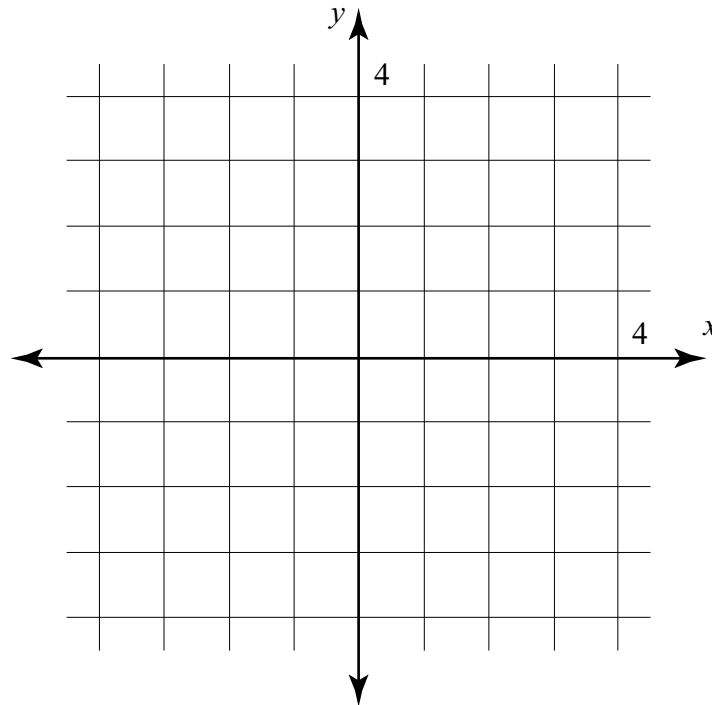
$$f(x) = \frac{x(x^2 - 5x + 6)(x - 3)^3}{(x - 2)^2(x - 3)^4}$$

to answer the following questions.

- (a) What is the domain of  $f(x)$ ?
  
  
  
  
  
  
  
  
  
  
- (b) What (if there are any) are the  $y$ -intercepts?
  
  
  
  
  
  
  
  
  
  
- (c) What (if there are any) are the  $x$ -intercepts?
  
  
  
  
  
  
  
  
  
  
- (d) What (if there are any) are the vertical asymptotes?
  
  
  
  
  
  
  
  
  
  
- (e) What (if there are any) are the horizontal asymptotes?
  
  
  
  
  
  
  
  
  
  
- (f) What (if there are any) are the holes?

8. [10pts] Sketch a graph of the following piecewise defined function:

$$f(x) = \begin{cases} (x + 2)^2, & \text{if } x < -1; \\ 3, & \text{if } x = -1; \\ -x, & \text{if } x > -1. \end{cases}$$



9. Find all solutions to the following equations.

(a) [5pts]

$$|x + 3| = 3x + 4.$$

(b) [10pts]

$$\ln(x + 3) = \ln(18) - \ln(x).$$

10. For the following questions use the functions

$$f(x) = \frac{\sqrt{3-x}}{x^2}, \quad g(x) = \log(x+1).$$

(a) [10pts] Is  $f(x)$  even, odd, or neither? (justify your answer)

(b) [7pts] What is  $(fg)(x)$ ?

(c) [10pts] Use interval notation to write the domain of  $(fg)(x)$ .

(d) [10pts] What is  $(f \circ g)(x)$ ?

11. [5pts each] Determine whether the following statements are true or false, AND explain your answer in **one coherent sentence**.

(a) TRUE/FALSE: The graph of a polynomial function with odd degree always crosses the  $x$ -axis.

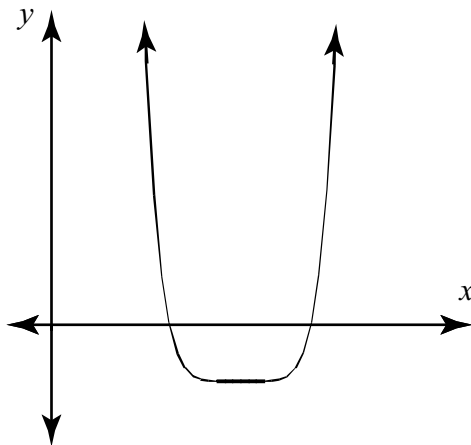
(b) TRUE/FALSE: For every real number  $k$ ,  $\log(e^k) = k$ .

(c) TRUE/FALSE: The intervals  $(0, \infty)$  and  $(-\infty, 0]$  have no points in common.

(d) TRUE/FALSE: If the points  $(2, 0)$  and  $(3, 0)$  are on the same graph, then the graph cannot be the graph of a function of  $x$ .

(e) TRUE/FALSE: If  $f(x)$  is an even function, then the point  $(x, f(-x))$  is on the graph of  $f(x)$ .

(f) TRUE/FALSE: The following could be the graph of a polynomial of degree 6.



12. [10pts] What is the maximal area of a rectangle with perimeter 30?