Unstable Inflation Targets*

William A. Branch
University of California, Irvine

George W. Evans
University of Oregon
University of St. Andrews

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Abstract

This paper studies long-run inflation targets and stability in an imperfect information environment. When central banks set an inflation target that is not fully communicated, agents draw inferences about inflation from recent data and remain alert to structural change in their econometric model by forming expectations from a forecasting model that is estimated via discounted least squares. Inflation targets can lead agents' beliefs to depart from rational expectations through two channels. First, implementing a higher inflation target can lead to overshooting of the target. Second, there can be nearly self-fulfilling inflation, disinflation, or deflation that arises as an endogenous response to shocks.

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Policy makers have generally chosen a 2 % (inflation rate target). But there was no very good reason to use 2 % rather than 4 %. Two percent doesn't mean price stability. Between 2 % and 4 %, there isn't much cost from inflation. If I were to choose an inflation target today, I'd strongly argue for 4%.

– Interview with Olivier Blanchard in WSJ 2/11/2010

In this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence and the ability of firms and households to make longer-term plans, while squandering the Fed's hard-won inflation credibility. Inflation expectations would also likely become significantly less stable, and risk premiums in asset markets–including inflation risk premiums–would rise.

–Chairman Ben S. Bernanke, remarks at the 2010 Jackson Hole Symposium

Since the economic crisis erupted, statistical properties of UK core CPI have swung markedly. From 2008 onwards, UK core inflation cannot be modelled as a stationary process. This shift may indicate that the MPC has become more tolerant of inflation deviating from target as a pragmatic reaction to the recession. However, it would also be consistent with the tolerance of persistently higher inflation – effectively raising the inflation target. We believe the former is the more accurate description of reality, but the danger is that wage and price setters take the latter view, and higher inflation expectations become self-fulfilling.

– Barclays Capital, Global Economics Weekly 10/15/2010

1 Introduction

The recent global economic crisis has renewed debate over the appropriate long-run inflation rate that monetary policymakers should target. Some have argued (see Krugman (1998), Summers (1991)) that in "normal" times monetary authorities should pursue a higher average inflation rate in order to provide a cushion that facilitates a lowering of nominal interest rates so as to avoid a liquidity trap due to a binding zero-lower-bound constraint on interest rates. Subsequent analyses, that are typically based in models with explicit micro-foundations, find little support for higher average inflation rates. For example, Woodford (2003), Schmitt-Grohe and Uribe (2011), Eggertsson and Woodford (2003) and Coibion, Gorodnichenko, and Wieland (2010) find that the optimal inflation rate in New Keynesian models are close to zero, even after accounting for the protection that higher inflation rates provide in avoiding the zero lower bound. Nevertheless, among many economists there is the view that the added stability that might be achieved from a higher inflation target outweighs any of the distortionary losses associated with inflation.

Policymakers, though, are reticent to depart from a commitment to low and stable rates of inflation. Chairman Bernanke, in the quote above, expresses the view that higher average inflation rates will lead to more volatile inflation and "inflation expectations would also likely become significantly less stable." Most research into optimal long-run inflation rates and optimal policy in the presence of a zero lower bound constraint do not have a channel through which higher average inflation leads to the disanchoring of inflation expectations. Conventional models assume perfect information (and rational expectations) which have the feature that, away from the zero lower bound, inflation will be a stationary process around the long-run target.[1]

This paper re-examines the stability of long-run inflation targets in an environment with imperfect information and adaptive learning. As a benchmark, the paper takes as its economic framework a simple dynamic Fisherian model where nominal interest rates are adjusted, in accordance with a Taylor rule, whenever inflation deviates from its long-run target rate. Private-sector agents in the economy know the form of the Taylor rule. However, they do not know – or harbor some doubt about – the precise values for the response coefficient or the value and/or timing of the long-run inflation target. Instead, agents draw inferences about the inflation process from recent data by adopting an econometric forecasting model whose reduced-form nests the full class of rational expectations equilibria. These agents are Bayesian and, because of their uncertainty about the inflation target, they place a prior on structural change in their econometric model. This imperfect information framework implies that private sector agents adopt a simple AR(1) forecasting model the parameters of which are updated in real time with a form of discounted least squares ("constant gain learning"). The priors of this model are specified in such a way that beliefs are, on average, close to rational expectations.

The primary results of this paper are as follows. First, although over time be-

[1] An important exception is Williams (2006) who examines whether higher inflation targets can make liquidity traps occur less often when rational expectations are replaced with a constant gain, or perpetual, learning rule.

liefs tend to converge toward rational expectations, the combination of constant gain learning and a positive inflation target can lead agents in the economy to temporarily believe that inflation follows a random walk without drift. Under these beliefs, agents will interpret recent innovations to inflation as permanent shifts in the mean inflation rate. Random walk beliefs arise for a very intuitive reason. The long-run inflation target, and imperfect information about that target, lead agents to estimate the mean inflation rate from historical data. As a thought experiment, suppose there is a slight (temporary) upward drift in the inflation rate. Agents' econometric models will pick up that drift, leading to higher inflation expectations that feed back into higher inflation rates. This process is self-reinforcing and in some cases can lead agents to believe that inflation follows a random walk.

Second, random-walk beliefs, as we will show below, are nearly self-fulfilling, and consequently such beliefs tend to persist for a substantial period of time. Furthermore these beliefs tend to generate considerable economic volatility, characterized by significant bursts of inflation, disinflation, and even deflation. Third, implementing a higher target – say by moving the target from 2% to 4% – will introduce just the type of drift in inflation that can lead to random-walk beliefs and cause a substantial overshooting of the inflation target.

An important feature of our analysis is that imperfect information of inflation targets can generate instability in inflation rates even though the departure, on average, from rational expectations is small. The framework employed here is related to an extensive literature that employs adaptive learning in macroeconomics. Most closely related are papers that incorporate constant gain learning in studies of monetary policy and asset pricing (see, for example, Branch and Evans (2011); Sargent (1999); Adam, Marcet, and Nicolini (2010); Orphanides and Williams (2005a); Cho, Williams, and Sargent (2002); Williams (2004); Cho and Kasa (2008); Eusepi and Preston (2010b)). Branch and Evans (2011), in particular, find that when risk-averse agents in an asset pricing model forecast both the risk and return of stock prices using a forecasting model whose parameters are updated with constant gain least squares then traders may also come to temporarily believe that stocks follow a random walk. These nearly self-fulfilling random walk beliefs lead to recurrent bubbles and crashes in stock prices. The intuition for why inflation targets are destabilizing in an adaptive learning environment is similar to the existence of bubbles and crashes in stock markets.

Is it reasonable to assume that the private-sector might have imperfect information, or doubts, about the long-run inflation target? The answer is yes, for a variety of reasons. First, the Federal Reserve Bank does not have a stated inflation target and, in fact, it faces a dual mandate legislated by congress. In recent

FOMC meetings, there has been debate about communicating an explicit long-run target, while ultimately concluding that the Summary of Economic Projections (reported once each quarter by the FOMC) conveys the central bank's target. However, there is considerable uncertainty about that target value due to diverse views in the composition of the FOMC. In the Summary of Economic Projections, the central tendency ranges from 1.5-2% (though recently more concentrated at 2%). In the Survey of Professional Forecasters, there is considerable disagreement about average annual inflation over a 10 year period. SPF participants expect a 2.5% long-run inflation rate and the dispersion across forecasters ranges from 0.4% to 0.8% in each survey quarter. Even in countries with an explicit target, such as England, the Barclays quote above shows that sophisticated market participants might hold doubts about the long-run target.

The results in the current paper provide a caution to proposals for higher long-run inflation targets as a safeguard against hitting the zero lower bound. In an extension of our basic analysis, we show that a New Keynesian model, closed with a standard Taylor rule and subject to imperfect information about the long-run target, may exhibit endogenous crashes in the inflation rate that bump up against the zero lower bound in nominal interest rates. These self-fulfilling paths exhibit rapid deflation and large negative output gaps. These liquidity traps only last for a finite period of time as the global stability of the rational expectations equilibrium eventually prevails.

There are important policy implications from these results. Only credibly and completely informing the private-sector about the long-run inflation target *and* the timing of when that target will be implemented can avoid the unstable dynamics associated with positive inflation targets. When agents know the mean inflation rate, and need only forecast its persistence, random-walk beliefs do not arise. This result is related to a finding by Eusepi and Preston (2010a), who show that central bank communication about the policy-setting process can affect the stability of rational expectations equilibria, and Orphanides and Williams (2005b), who emphasize the importance of a credible inflation target.[2] The analysis below also considers other popular policy proposals, such as price-level targeting and optimal discretionary policy rules, and demonstrates that these policies are not immune to instability. The key message is that environments with imperfect information that cause agents to forecast *both* the mean and persistence of inflation will lead to unstable inflation dynamics.

Finally, the results in this paper are also related to the literature on global Taylor rules and liquidity traps (e.g. Benhabib, Schmitt-Grohe, and Uribe (2001)).

[2]Neither of these papers looks at the impact under learning of a change in an imperfectly communicated inflation target.

In this literature, the zero lower bound typically implies the existence of two steady states, one corresponding to the inflation target and the other associated with a liquidity trap. Evans, Guse, and Honkapohja (2008) examine the learning dynamics in this setting and find that if a pessimistic expectations shock moves the economy far enough from the targeted steady state, then the trajectory diverges to a liquidity trap with stagnation and deflation. Evans, et al. focus on the global learning dynamics that result from large expectations shocks, while the current paper focuses on learning rules that are, on average, close to the targeted equilibrium but that are subject to occasional departures from equilibrium.

This paper proceeds as follows. Section 2 presents the main results in a simple Fisherian model. Although, the basic result is far more general, and likely to arise in any linear, forward-looking model, the Fisherian model can illustrate the mechanics of the learning process in a transparent manner. Section 3 considers extensions and policy implications and, in particular, demonstrates that the main qualitative results from Section 2 carry over to the New Keynesian model.

2 A Simple Model of Inflation Targets and Imperfect Information

This paper begins by considering a simple Fisherian model, which can emerge as a special case from richer models that incorporate real and nominal frictions. The Fisherian model illustrates the main points and provides analytic results. As will be seen below, our results can arise in more practical models, such as the New Keynesian model.

2.1 Fisherian Model and Imperfect Information

The Fisherian model emerges from a constant endowment economy that abstracts from frictions. The Fisher relation arises from a household's first-order condition that prices one period nominal bonds. Monetary policy adjusts nominal interest rates according to a Taylor rule. In this simple (log-linearized) environment, inflation is determined by the following two equations

$$i_t = \hat{E}_t \left(\pi_{t+1} - \bar{\pi}\right) + r_t \quad (1)$$

$$i_t = \alpha \left(\pi_t - \bar{\pi}\right) \quad (2)$$

where i_t is the nominal interest rate in log deviation from steady-state form, π_{t+1} is the inflation rate and $\bar{\pi}$ is the central bank's long-run inflation target. For simplicity the exogenous shock r_t is assumed to be (unobserved) white noise. Equation (1) is the Fisherian relation and (2) is the Taylor rule. The operator $\hat{E}$ is the (possibly) non-rational expectations operator, highlighting that imperfect information can affect the economy through the self-referential nature of the asset pricing equation (1).

Combining (1) with (2) leads to an expectational difference equation that determines the path for inflation:

$$\pi_t = \frac{(\alpha - 1)}{\alpha}\bar{\pi} + \alpha^{-1}\hat{E}_t\pi_{t+1} + \alpha^{-1}r_t \tag{3}$$

A rational expectations equilibrium is any (non-explosive) solution to (3). Under rational expectations, $\hat{E} = E$, there are two classes of equilibria that satisfy (3), the minimum state variable solution

$$\pi_t = \bar{\pi} + \alpha^{-1}r_t$$

and a non-fundamentals solution

$$\pi_t = (1 - \alpha)\bar{\pi} + \alpha\pi_{t-1} - r_{t-1} + \xi_t$$

where ξ_t is a martingale difference sequence, i.e. $E_{t-1}\xi_t = 0$. Provided that $\alpha > 1$, the non-fundamentals solution is explosive, and the MSV is the unique (non-explosive) rational expectations equilibrium. The condition $\alpha > 1$ is often referred to as the "Taylor principle" as it prescribes nominal interest rates to be adjusted more than one-for-one when inflation deviates from target, and in many models it is a key condition ensuring equilibrium determinacy. Throughout this paper, we focus on policy rules that satisfy the Taylor principle.

Households must have full information about the distribution of the endogenous variables in order to form rational expectations. This includes knowing the details of the policy rule, such as the long-run target $\bar{\pi}$ as well as the reaction coefficient α. An alternative to rational expectations is to assume that agents behave like econometricians who hold a (correctly) specified model of the economy but they must recover the parameters, in real time, from data. An extensive literature studies the conditions under which an economy with adaptive learning will converge to a rational expectations equilibrium (see Evans and Honkapohja (2001)). The imperfect information approach adopted in this paper builds on this approach.

When there is imperfect information about the economy, it is reasonable to expect households to form forecasts in the same way as an econometrician. Following this logic, agents are assumed to form expectations based on a perceived model of the economy that in effect nests the full class of rational expectations equilibria:

$$\pi_t = a + b\pi_{t-1} + \varepsilon_t \tag{4}$$

where ε_t is a (perceived) white noise error. Notice that for suitable parameter values (a, b) this perceived model of the economy coincides with either class of equilibria. Given the forecasting model (4), conditional expectations are formed

$$\hat{E}_t\pi_{t+1} = a(1 + b) + b^2\pi_{t-1} \tag{5}$$

To calculate these expectations it is assumed that endogenous variables are not observed contemporaneously.[3] With expectations in hand, inflation is determined by plugging expectations (5) into the temporary equilibrium equation (3),

$$\begin{aligned} \pi_t &= \alpha^{-1}(\alpha - 1)\bar{\pi} + \alpha^{-1}a(1 + b) + \alpha^{-1}b^2\pi_{t-1} + \alpha^{-1}r_t && (6) \\ &\equiv T(a, b)'X_{t-1} + \alpha^{-1}r_t && (7) \end{aligned}$$

where $T(a, b)' = \left(\frac{(\alpha-1)}{\alpha}\bar{\pi} + \alpha^{-1}a(1 + b), \alpha^{-1}b^2\right)$ and $X' = (1, \pi)$. It is straightforward to verify that a fixed point to $T(a, b)$ is a rational expectations equilibrium.

The map T, which takes perceived coefficients (a, b) to the actual coefficients in (6), plays a prominent role in analyses of the expectational stability ("E-stability") of rational expectations equilibria. The T-map can be interpreted in the following way. If agents held beliefs in the form of the perceived law of motion (4), with parameters (a, b) held constant over time at (possibly) non-RE values, then their forecast rule would be (5). The stochastic process for inflation (6) takes the same form as the perceived process, but with the coefficients $T(a, b)$ replacing (a, b) in (4). Since a rational expectations equilibrium aligns perceptions with outcomes, it is not surprising that a rational expectations equilibrium is a fixed point of the T-map.

Under real-time learning the parameters (a, b) are updated over time, e.g. with least squares, in response to new data. Evans and Honkapohja (2001) has shown that one can easily compute from the T-map a stability condition, E-stability, which governs whether the rational expectations equilibrium is locally stable under learning and that the ordinary differential equation, used to define E-stability, also provides information on the global learning dynamics. More formally, the E-stability Principle states that Lyapunov stable rest points of the E-stability ordinary differential equation

$$\frac{d(a, b)'}{d\tau} = (T(a, b) - (a, b))' \tag{8}$$

[3] A frequently-used timing convention in adaptive learning models is that agents cannot observe contemporaneous endogenous variables. This timing protocol eliminates the simultaneity of inflation and expected inflation.

are locally stable under least squares learning and other closely related learning algorithms. Here τ denotes "notional" time, which can, however, be linked to real time t.

That the E-stability condition governs stability of an equilibrium under learning is intuitive, since (8) states that the estimated coefficients (a, b) should be adjusted in the direction of the actual law of motion parameters that generate the data. Local stability of (8) thus addresses whether a small perturbation in the perceived coefficients (a, b) will return to their rational expectations equilibrium values.

In the Fisherian model, it is fairly simple to compute the E-stability of the rational expectations equilibrium. A rational expectations equilibrium will be E-stable provided the roots of $DT(a, b)$, evaluated at their equilibrium values, have real parts less than one. In the current case, it is straightforward to verify that provided the Taylor principle is satisfied, i.e. $\alpha > 1$, the unique rational expectations equilibrium is E-stable. Figure 1 illustrates the intuition by plotting the resting points of the E-stability ODE and the associated vector field. The solid lines indicate the values for which $\dot{a} = 0, \dot{b} = 0$ and the arrows indicate the direction of adjustment in (8). The figure illustrates the two rational expectations equilibria. The $b = \alpha$ equilibrium is explosive and is also unstable under the E-stability dynamics. In contrast, the unique non-explosive rational expectations equilibrium is a sink under learning.

Figure 1 illustrates two further features. First, the fundamentals equilibrium is E-stable and it's basin of attraction includes all initial conditions with $b < \alpha$. Second, most analyses of policy under learning focus on the E-stability properties of a particular equilibrium. The figure also demonstrates that the transitional dynamics might be of independent interest. The vector field indicates that some transitional paths may include non-linear paths to the rational expectations equilibrium.

2.2 Inflation Targets and the Dynamics of Imperfect Information

The E-stability dynamics govern the stability of the rational expectations equilibrium. However, they do not give the full picture of global learning dynamics. This subsection details the learning dynamics and illustrates how long-run inflation targets can alter the qualitative nature of learning dynamics. The central idea is the following: private sector agents are aware of the form of the policy rule but the specifics, such as the size of the reaction coefficient and/or the value and timing of the long-run inflation target, are unknown. An agent in this setting would be wise to remain alert to potential changes in the size and timing of the implementation of the long-run inflation target. Such an agent will then place a prior probability on drifting coefficients in their forecasting model. There are two central ingredients to the results that come below: a positive long-run inflation target that is imperfectly known by agents, and a prior belief of possible structural change.

Let $\theta' = (a, b), X' = (1, \pi)$. Agents are assumed to update their parameter estimates according to the following recursive algorithm

$$\theta_t = \theta_{t-1} + \gamma S_{t-1} X_{t-1} \left(\pi_t - \theta'_{t-1} X_{t-1}\right)' \tag{9}$$

$$S_t = S_{t-1} + \gamma \left(X_t X'_t - S_{t-1}\right) \tag{10}$$

The equations in (9)-(10) are the updating equations for recursive least squares where the data are discounted by a constant "gain" γ. Here S_t is an estimate of $EX_tX'_t$, the second moment matrix of the regressors. Least-squares updating arises when the constant gain γ is replaced by a decreasing sequence $\gamma_t = t^{-1}$.

Sargent and Williams (2005) demonstrate that constant gain least squares can arise from an (approximate) Kalman Filter when agents believe that the process for the drifting coefficients θ_t follows a random walk, a standard assumption in applied econometric work. Specifically, the constant gain learning equations (9)-(10) arise from an approximate Bayesian learning process in which the prior on parameter drift is proportional to the ratio of observation noise variance to the covariance of the regressors, with the speed of drift controlled by the constant gain γ. An alternative interpretation of (9)-(10) is that agents use least squares modified to discount past data due to a concern about possible structural change of an unknown form.

The asymptotic behavior of θ_t is a non-trivial issue because the model is self-referential. It turns out, though, that for small gains γ it is possible to obtain results on the asymptotics by studying a continuous time approximation to the recursive algorithm. More specifically, results from stochastic approximation theory show that asymptotically the dynamics are governed by the "mean dynamics" ordinary differential equation (ODE)

$$\frac{d\theta}{d\tau} = S^{-1} M(\theta) \left(T(\theta) - \theta\right) \tag{11}$$

$$\frac{dS}{d\tau} = M(\theta) - S \tag{12}$$

where $\tau = \gamma t$, $M(\theta)$ is the unconditional covariance matrix of the regressors holding θ fixed. The ordinary differential equation governing the evolution of

θ is identical to the E-stability differential equation with the exception that it includes weighting terms that depend on estimates of the regressors covariance matrix. See the Appendix for further details on the derivation of the ODE. It is straightforward to see that a rational expectations equilibrium is a rest point of the ODE, and in fact the fundamentals REE $(\bar{a}, \bar{b}) = (\bar{\pi}, 0)$ is a locally stable rest point provided $\alpha > 1$. Thus, the stability of a rational expectations equilibrium can be determined by the local stability of rest points to the ODE.

Under decreasing gain learning γ is replaced with $1/t$ and it can be shown that provided the fundamentals REE is E-stable, i.e. $\alpha > 1$, then in the limit as $t \to \infty$ the learning dynamics converge with probability one to the rational expectations equilibrium. This paper focuses on constant gain learning, in which parameter estimates weight recent data more heavily than past. We next summarize the analytical results for constant gain learning.

2.3 Analytic Results

The first result establishes that for a sufficiently small constant gain the perceived coefficients θ_t will be an approximately normal random variable with a mean equal to its rational expectations values and a variance that depends on both the constant gain and the long-run inflation target $\bar{\pi}$. The second result shows that from a given initial condition (θ_0, S_0) the solution to the "mean dynamics" of the ODE (11)-(12) give the expected transition path to the rational expectations values.

Proposition 1 *Let* $|\alpha| > 1$*. The belief parameters* θ_t *are approximately distributed as* $\theta_t \sim N\left(\bar{\theta}, \gamma V\right)$ *for small* $\gamma > 0$ *and large* t*, where* $\bar{\theta} = (\bar{\pi}, 0)'$ *and*

$$V = \begin{pmatrix} (\alpha - 1)\bar{\pi}^2 + \alpha\sigma_r^2 & -\bar{\pi}/2 \\ -\bar{\pi}/2 & 1/2 \end{pmatrix}$$

Proposition 2 *Let* $|\alpha| > 1$ *and define* $\phi_t = (\theta_t, vec(S_t))'$*. For any* ϕ_0 *within a suitable neighborhood of the unique, non-explosive rational expectations equilibrium, define* $\tilde{\phi}(\tau, \phi_0)$ *as the solution to the differential equation (11)-(12), with initial condition* ϕ_0*. Fix* $T > 0$*. The mean dynamics of (9)-(10) satisfy* $E\phi_t \approx \tilde{\phi}(\gamma t, \phi_0)$ *for* γ *sufficiently small and* $0 \leq t < T/\gamma$*.*

There are several important consequences from Proposition 1. First, the rational expectations equilibrium provides a benchmark solution in the sense that the coefficients for the forecast rule under learning are centered on the rational expectations values. Second, for a constant gain $\gamma \to 0$, the learning dynamics are close to the rational expectations equilibrium with high probability. Third, to gain insight into the global learning dynamics, for finite periods of time, one can study the solution paths to the mean dynamics differential equation, given initial conditions. The remainder of the paper uses these tools to study the implications for learning dynamics of inflation targets.

2.4 Learning Dynamics and Random-Walk Beliefs

Under constant gain learning there can be significant, temporary departures from RE. These departures can arise either by a (imperfectly announced) change in the long-run target or as an endogenous response to exogenous shocks. This section illustrates these possibilities in the Fisherian model with constant gain learning.

Proposition 1 shows that the real-time estimates θ_t are approximately normal with a mean equal to the rational expectations equilibrium and a standard deviation that is increasing in the constant gain γ. To illustrate the implications this has for learning dynamics, Figure 2 plots the 95% confidence ellipses around the REE of the constant gain learning coefficients for various values of the long-run inflation target $\bar{\pi}$. This figure was generated by setting $\alpha = 1.1, \sigma_r^2 = 0.1$ and a constant gain $\gamma = 0.05$, though the qualitative results hold for alternative parameterizations

Figure 2 demonstrates the finding in Proposition 1 that the constant gain parameter estimates are distributed around the rational expectations equilibrium, which is $(\bar{a}, \bar{b}) = (\bar{\pi}, 0)$. For small long-run inflation targets, the principal axis of the confidence ellipse is close to horizontal. For higher inflation targets, the confidence ellipses feature a decreasing principal axis. The slope of the principal axis is important since one can expect many trajectories moving in the direction of the axis. Note that even for high inflation targets, the ellipses are pointed in the direction of a random walk without drift, with larger values of b associated with smaller values of a. The relative size of these ellipses depends on the sizes of the constant gain; however, the direction in which the ellipses point depend on the size of the long-run inflation target. The confidence ellipses pointing toward a random walk without drift does not imply that actual learning dynamics will converge to a random walk model. The slope of the principal axes suggest that one can expect many trajectories moving in the direction of a random walk. Then the mean dynamics can help illustrate what happens subsequently for trajectories that move along the principal axis.

Proposition 2 shows that for any initial condition, and finite period of time, the expected transition path to the rational expectations equilibrium will be the solution path to the mean dynamics. One can think of constant gain learning dynamics as re-initializing the mean dynamics. Figure 3 plots representative mean dynamics where initial values for $a, b > 0$ are selected from the principal axis of the confidence ellipse in Figure 2 for a 4% inflation target. The initial values $a = 3.3$ and $b = 0.55$ correspond to an increase in the perceived mean and perceived serial correlation in inflation. The top panel plots the perceived value for the mean of inflation, a, while the bottom panel plots the perceived lag coefficient b.

The fundamental rational expectations equilibrium is a stable rest point of the mean dynamics, implying that along a learning path the mean dynamics will converge to the rational expectations equilibrium. Additionally, as anticipated in Figure 2, the transition path for the mean dynamics is interesting in its own right. At first the estimate for b moves toward the rational expectations equilibrium, slightly overshooting $b = 0$, but then reverses course and increases to a value of $b = 1$, where it remains for some time before returning to its rational expectations equilibrium value. At the same time, the value of a increases before abruptly decreasing to zero and then converging to its equilibrium value as b converges to zero. Note, in particular, that $a \approx 0$ at the same time that $b \approx 1$. Therefore, the mean dynamics show that private-sector agents come to believe temporarily that the inflation process is approximately a random walk. Importantly, while there is a path to $b \approx 1$ for initial $b = 0.55$ and a drawn from the principal axis for a 4% inflation target, there is no such path for a lower 2% inflation target.

Random-walk beliefs play a key role in the learning dynamics. In essence, agents come to believe that recent innovations in inflation are permanent shifts and not mean-reverting fluctuations. These random-walk beliefs are nearly self-fulfilling. A detailed argument is presented in Branch and Evans (2011), but an overview of the argument is useful. Suppose that agents hold random walk beliefs in terms of a forecasting model of the form

$$\pi_t = \pi_{t-1} + \varepsilon_t$$

which will arise in the learning model when $a = 0, b = 1$. Given these beliefs, actual inflation outcomes will be

$$\pi_t = \alpha^{-1}(\alpha - 1)\bar{\pi} + \alpha^{-1}\pi_{t-1} + \alpha^{-1}r_t$$

If $\alpha > 1$ is close to $\alpha = 1$, then the actual law of motion for inflation is a stationary but highly persistent process that is difficult to distinguish from a random walk.[4] Also, the mean inflation rate is the same under random walk beliefs

[4]For larger values of α, the random-walk model provides a progressively worse approxima-

as it is in the unique REE. That random-walk beliefs are nearly self-fulfilling has been pointed out in other settings by Sargent (1999) and Lansing (2009). Random walk beliefs introduce serial correlation into a model that is not serially correlated under rational expectations, as the random walk model uses higher order moments to track low frequency drift in inflation.

The mean dynamics show that random walk beliefs only can last for finite stretches of time. However, because the random walk beliefs are nearly self-fulfilling, it is difficult to detect the misspecification except using a long history of data. Most importantly, the random walk model provides a robust way to capture a time-varying conditional mean. When this drift is large enough then random-walk beliefs will fit the data well. Thus, random-walk beliefs can be long-lasting and, as will be seen below, they have important implications for the dynamics of inflation.

2.5 Implications of Inflation Targets

Having established the possibility of random-walk beliefs emerging under learning, we turn briefly to real-time simulations.

Consider the following experiment. The central bank is going to implement an increase in its long-run annual target from 2% to 3%. Assume that the economy is initially in a rational expectations equilibrium, but the private-sector has imperfect information about when the central bank will implement its new target and is unsure about the central bank's commitment to the new target. Figure 4 plots the resulting dynamics.[5] At time 0, the central bank's target $\bar{\pi}$ increases and leads to an increase in inflation without a corresponding increase in inflation expectations (which are determined by the adaptive learning rule). Then initially inflation is below target and the central bank begins reducing nominal interest rates in order to bring inflation up to target. The increase in the inflation rate is tracked by agents' econometric model as an increase in the persistence of inflation. As the mean dynamics predict, eventually agents' beliefs are that inflation follows a random walk. At this point, there is a burst as inflation increases to nearly 8% before returning to its new long-run value. Thus, implementing a higher target, as many observers have recommended, can lead to an overshooting of the new

tion to actual inflation dynamics. Thus, for α sufficiently, large random-walk beliefs will not emerge from the learning dynamics. A somewhat related point has been made by Orphanides and Williams (2005b). However, in the New Keynesian model below, inflation persistence is increasing in the target rate of inflation.

[5]This figure was generated as the average time-path across 1000 stochastic simulations of length 1000.

target. This overshooting arises because the initial upward drift in inflation, as the central bank implements its new target, leads to a nearly self-fulfilling belief that inflation follows a random walk.[6]

Even without a change in the long-run target, inflation may deviate substantially from its rational expectations equilibrium value as an endogenous response to fundamental shocks. For example, Figure 5 plots two real-time simulations of inflation dynamics in the Fisherian model with a 4% inflation target. As before, the figure is computed setting $\alpha = 1.1, \sigma_r^2 = .1$, and as in Figure 2 the constant gain is set $\gamma = .05$. To generate this figure the model is initialized at the REE, expectations are generated according to (5) with parameters updated via constant gain least-squares, and inflation is determined by (3). Figure 5 plots the results for two different typical simulations. The top two panels plots a_t, b_t, respectively, and the bottom panel plots inflation. The left panels are for the case where the deviation from equilibrium results in a burst of inflation, while the right panels show a rapid disinflation.

Under constant gain learning the economy hovers around its rational expectations equilibrium value. Then there is an abrupt qualitative change in the dynamics with bursts of inflation or disinflation before returning to a neighborhood of the rational expectations equilibrium. The pattern of beliefs correspond with what was observed in Figure 3, and Proposition 2, in that for finite stretches of time private-sector agents believe inflation follows a random walk. In simulations, these large deviations from rational expectations are recurrent.

Unlike Figure 4, the deviations away from the rational expectations equilibrium in Figure 5 are an endogenous response to fundamentals rather than to a change in the long-run inflation target of the central bank. Using techniques employed by Cho, Williams, and Sargent (2002), it is possible to examine which "escape paths" are most likely to drive the system away from the REE and to generate random walk beliefs by looking for the "most likely unlikely sequences" of shocks that move the system a given distance away from the equilibrium. In principle one can compute these escape paths analytically in special cases, but more typically it is necessary to resort to simulations.

However, it is intuitive that random walk beliefs can arise for the right sequence of shocks. Take the case of positive inflationary shocks. These shocks place inflation on an upward path leading agents' econometric model to pick up this trend with higher estimated values of b_t and lower values of a_t. In turn, inflation expectations will increase leading to a further upward drift in inflation, higher

[6]These results are reminiscent of McGough (2006) who examines changes to the natural rate of unemployment in the model of policymaker learning developed in Sargent (1999).

estimated values of b_t, until the estimated coefficients arrive at a random walk model which, as argued above, is nearly self-confirming. Moreover, the mean dynamics predict that even if beliefs are, on average, close to rational expectations this is the expected transition path following a series of these "most likely unlikely" sequences of shocks.

There is one strong conclusion to draw from Figures 4-5: in the Fisherian model, when the central bank implements a long-run inflation target with imperfect information, then inflation will deviate significantly from its equilibrium values (i) when the target is first implemented and (ii) as an endogenous response to certain sequences of shocks. The remainder of the paper demonstrates that these results are found in standard New Keynesian models under a wide range of policy rules.

3 Application to the New Keynesian Model

The previous section adopted a simple Fisherian model of inflation to illustrate that setting policy to implement a long-run inflation target in an imperfect information environment can lead to substantial deviations of inflation from its rational expectations equilibrium value. We now show that similar results obtain in the New Keynesian model with trend inflation (see Ascari and Ropele (2007)). The richer setting of the New Keynesian model facilitates a wider exploration of the generality of the results as well as policy implications.

3.1 A New Keynesian Model with Imperfect Information

Ascari and Ropele (2007) take a standard New Keynesian setting log-linearized around a non-zero steady-state rate of inflation. (See Appendix for details.) They show that this leads to the following equations that determine aggregate output and inflation

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma^{-1}\left(i_t - E_t\hat{\pi}_{t+1} - r_t^n\right) \quad (13)$$

$$\hat{\pi}_t = \theta_1\hat{x}_t + \theta_2 E_t\hat{\pi}_{t+1} + \sum_{j\geq 0}\xi_2^j\left(\theta_3 E_t\hat{x}_{t+1+j} + \theta_4 E_t\hat{\pi}_{t+1+j}\right) + u_t \quad (14)$$

where the reduced-form parameters $\theta_k, k = 1, ..., 4$ and ξ_2 are complicated expressions that depend on the underlying structural parameters. (See the Appendix for details). $\hat{x}_t, i_t, \hat{\pi}_t$ are log deviations of the output gap and the inflation rate, respectively, from steady-state. The shocks r_t^n, u_t are assumed for simplicity to be zero-mean iid with variance σ_r^2, σ_u^2. The model is closed by assuming that monetary policy sets nominal interest rates according to the Taylor rule:

$$i_t = \alpha_\pi (\pi_t - \bar{\pi}) + \alpha_x (x_t - \bar{x})$$

Stacking these equations, it is possible to represent the economy in vector form

$$Y_t = H + FE_t Y_{t+1} + M \sum_{j=0}^{\infty} \xi_2^j E_t Y_{t+1+j} + G z_t$$

with $Y = (\hat{x}, \hat{\pi})'$ and $z = (r^n, u)'$. When the inflation factor $\Pi = 1$, i.e. when there is zero steady-state inflation, these equations reduce to the benchmark New Keynesian model:

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1} - r_t^n) & (15) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t & (16) \end{aligned}$$

Ascari and Ropele demonstrate that, under most parameterizations, steady-state inflation lowers steady-state output and it alters the determinacy conditions when monetary policy follows a Taylor rule. Kobayashi and Muto (2010) show that E-stability properties of the model can also differ from the New Keynesian model linearized around a zero steady-state inflation rate. Throughout, monetary policy will be assumed to guarantee the model is determinate and E-stable.

As in the previous section, it is assumed that there is imperfect information about the economy, and so private-sector agents base forecasts on a perceived law of motion whose reduced-form is consistent with a rational expectations equilibrium. In the bivariate New Keynesian model, agents are assumed to forecast based on a simple VAR(1) model:

$$Y_t = A + BY_{t-1} + \varepsilon_t \quad (17)$$

where ε_t is a perceived white noise error. From this perceived law of motion, conditional expectations can be computed. As before, expectations conditional on the forecast model (17) are

$$\hat{E}_t Y_{t+1} = (I + B)A + B^2 Y_{t-1}. \quad (18)$$

As in the Fisherian model, the actual law of motion can be found by substituting the forecasts (18) into the expectational difference equation:

$$\begin{aligned} Y_t &= H + F(I+B)A + (1-\xi_2)^{-1} M \left[I + (I - \xi_2 B)^{-1} B \right] A & (19) \\ &+ \left[F + M(I - \xi_2 B)^{-1} \right] B^2 Y_{t-1} + G z_t & (20) \\ &\equiv T(A_{t-1}, B_{t-1})' \begin{bmatrix} 1 \\ Y_{t-1} \end{bmatrix} + G z_t & (21) \end{aligned}$$

Analytic results on E-stability, and convergence of constant gain learning, are unavailable in the New Keynesian model with trend inflation. Instead, the following sections focus on a calibrated version of the model and present numerical results that illustrate the theoretical possibilities. The model's parameters are calibrated in Table 1. The parameter values are chosen so that a time period corresponds to a quarter. The value of ζ implies a mark-up of 11%, the policy coefficients are equivalent to Taylor (1993) original policy prescription, the frequency of price adjustment α is in line with most empirical studies, while the values of σ_r^2, σ_u^2 come from Smets and Wouters (2007). The basic qualitative results do not hinge on the specifics of the calibration. The necessary ingredients are a non-zero inflation target, imperfect information, and a sufficiently strong feedback from expectations.[7]

3.2 Inflation Dynamics in the New Keynesian Model with Learning

The "mean dynamics" for the New Keynesian model take the same form as in the Fisherian model with the exception that the multivariate model complicates the expressions. Nevertheless, as in the Fisherian model a great deal can be learned about learning dynamics by examining the mean dynamics for the calibrated New Keynesian model. The mean dynamics are the solution path to the ordinary differential equation (11)-(12) where $\theta' = (A, B)$.

Figure 6 plots the mean dynamics, with the parameters calibrated as in Table 1, and a 4% inflation target. The initial values for all coefficients, except the constant and the own lag coefficient in the inflation component of the forecasting model, are set to their REE values. The remaining initial conditions were chosen so that the mean inflation rate is above its equilibrium value. There are 6 coefficients in θ, the two constants and the four coefficients in the lag matrix B. Each panel plots a different component of θ, while the bottom two panels plot the roots of the matrix B along the mean learning path. The key result to notice in Figure 6 are the first two panels on the right hand side of the figure. These plots are equivalent to the mean dynamics plots in the Fisherian model. Notice that agents come to believe that the process for inflation, but not the output gap, is a random-walk without drift. Private-sector agents can hold these beliefs temporarily before they converge to the rational expectations equilibrium. The bottom panel on the right shows that along the mean path there is, in fact, a unit root in the perceived

[7] In the Fisherian model, for sufficiently large values of α, a random-walk forecasting model provides a poor approximation to the implied inflation dynamics. In the New Keynesian model with trend inflation, similar results obtain for large values of α_π and small inflation targets.

coefficient matrix B.

Figure 7 solves the mean dynamics just as in Figure 6 for long-run targets of $0\%, 1\%, 2\%$, and 4%, with the same starting values for the coefficient matrix B.[8] The figure plots just the constant and the own lag coefficient from the inflation forecasting equation, while suppressing the other coefficient paths. As can be seen in the figure, when there is a zero inflation target the beliefs converge monotonically and rapidly to the rational expectations equilibrium. For successively larger values of the inflation target, the estimates for $B(2,2)$ begin to move towards the REE value and then reverses course and approaches $B(2,2) \approx 1$ when the target $\bar{\pi} = 4\%$. At the same time, the estimated $A(2)$ moves towards zero before converging to the mean value. When the inflation target is 4% random walk beliefs emerge.

One view of the US Federal Reserve Bank is that it has an implicit inflation target of approximately 2%. One popular argument to avoid the possibility that in the future the zero lower bound will again bind, is to increase the target to 4%. Figure 8 conducts this experiment in the New Keynesian model with imperfect information. The model is calibrated according to Table 1, initialized in the unique REE when $\bar{\pi} = 2\%$. The central bank then increases its target to 4% and the private-sector must learn in real time about the new higher average rate of inflation. The figure plots the time-series averaged across 1000 stochastic simulations. The figure demonstrates that, as in the Fisherian model, there is a significant overshooting of inflation as agents come to believe that inflation follows a random walk. In this experiment inflation increases to above 10% per annum before returning to the new targeted value of 4%.

Many of those who advocate higher inflation targets do so in order to avoid abrupt disinflationary episodes. The results from the Fisherian model suggest that this may not be the case with imperfect information. Figure 9 simulates the real-time learning of the calibrated New Keynesian model with a 4% inflation target for two separate simulations. In each panel, the model is initialized in a rational expectations equilibrium, within which inflation will always remain bounded in a small neighborhood of the inflation target. Under constant gain learning, however, there is an abrupt qualitative change as agents come to believe that inflation follows a random walk and there is a rapid disinflation, in the left panel, and an abrupt inflationary episode in the right panel. These types of destabilizing dynamics are recurrent in the model when the central bank sets a sufficiently large inflation target. In this case, sufficiently large is 4%.

[8]The remaining coefficients A, S were initialized at their REE values.

3.3 Zero Lower Bound

One of the primary reasons that some economists and policymakers advocate for a higher long-run inflation target is to avoid the zero lower bound from binding. In New Keynesian models, e.g. Eggertsson and Woodford (2003), Eggertsson (2008), the zero lower bound on nominal interest rates can bind when there are sufficiently large and persistent negative shocks. A liquidity trap with deflation can arise, under rational expectations, since the persistent shocks generate expectations of deflation. Under learning Williams (2006) and Williams and Reifschneider (2000), shows that the zero lower bound may be reached even more often. In both cases, increasing the inflation target makes it less likely that the zero lower bound will bind.

So far, we have not analyzed the issue of the zero lower bound, since the main message of this paper is the destabilizing learning dynamics that can arise under long-run inflation targets. However, it is straightforward to extend the framework to incorporate a zero lower bound and examine whether a higher inflation target can rule out a binding zero lower bound on nominal interest rates. To a certain extent, this is not a fair comparison: in our model, under rational expectations, the zero lower bound will almost never bind since all shocks are iid with small variances. However, if there can be deflationary spirals under learning when there is no liquidity trap under rational expectations, there is even greater reason to suspect that inflation targets can lead to destabilizing economic dynamics.

To study this issue consider the "benchmark" version of the New Keynesian model:

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t, \end{aligned}$$

where the central bank employs the Taylor rule subject to the zero lower bound:

$$i_t = \max\{\bar{i} + \alpha_\pi \left(\pi_t - \bar{\pi} \right) + \alpha_x \left(x_t - \bar{x} \right), 0\}$$

and where $\bar{x} = (1 - \beta)\bar{\pi}/\kappa$ ensures the steady-state inflation rate equals the central bank's target. This is the framework adopted in Eggertsson and Woodford (2003), though here it is assumed r_t^n are iid shocks, and can be derived from the New Keynesian model above by setting the steady-state inflation equal to zero. Woodford (2003) argues that these equations are valid approximations in a low inflation environment so long as the central bank's inflation target is not too large.

Calibrating the model as above, which implies a value for $\kappa \approx 0.16$, and simulating it under constant gain learning leads to occasional departures from a neighborhood of the rational expectations equilibrium and a deflationary spiral.[9] At first, the economy fluctuates around the long-run inflation target. Then a qualitative change in the dynamics arises as agents come to believe inflation follows a random walk and inflation (and expected inflation) rapidly disinflates as was seen earlier. However now, when the zero lower bound binds, rapid deflation sets in, resulting in a severe recession, with output gaps approaching -10%. The spiral does not persist forever as the mean dynamics take over and the economy returns to a neighborhood of the steady-state.[10]

The deflation episode in Figure 10 suggests that a higher inflation target cannot rule out paths that collapse to the zero lower bound. However, it is not obvious whether a higher target will lead to a binding zero lower bound more or less often than a lower target. To shed light on this question, we simulated the model 1000 times for 5000 periods in each simulation and recorded the first time in which the economy was at the zero lower bound and the average time spent at the zero lower bound. We did this exercise for a 2% inflation target and a 3% inflation target. Table 2 reports the results. While the quantitative difference between a 2% and 3% target is slight, these results indicate that a higher inflation target may not be useful in avoiding the liquidity trap.

4 Extensions

The paper concludes with several important extensions to the New Keynesian model with learning.

4.1 Implications for Policy Communication

The previous results assumed that monetary policy is set to satisfy the Taylor rule. The results do not hinge on this assumption: the key is that the target is not perfectly communicated. This section considers implications for policy communication of the present learning framework with long-run inflation targets.

The assumption maintained in this paper is that the central bank cannot completely communicate the long-run inflation target. The private-sector has im-

[9] These deflationary spirals are similar to the paths in Evans, Guse, and Honkapohja (2008).

[10] To prevent deflationary paths that fall without bound, we impose the following restrictions on the learning and economic dynamics: (i) agents' only update their parameter estimates of B if the roots lie inside the unit circle, and (ii) the deflation rate and the output gap have ceilings of 8% and 10% per annum, respectively. The latter restriction is consistent with Evans (2011) and Bullard and Cho (2005).

perfect information about the economic environment and the policy target. It suffices that they do not know the timing of when the central bank will implement its target. If there is imperfect information about the economy, but perfect information about the long-run inflation target then the random-walk beliefs will not arise and the instability witnessed in the previous sections will not exist.

To illustrate this point, assume that agents know the mean values for inflation and the output gap. Thus, their perceived law of motion does not require them to estimate the mean rates of inflation and the output gap, but just the lag coefficients. That is, assume a perceived law of motion of the form:

$$\hat{Y} = B\hat{Y}_{t-1} + \varepsilon_t$$

where the perceived law of motion is written in deviation from mean form, $\hat{Y}$. It is straightforward to verify that the actual law of motion will be of a similar form, except written in deviation from mean form. Thus, the B component of the T-map is unchanged. Figure 11 plots the mean dynamics path for the calibrated model and various long-run inflation targets when the long-run average inflation rate is perfectly communicated. The figure clearly demonstrates that in this case the long-run inflation target does not have much impact on the learning dynamics and that random-walk beliefs do not arise.

4.2 Optimal Discretionary Policy

The Taylor rules considered previously prescribe that nominal interest rates should be adjusted whenever inflation deviates from its long-run target value. One might instead imagine a central bank facing a dual mandate of stabilizing both prices and output, with deviations from long-run targets made explicit according to an optimal policy problem. This subsection demonstrates that random-walk beliefs also arise in such a setting under optimal discretionary policy, i.e. optimal policy without commitment. The next subsection will show that even with commitment it is possible for random-walk beliefs to arise.

For simplicity, assume the economy can be represented by the benchmark New Keynesian equations (15)-(16). The objective function of the central bank is

$$\max_{\pi_t, x_t} -(1/2)E_0 \sum_{t\geq 0} \beta^t \left[\lambda(x_t - \bar{x})^2 + (\pi_t - \bar{\pi})^2\right]$$

where $\bar{x}$ is the long-run output gap consistent with the inflation target $\bar{\pi}$. The central bank takes the New Keynesian Phillips Curve (16) as its constraint. Without commitment, the central will set policy to satisfy it's first order condition

$$\pi_t - \bar{\pi} = -\frac{\lambda}{\kappa}(x_t - \bar{x}) \tag{22}$$

Combining (22) with (16) gives an expectational difference equation that generates the stochastic process for inflation

$$\pi_t = \alpha_0 + \alpha_1 \hat{E}_t \pi_{t+1} + \nu_t \tag{23}$$

where $\alpha_0 = \frac{\kappa^2+\lambda(1-\beta)}{\lambda+\kappa^2}\bar{\pi}$, $\alpha_1 = \frac{\beta\lambda}{\lambda+\kappa^2}$, and ν_t is an appropriately defined white-noise shock. The reduced form (23) is identical to the Fisherian model of section 2 with α_1 determined by β, λ, κ. Proceeding in the same manner as in section 2, agents form their expectations from an AR(1) model of inflation. Figure 12 plots the resulting mean dynamics.

Figure 12 was created by solving the mean dynamics under the following parameterization: $\beta = 0.99, \kappa = .14, \lambda = .15, \sigma_u = .05$. As before, the learning coefficients a, b, S were initialized above their rational expectations equilibrium values, and then the mean dynamics are solved to find the transition path back to the unique rational expectations equilibrium. The transition path leads temporarily to random-walk beliefs despite the fact that policy is now set optimally.[11] We remark, however, that small values of λ yield small values of α_1, which will make random-walk beliefs less likely. Since smaller values of λ implies a greater relative weight on inflation stabilization, this last remark is consistent with Orphanides and Williams (2005b) who show that policy that optimal policy when private agents are learning should place greater emphasis on inflation stabilization.

4.3 Price-level Targeting

The results in this paper demonstrate that, in addition to the other consequences of having a higher long-run inflation target, the target will be destabilizing in an imperfect information environment. One popular alternative to inflation targeting is price-level targeting. Woodford (2003) and Vestin (2006) shows that a policy to target a price-level path can implement the optimal policy with commitment.

[11] Evans and Honkapohja (2003) show that the "fundamentals-based" nominal interest-rate rule, which aims to implement the optimal discretionary equilibrium using a rule that *assumes* rational expectations, will lead to indeterminacy and instability under learning. They also show that if policymakers use a suitable "expectations-based" rule consistent with optimal policy, then the optimal rational expactations equilibrium will be determinate and stable under learning. However, random-walk beliefs can still sometimes arise under their policy rule if agents use constant-gain learning.

Eggertsson and Woodford (2003) show that a price-level target can be an effective policy for pulling an economy away from the zero-lower bound.

However, what if, as with the inflation target, the central bank is unable to perfectly communicate the precise value or timing for the price-level target? Can price-level targeting lead to temporarily unstable learning dynamics just as in the case of long-run inflation targets? To address this issue, this section considers a central bank that acts in accordance with the following price-targeting rule:

$$\kappa p_t + \lambda x_t = p^* \tag{24}$$

where p_t is the (log) price-level and p^* is the target value for the (log) price-level. This policy rule will implement the optimal policy under commitment for a zero long-run inflation target. The qualitative results below carry over to the case where the central bank targets a price-level path consistent with a non-zero long-run inflation target.

Using the identity $\pi_t = p_t - p_{t-1}$, plugging (24) into the benchmark NK aggregate supply equation (16) leads to the following equation for the price-level

$$p_t = \alpha_0 + \alpha_1 \hat{E}_t \left(p_{t+1} - p_t\right) + \alpha_2 p_{t-1} + \eta_t \tag{25}$$

where $\alpha_0 = (\kappa^2/(\lambda + \kappa^2))\bar{\pi}, \alpha_1 = \beta\lambda/(\lambda + \kappa^2), \alpha_2 = \lambda/(\lambda + \kappa^2)$. Notice that p_t depends on $\hat{E}_t p_t$ under imperfect information because we assume that p_t is not contemporaneously observable. Again, suppose that private-sector agents forecast the price level according to the forecasting model

$$p_t = a + bp_{t-1} + \varepsilon_t \Rightarrow E_t p_t = a + bp_{t-1},\ E_t p_{t+1} = a(1+b) + b^2 p_{t-1}.$$

The actual price-level process is found by plugging these expectations into (25), yielding

$$p_t = T(a,b)' X_{t-1} + \nu_t$$

where $T(a,b)' = (\alpha_0 + \alpha_1 ab, \alpha_1 b(b-1) + \alpha_2)$.

Figure 13 plots the mean dynamics for the price-level targeting rule case under the same calibrated parameter values as the previous subsection (Figure 12) and $p^* = 10$. A key difference with the price-target rule, compared to the rules considered earlier, is that the rational expectations equilibrium exhibits non-zero serial correlation. For the chosen parameter values the REE value of b is approximately 0.7. Figure 13 initializes the learning coefficients at 0.77. The transition path first leads away from the rational expectations equilibrium, and then abruptly changes course heading towards a random-walk model for the price-level before finally converging to the rational expectations equilibrium. Thus, the results of this section demonstrate that if there is imperfect information about the price-level target, then price-level targeting policy rules can also lead to temporarily unstable inflation dynamics.

5 Conclusion

Long-run inflation targets, on the order of 4% per annum, have sometimes been recommended to guard against liquidity traps and binding zero constraints on nominal interest rates. These recommendations persist even though many welfare analyses caution against this approach since the distortions resulting from higher average inflation are often found to outweigh any gains from stabilizing inflation. Both the arguments for and against higher inflation targets have typically been made under the rational expectations assumption. This paper has revisited the issue of raising the inflation target, focusing on the question of whether higher targets do, in fact, lead to greater stability.

The primary results of this paper are as follows. First, although over time beliefs converge toward rational expectations, the combination of constant gain learning and a positive inflation target can lead agents in the economy to temporarily believe that the inflation process follows a random walk without drift. Such beliefs are temporarily (almost) self-confirming. When agents perceive the inflation process to be a random walk they will interpret recent innovations to inflation as permanent shifts in the mean inflation rate. These random walk beliefs arise for a very intuitive reason. The long-run inflation target, and imperfect information about that target, lead agents to estimate the mean inflation rate from real-time data. If data lead to a slight upward drift in the inflation rate, agents' econometric model will pick up that drift, leading to higher inflation expectations that feed back into higher inflation rates. This process is self-reinforcing and in some cases agents eventually come to believe that inflation follows a random walk. Crucially, we have shown that these beliefs are nearly self-fulfilling.

Implementing a higher target – say by moving the target from 2% to 4% – will introduce just the type of drift in inflation that can lead to random walk beliefs. These random walk beliefs cause a substantial overshooting of the inflation target. Finally, occasional "unlikely" sequences of shocks can introduce drift to the inflation process that trigger random-walk beliefs and large deviations from the rational expectations equilibrium. Such departures from rational expectations can generate significant bursts of inflation, disinflation, and even deflation, and these are more likely at higher inflation targets. In summary, higher inflation targets, in an imperfect information environment, increases the chances of unstable inflation dynamics.

Appendix

Proof to Propositions 1-2.

Propositions 1 and 2 provide asymptotic approximations to the learning algorithm

$$\begin{aligned} \theta_t &= \theta_{t-1} + \gamma S_{t-1} X_{t-1} \left(\pi_t - \theta'_{t-1} X_{t-1}\right)' \\ S_t &= S_{t-1} + \gamma \left(X_t X'_t - S_{t-1}\right) \end{aligned}$$

and where $\pi_t = T(\theta_{t-1})' X_{t-1} + \alpha^{-1} r_t$. It is possible to re-write the equations for real-time learning in the form

$$\phi^\gamma_t = \phi^\gamma_{t-1} + \gamma \mathcal{H}(\phi^\gamma_{t-1}, \bar{X}_t)$$

where $\bar{X}_t = (1, p_t, p_{t-1}, r_t)'$. Verifying many of the technical conditions required for convergence of the learning algorithm is simplified by the fact that the state dynamics are conditionally linear and can be written as

$$\bar{X}_t \equiv \begin{bmatrix} X_t \\ X_{t-1} \\ r_t \end{bmatrix} = \begin{bmatrix} A(\phi_{t-1}) & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \bar{X}_{t-1} + \begin{bmatrix} B & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} W_t$$

where $I, 0$ are conformable matrices, and

$$X_t = A(\phi_{t-1}) X_{t-1} + B W_t$$

Here $X_t = (1, p_t)'$ and $W_t = (1, r_t)'$. The superscript γ highlights the dependence of the parameter estimates on γ. The stochastic approximation approach is to compare the solutions to the continuous time ODE and the discrete time algorithm, and then study the convergence of the continuous time approximating ODE. Thus, define the corresponding continuous time sequence for ϕ^γ_t as $\phi^\gamma_t = \phi^\gamma_t$ if $\tau^\gamma_t \leq \tau < \tau^\gamma_{t+1}$ where $\tau^\gamma_t = \gamma t$.

This Appendix sketches the proof to the propositions by making use of Propositions 7.8 and 7.9 of Evans and Honkapohja, and using arguments in Chapter 14 of Evans and Honkapohja and Branch and Evans (2011). The "mean dynamics" are the solution to the ODE

$$\frac{d\phi}{d\tau} = h(\phi)$$

where $h(\phi) = E\mathcal{H}(\phi, \bar{X}_t)$. Notice, in particular, that this is the mean dynamics ODE given in the text:

$$\begin{aligned} \frac{d\theta}{d\tau} &= S^{-1} M(\phi) \left(T(\theta) - \theta\right) \\ \frac{dS}{d\tau} &= M(\phi) - S \end{aligned}$$

Let $\tilde{\phi}(\tau, \phi_0)$ be the solution to the mean dynamics differential equation $\dot{\phi} = h(\phi)$ from an initial condition ϕ_0. Define $U^\gamma(\tau) = \gamma^{-1/2}\left(\phi^\gamma(\tau) - \tilde{\phi}(\tau, \phi_0)\right)$. The two propositions in the text are based on U^γ converging to a Gaussian variable, in a sense made precise below. In particular, for small γ the probability distribution of $U^\gamma(\tau)$ converges to the probability distribution of the solution $U(t)$ to the differential equation

$$dU(\tau) = D_\phi h(\tilde{\phi}(\tau, \phi_0))U(\tau)d\tau + \mathcal{R}^{1/2}(\tilde{\phi}(\tau, \phi_0))dW(\tau)$$

The results below establish that $EU(\tau) = 0$ so that, as $\gamma \to 0, E\phi^\gamma(\tau) = \tilde{\phi}(\tau, \phi_0)$ and $\lim_{\tau\to\infty} \tilde{\phi}(\tau, \phi_0) = \phi^*$. Thus, key properties of the learning dynamics arise from a study of (i.) the asymptotic distribution for θ_t around the rational expectations equilibrium and (ii.) the mean dynamic path $\tilde{\phi}(\tau, \phi_0)$ where ϕ_0 are drawn from the asymptotic distribution.

The validity of the propositions in the text depend on verifying a set of technical conditions. The conditions required for Proposition 2 can be verified by using the arguments in Branch and Evans (2011), and so they are omitted here.

Proposition 2 uses the following result from Evans and Honkapohja (2001):

Proposition 3 (EH(2001)) *Consider the normalized random variables* $U^\gamma(\tau) = \gamma^{-1/2}\left(\phi^\gamma(\tau) - \tilde{\phi}(\tau, \phi_0)\right)$. *As* $\gamma \to 0$, *the process* $U^\gamma(\tau)$, $0 \le \tau \le T$, *converges weakly to the solution* $U(\tau)$ *of the stochastic differential equation*

$$dU(\tau) = D_\phi h(\tilde{\phi}(\tau, \phi_0))U(\tau)d\tau + \mathcal{R}^{1/2}(\tilde{\phi}(\tau, \phi_0))dW(\tau)$$

with initial condition $U(0) = 0$, *where* $W(\tau)$ *is a standard vector Wiener process, and* $\mathcal{R}$ *is a covariance matrix whose* i, j*th elements are*

$$\mathcal{R}^{ij}(\phi) = \sum_{k=-\infty}^{\infty} Cov\left[\mathcal{H}^i(\phi, \bar{X}_k^\phi), \mathcal{H}^j(\phi, \bar{X}_0^\phi)\right]$$

Moreover, the solution to the stochastic differential equation has the following properties

$$EU(\tau) = 0 \tag{26}$$
$$\frac{dVar(U(\tau))}{d\tau} = D_\phi h(\tilde{\phi}(\tau, \phi_0))V_u(\tau) + V_u D_\phi h(\tilde{\phi}(\tau, \phi_0))' + \mathcal{R}(\tilde{\phi}(\tau, \phi_0)), \tag{27}$$

where $V_u = Var(U(\tau))$. This result indicates that, for finite periods of time, the learning dynamics weakly converge to the solution of the ODE $\dot{\theta} = h(\theta)$, thus establishing Proposition 2.

Proposition 1 relies on the stochastic differential equation in the above result to have a stationary distribution asymptotically. Establishing this result requires stronger conditions. In particular,

A1 ϕ^* is a globally asymptotically stable resting point of the ODE $\dot{\phi} = h(\phi)$.

A2 $D_\phi h(\phi)$ is Lipschitz and all of the eigenvalues of $D_\phi h(\phi^*)$ have strictly negative real parts.

A3 There exist $q_1, q_2, q_3 \geq 0$ such that, for all $q > 0$ and all compact sets Q, there is a constant $\mu(q, Q)$ such that for all $x \in \mathbb{R}^d, a \in Q$,

i. $\sup_n E_{x,a}(1 + |\bar{X}_n|^q) \leq \mu(1 + |x|^q)$,

ii. $\sup_n E_{x,a}(|\mathcal{H}(\phi_n^\gamma, \bar{X}_{n+1})|^2) \leq \mu(1 + |x|^{q_1})$,

iii. $\sup_n E_{x,a}(|\nu_{\phi_n^\gamma}(\bar{X}_{n+1})|^2) \leq \mu(1 + |x|^{q_2})$, where $\nu_\phi = \sum_{k \geq 0}(\Pi_\phi^k \mathcal{H}_\phi - h(\phi))(y)$, and Π_ϕ is the stationary transition probability associated to the stationary Markov process $\bar{X}_n$,

iiii. $\sup_n E_{x,a}(|\phi_n^\gamma|^2) \leq \mu(1 + |x|^{q_3})$.

As noted in the text, there are two resting points to the ODE $\dot{\phi} = h(\phi)$, corresponding to the two REE one with $b = 0$ the other with $b = \alpha$. The $b = \alpha$ REE is unstable under learning, and for some values of ϕ_t^γ the dynamics are explosive. For initial conditions sufficiently close to $b = 0$, and sufficiently small gain parameters γ, then the MSV REE is a stable resting point to the learning dynamics. However, to apply the approximation theorem below, the algorithm needs to rule out trajectories in the explosive region. Thus, the learning algorithm is supplemented with a "projection facility" that projects the iterates ϕ_t^γ into a confined set (see Evans and Honkapohja (2001) and Kushner and Yin (1997)). As a result of these assumptions the RE solution $(\bar{a}, \bar{b}) = (\bar{\pi}, 0)$ is a globally stable resting point of the ODE that satisfies (A1)-(A2).

It remains to verify (A3). Write $\bar{X}_n = \bar{A}(\phi_{n-1})\bar{X}_{n-1} + \bar{B}W_t$, where the expressions for $\bar{A}, \bar{B}$ are given above. The eigenvalues of $\bar{A}$ are zero and A, and the projection facility along with the conditional linearity ensures that $\bar{X}_n$ remains in a compact subset of D, an open set around the REE $(\bar{\pi}, 0)$, which has a unique resting point to $\dot{\phi} = h(\phi)$. Thus (A3.i) is immediate. Verifying conditions (A.ii)-(A.iv) is tedious, but given a projection facility that constrains ϕ_t to lie in a compact subset of D, it is straightforward to extend the arguments in Evans and Honkapohja (2001) (pg.335-336) for the Cobweb model to the present setting.

Proposition 1 arises from the following result in Evans and Honkapohja:

Proposition 4 (EH(2001)) *Consider the normalized random variables* $U^{\gamma_k}(\tau) = \gamma_k^{-1/2}(\phi^{\gamma_k}(\tau) - \phi^*)$. *For any sequences* $\tau_k \to \infty, \gamma_k \to 0$, *the sequence of random variables* $(U^{\gamma_k}(\tau_k))_{k\geq 0}$ *converges in distribution to a normal random variable with zero mean and covariance matrix*

$$C = \int_0^\infty e^{sB}\mathcal{R}(\theta^*)e^{sB'}ds,$$

where $B = D_\phi h(\phi^*)$.

It follows then that $\theta_t \sim N(\theta^*, \gamma C)$ for small γ and large t. Using arguments in Evans and Honkapohja (2001), Chapter 14.4, C is the solution to the matrix Riccati equation

$$D_\theta h(\phi^*)C + C\left(D_\theta h(\phi^*)\right)' = -\mathcal{R}_\theta(\phi^*)$$

where $\mathcal{R} = E\mathcal{H}(\phi^*, \bar{X})\mathcal{H}(\phi^*, \bar{X})'$. Straightforward calculations then lead to the expression for V in the text.

Overview of the New Keynesian Model with Trend Inflation.

The reduced-form equations (13)-(14) were derived by Ascari and Ropele (2007) from a standard New Keynesian framework and log-linearized around a non-zero steady-state inflation rate. This Appendix provides a brief overview of the model in Ascari and Ropele (2007).

There are a continuum of (identical) households whose flow utility is given by

$$U(C, N) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi N_t$$

Households maximize lifetime utility subject to the constraint,

$$P_t C_t + B_t \leq P_t w_t N_t + (1 + i_{t-1}B_{t-1} + \Pi_t + T_t$$

where P_t is the price of the final good, B_t are risk-free one period bonds with nominal net return i_{t-1}, Π_t are profits returned to households and T_t are lump-sum transfers. This formulation assumes the "cashless limit" that abstracts from money balances in the household's problem. The household will select sequences of consumption, labor hours, and bond holdings to satisfy the first-order conditions

$$C_t^{-\sigma} = \beta\hat{E}_t\left(C_{t+1}^{-\sigma}(1+i_t)\frac{P_t}{P_{t+1}}\right) \quad (28)$$

$$\chi C_t^{\sigma} = w_t \quad (29)$$

When $\hat{E} = E$, i.e. agents hold rational expectations, the conditions (28)-(29) have the usual interpretation. When $\hat{E} \neq E$, then (28) is a behavioral relation that dictates that boundedly rational households will choose their consumption holdings so as to equate their expected marginal rate of substitution with the marginal rate of transformation. This is called Euler equation learning and is the benchmark approach in the learning literature. An alternative approach has been advanced by Preston (2006) where boundedly rational agents solve their perceived dynamic programming problem, assuming that their beliefs will not change over time. The infinite horizon approach implies a reduced-form IS equation that depends on expectations of interest rates and inflation over all future horizons. The reduced-form equation (13) only requires boundedly rational agents to forecast one period ahead. This assumption was made for technical convenience. The results in that section do not hinge on the assumption of Euler equation learning.

The final good Y_t is produced by perfectly competitive firms using intermediate goods $Y_t(i)$ produced using a CES production function $Y_t = \left(\int_0^1 Y_t(i)^{(\zeta-1)/\zeta} di\right)^{\zeta/(\zeta-1)}$, $\zeta > 1$. The final goods firms choose their inputs to maximize profits, taking prices as given, resulting in the demand for input i $Y_t(i) = (P_t(i)/P_t)^{-\zeta} Y_t$. Intermediate goods are produced by a continuum of firms with technology $Y_t(i) + N_t(i)$. Intermediate goods producers take the demand for their good as given when setting prices optimally. However, they also face the Calvo risk where with probability α the firm's price will remain unchanged each period. This leads to an expression for price setting that is identical to that of Woodford, except that the optima re-set price also depends on the cumulative gross inflation rates over the period that a price might remain fixed.

Ascari and Ropele (2007) show that the steady-state properties depend on the trend inflation rate and, in particular, under most plausible parameterizations positive trend inflation leads to a lower steady-state output. Ascari and Ropele then demonstrate that a log-linearization, around a steady-state with gross inflation Π, of the equilibrium conditions lead to the following reduced-form equations:

$$\begin{aligned}
\hat{x}_t &= E_t \hat{x}_{t+1} - \sigma^{-1}\left(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t\right) \\
\hat{\pi}_t &= \kappa \hat{x}_t + \beta \Pi E_t \hat{\pi}_{t+1} + (\Pi - 1)\beta(1 - \alpha \Pi^{\zeta-1}) E_t \left((\zeta - 1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right), \\
\hat{\phi}_t &= (1 - \alpha\beta\Pi^{\zeta-1})(1 - \sigma)\hat{x}_t + \alpha\beta\Pi^{\zeta-1} E_t \left((\zeta - 1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right)
\end{aligned}$$

where $\hat{x}, \hat{\pi}, \hat{i}$ are log deviations from a steady-state with gross inflation factor Π. Iterating forward on the ϕ equation leads to the equations in the text. Ascari and Ropele show that $\kappa = (\Pi - 1)(\sigma - 1)\beta(1 - \alpha\Pi^{\zeta-1}) + \sigma\lambda(\Pi), \lambda(\Pi) = (1 - \alpha\Pi^{\zeta-1})(1 - \alpha\beta\Pi^{\zeta})/\alpha\Pi^{\zeta-1}$.

By setting $\Pi = 1$, i.e. linearizing around a zero inflation steady-state, these equations reduce to the benchmark New Keynesian model

$$\begin{aligned} \hat{x}_t &= E_t \hat{x}_{t+1} - \sigma^{-1} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} - r_t \right) \\ \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t \end{aligned}$$

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Table 1: Calibration. Note: For definitions of parameters, see Appendix.

β	0.995
θ	10
α_π	1.5
α_x	0.125
α	0.67
σ	1.00
χ	1.00
σ_r^2	0.1
σ_u^2	0.003

Table 2: Zero lower bound frequencies.

$\bar{\pi}$	1st time to ZLB	mean time at ZLB
2%	393.38	3%
3%	347.34	14%

Figure 1: T-map dynamics in the Fisherian model.

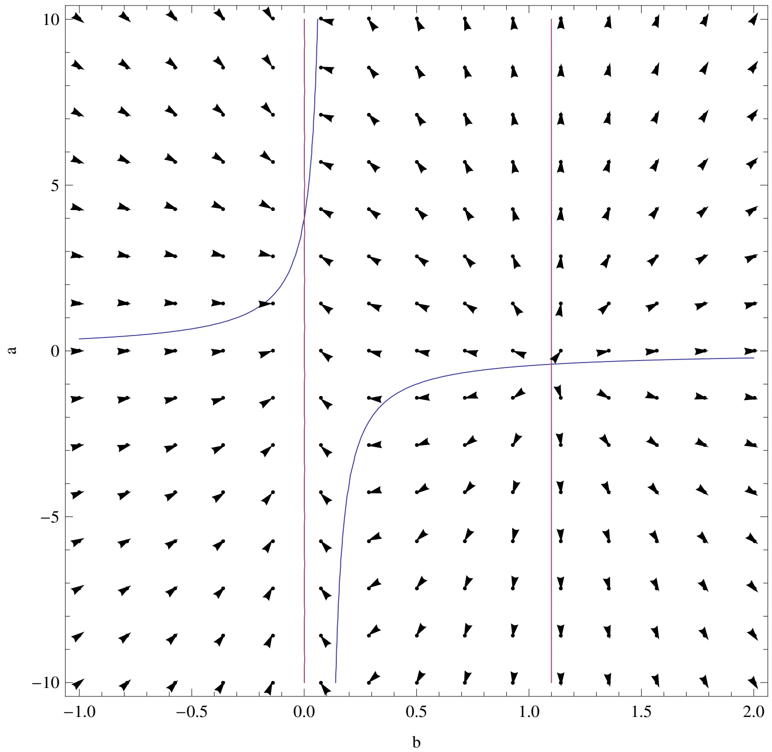

Figure 2: Confidence Ellipses around REE for constant gain learning. Each ellipse corresponds to a different inflation target. The targets are $.5\%, 1\%, 2\%, 3\%, 4\%, 5\%$, expressed in annualized rates.

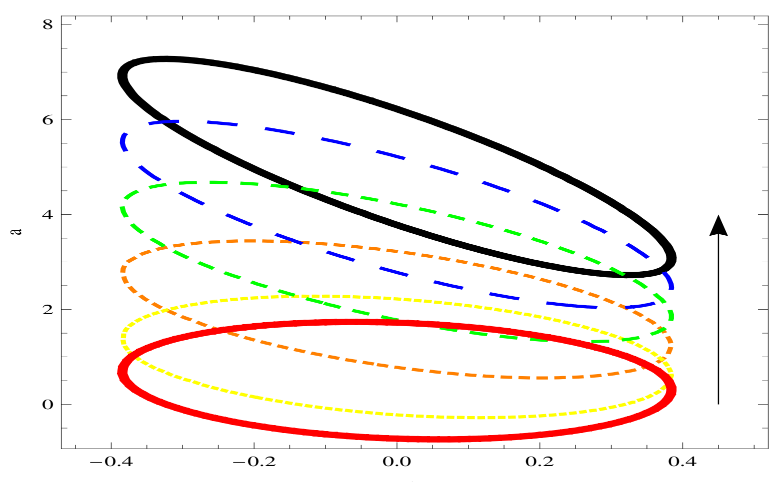

Figure 3: Mean Dynamics in the Fisherian Model. Initial conditions are drawn from the confidence ellipse.

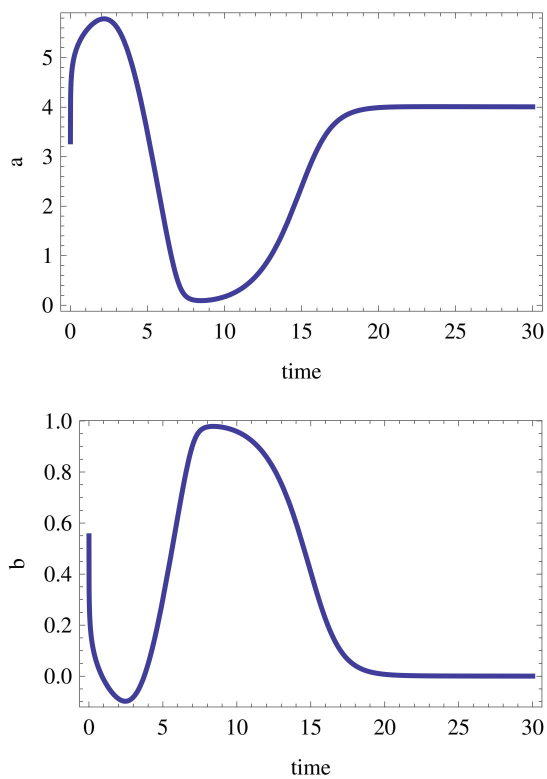

Figure 4: Change in inflation target from 2% to 3%. Economy begins in the REE. $\alpha = 1.1, \sigma_r^2 = .003, \gamma = .02$.

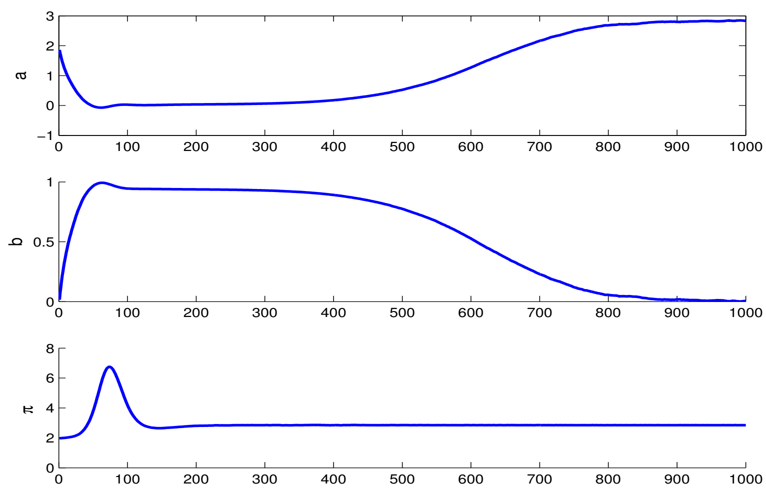

Figure 5: Fisherian inflation dynamics with a 4% target. Left panel plots an inflationary episode, right panel plots a disinflationary episode.

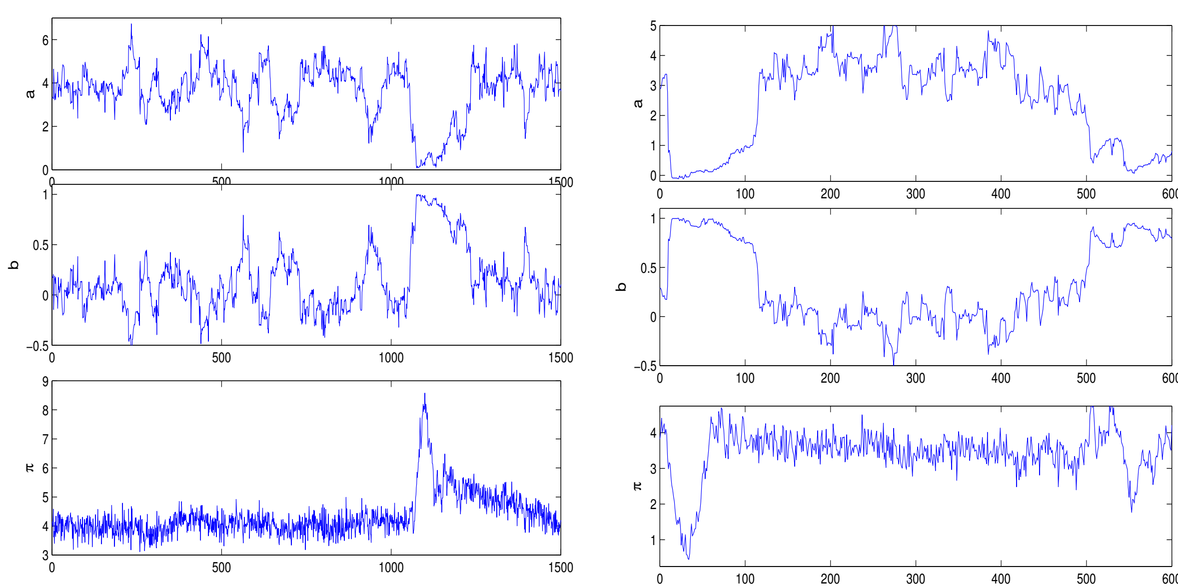

Figure 6: Mean dynamics in NK Model with a 4% target.

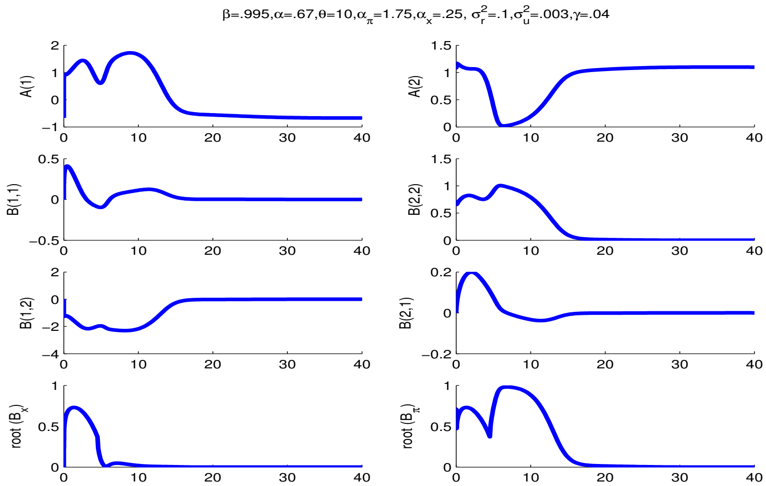

Figure 7: Mean dynamics in NK Model with alternative inflation targets.

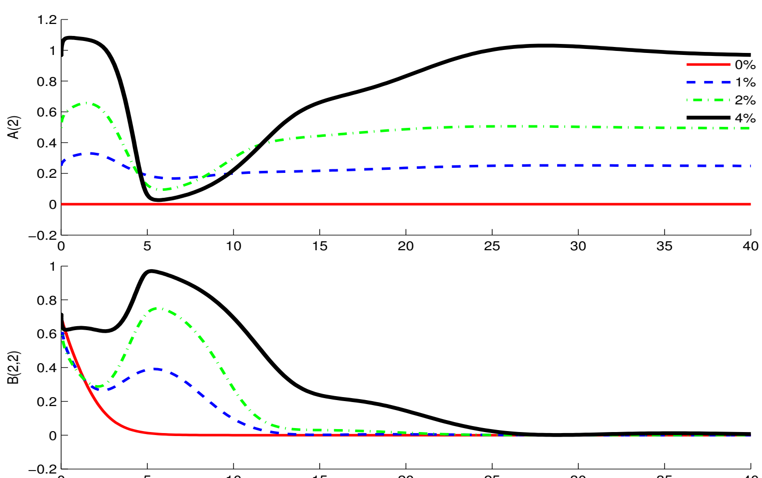

Figure 8: Increasing the Inflation target from 2% to 4%.

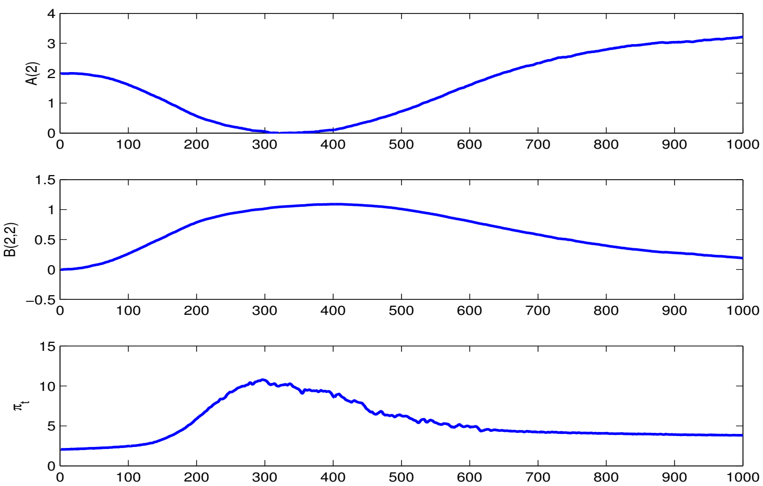

Figure 9: Inflation dynamics in NK Model with a 4% target.

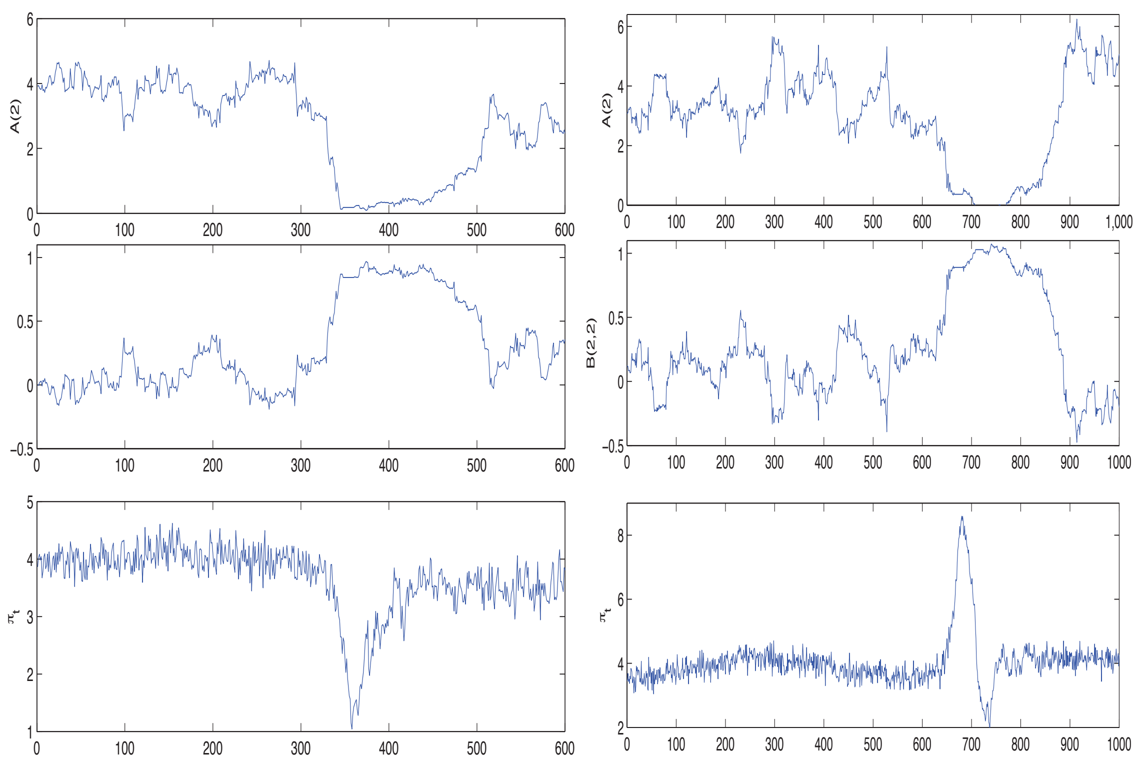

Figure 10: Inflation dynamics in NK Model with a 4% target and a zero lower bound.

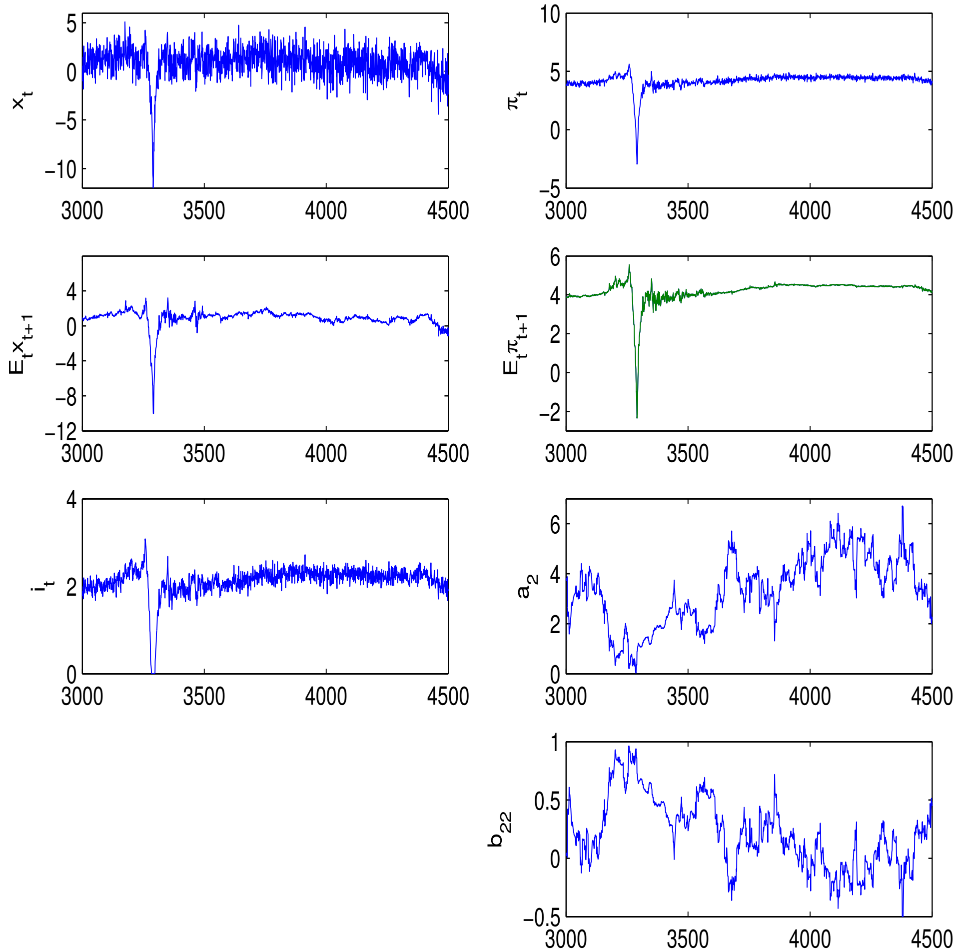

Figure 11: Policy target communication.

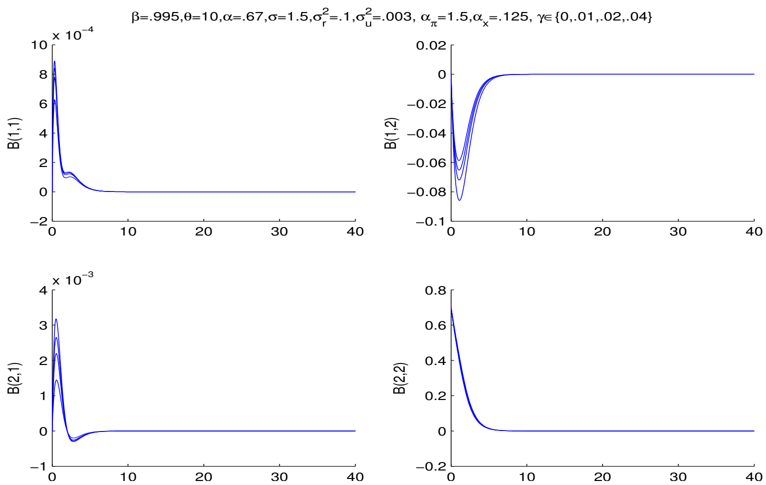

Figure 12: Mean dynamics in NK Model with a 4% target and optimal policy without commitment.

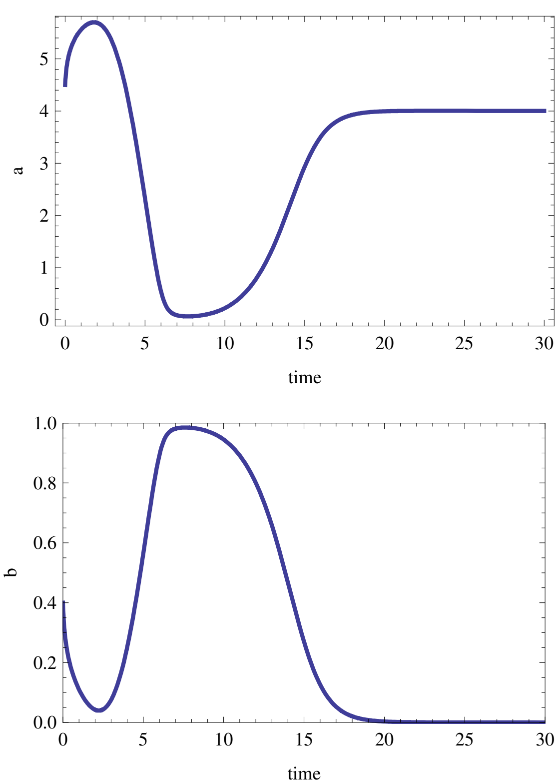

Figure 13: Mean dynamics in NK Model with a price-level targeting rule.