Expectations in Macroeconomics: Adaptive versus Eductive Learning[1]

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Rational expectations solutions to macroeconomic models are equilibria requiring the coordination of expectations, and one can investigate the local stability of these solutions under alternative learning rules. Eductive (mental) approaches to learning have somewhat stricter stability conditions than do adaptive (statistical) approaches, as we illustrate using three economic models. The expectational stability principle can be used to understand the relationship between the corresponding stability conditions. This principle also provides insight into the persistent learning dynamics that can arise under modified adaptive learning rules.

Les Anticipations Macroéconomiques: l'Apprentissage Adaptatife et Divinatoire

Les solutions à anticipations rationales dans les modèles macroeconomiques sont des equilibres qui exigeant la coordination des anticipations, et on peut examiner la stabilité locale des cettes solutions selon la choix des regles d'apprentissage. Les approches divinatoires (mentales) à l'apprentissage conduisent à des conditions plus strictes que celles des approches adaptatifes (statistiques), et on illustre cette proposition avec trois modelès economiques. Le principe de "expectational stability" démontre la connexion entre les conditions correspondantes de stabilité. Ce principe éclaire les dynamiques d'apprentissage persistantes qui se produiraient avec des regles modifiés d'apprentissage adaptatife.

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1 Introduction

The current standard paradigm for modeling expectations in macroeconomics is rational expectations (RE). There are obvious attractions. Under RE

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there are no incentives to change the forecast rule, while under alternative assumptions an individual agent would be able to obtain higher utility or profits by adopting a different forecasting procedures. However, implicit in the RE hypothesis are strong assumptions about the knowledge of the agents about the model structure. Furthermore, in models in which expectations affect aggregate outcomes, a rational expectations equilibrium (REE) must be thought of as a Nash equilibrium in strategies. Consequently, from the time of its introduction, questions were raised about whether, and if so how, an REE could be reached by real world agents.

There is now a substantial literature on the question of whether REE can be attained as the outcome of a learning process. The approaches fall into two broad groups: eductive and adaptive. Eductive processes operate in mental time, describing a process of reasoning. The issue is whether rational agents, following a reasoning process, can deduce that the equilibrium path must given by a particular REE, leading to coordination on that REE. Adaptive learning instead operates in real time. Agents have a forecast rule, which they update in response to forecast errors. A standard approach is to assume they update parameter estimates according to least squares or other statistical procedures. The main issue is whether over time expectations converge to some REE.

There are strong connections between these two approaches but the stability conditions are not identical. The expectational stability principle, based on the mapping from the perceived law of motion to the actual law of motion, provides a way of understanding the connections: in a wide range of economic models, expectational stability gives the stability conditions under adaptive learning, while the stricter iterative expectational stability conditions are necessary for stability under eductive learning. This paper reviews these connections in three types of model. We also discuss an adaptive rule that does not fully converge to RE.

Most of the material on adaptive learning is based on my forthcoming book with Seppo Honkapohja (Evans and Honkapohja 2001). The discussion of eductive learning is mainly based on joint work with Roger Guesnerie, as well as on his earlier research.

2 The Cobweb Model

We begin with the simplest model, the well-known cobweb market model. Demand for a product depends on price, p_t. Supply depends linearly on $E^*_{t-1}p_t$, the price expected the previous period, when production decisions are made. Both supply and demand also depend on unobserved white noise random shocks. Assuming market clearing yields the reduced form

$$p_t = \mu + \alpha E^*_{t-1}p_t + \eta_t \tag{1}$$

where $\eta_t \sim iid(0, \sigma^2_\eta)$. In the basic cobweb model $\alpha < 0$, but related models with the same reduced form have $\alpha > 0$, so we leave α unrestricted.

Under rational expectations $E^*_{t-1}p_t = E_{t-1}p_t$ and there is a unique REE (rational expectations equilibrium) given by

$$p_t = \bar{a} + \eta_t \text{ and } E_{t-1}p_t = \bar{a}, \text{ where } \bar{a} = (1-\alpha)^{-1}\mu.$$

2.1 Adaptive learning

A simple and natural adaptive learning scheme is to set expectations equal to the sample mean of the data:

$$E^*_{t-1}p_t = a_{t-1}, \text{ where } a_t = t^{-1}\sum_{i=1}^{t} p_i.$$

Note that this is a special case of Least Squares in which p_t is regressed on a constant. The formula for a_t can be written recursively as

$$a_t = a_{t-1} + \gamma_t(p_t - a_{t-1}), \text{ where } \gamma_t = 1/t.$$

The sensitivity to forecast errors γ_t (the *gain* sequence) declines to zero as $t \to \infty$. In a stochastic context this "decreasing gain" assumption is entirely natural as it is a requirement for the consistency of an estimator in standard econometric frameworks. The classical "adaptive expectations" assumption of a fixed gain $0 < \gamma_t = \gamma < 1$ cannot converge to RE as it would remain stochastic in the limit. For the decreasing gain rule we have:

Proposition 1 *If* $\alpha < 1$ *then* $a_t \to \bar{a}$ *with probability* 1. *If* $\alpha > 1$ *then* a_t *converges with probability* 0.

See (Evans and Honkapohja 2001), and the references therein, for these and related results on least squares learning in the cobweb model. The results can be generalized in many ways. For example, suppose supply to depend on a vector of exogenous, stochastically stationary observable shocks w_{t-1}, so that the reduced form is

$$p_t = \mu + \alpha E^*_{t-1} p_t + \delta' w_{t-1} + \eta_t.$$

The unique REE takes the form $p_t = \bar{a} + \bar{b}' w_{t-1} + \eta_t$ and under adaptive learning a natural forecast procedure would be to set $E^*_{t-1} p_t = a_{t-1} + b'_{t-1} w_{t-1}$, where (a_t, b'_t) is updated each period using a Least Squares regression of p_s on an intercept and w_{s-1}, using data from $s = 1, \ldots, t$. Then it can be shown that $(a_t, b'_t) \to (\bar{a}, \bar{b}')$ with probability 1 if $\alpha < 1$. One can also allow for heterogeneous expectations of agents in various ways.

2.2 Eductive learning

We now take up the eductive viewpoint for the model (1). The argument here was initially given in (Guesnerie 1992). See (Evans and Guesnerie 1993) for the multivariate formulation and (Guesnerie 1999) for a comprehensive discussion and a survey of current research in macroeconomics using the eductive approach.

The cobweb model can be reformulated as a producers' game in which the strategy of each firm is its output and the optimal choice of output depends on expected price. We assume that firms have identical costs. We allow for heterogeneous expectations, however, so that the equilibrium market price is given by

$$p_t = \mu + \alpha \int E^*_{t-1} p_t(\omega) d\omega + \eta_t,$$

where we now assume a continuum of agents indexed by ω and $E^*_{t-1} p_t(\omega)$ is the expectation of the market price held by agent ω. The REE is unchanged.

The eductive argument works as follows. Let $S(\bar{a})$ denote a neighborhood of $\bar{a}$. Suppose it is common knowledge (CK) that $E^*_{t-1} p_t(\omega) \in S(\bar{a})$ for all ω. Then it follows that it is CK that $Ep_t \in |\alpha|\, S(\bar{a})$. Hence, assuming individual rationality, it follows that it is CK that $E^*_{t-1} p_t(\omega) \in |\alpha|\, S(\bar{a})$ for all ω. If $|\alpha| < 1$ then this reinforces and tightens the CK assumption. Iterating this argument it follows that $E^*_{t-1} p_t(\omega) \in |\alpha|^N S(\bar{a})$ for all $N = 0, 1, 2, \ldots,$ and hence the REE $Ep_t = \bar{a}$ is itself CK. Guesnerie calls such an REE "strongly rational." We also use the equivalent terminology that the REE is "eductively stable" or "stable under eductive learning."

Proposition 2 *If* $|\alpha| < 1$ *then the REE is stable under eductive learning, while if* $|\alpha| > 1$ *the REE is not eductively stable.*

Note two crucial differences from the adaptive learning results. First, the learning here takes place in mental time, not real time. In principle, given the CK assumptions and applying the full power of their reasoning abilities, rational agents would coordinate instantaneously on the REE when $|\alpha| < 1$. Second, when $\alpha < -1$ the REE is not eductively stable, but is asymptotically stable under adaptive learning.

2.3 E-stability and Iterative E-stability

The results for stability under adaptive and eductive learning can be obtained using the expectational stability principle. (See (Evans 1985), (Evans and Guesnerie 1993) and (Evans and Honkapohja 2001)). Suppose all agents have the homogeneous expectation $E^*_{t-1}p_t = a$ corresponding to a perceived law of motion for prices $p_t = a + \eta_t$. Then the actual law of motion for prices would be given by $p_t = (\mu + \alpha a) + \eta_t$ and the corresponding true expected price would be $\mu + \alpha a$. We thus have a mapping from the perceived law of motion to the actual law of motion given by

$$T(a) = \mu + \alpha a.$$

The REE $\bar{a}$ is the fixed point of this map. If $\lim_{k\to\infty} T^k(a) = \bar{a}$ for $a \neq \bar{a}$, so that $\bar{a}$ is stable under iterations of the T- map, the REE is said to be *iterative expectationally stable* (or IE-stable). Clearly this is given by the condition $|\alpha| < 1$, which corresponds to stability under eductive learning.

Stability of $\bar{a}$ under the differential equation

$$da/d\tau = T(a(\tau)) - a(\tau)$$

determines whether the REE is said to be *expectationally stable* (or E-stable). For the cobweb model the condition is $\alpha < 1$, which corresponds to stability under adaptive learning. Intuitively, E-stability describes a "slow adjustment" of a toward the true expected value generated by a.

We have described E-stability and IE-stability in the context of the simple cobweb model, but the approach can easily be generalized. For a given economic model, let ϕ be a parameter vector specifying a stochastic process believed by agents to describe the solution. Suppose that $T(\phi)$ is the parameter vector that characterizes the ALM (actual law of motion) generated by the PLM (perceived law of motion) parameter vector ϕ. Then IE stability of a fixed point $\bar{\phi}$ of $T(\phi)$ is defined by local stability under iterations of T, while E-stability is defined as local stability under the differential equation $d\phi/d\tau = T(\phi) - \phi$.

How general are the links between E-stability and adaptive learning (e.g. least squares learning), and between IE-stability and eductive learning? For a wide range of models it can be demonstrated that E-stability governs local stability of least squares learning. This is discussed extensively in (Evans and Honkapohja 2001), which also outlines several types of assumption that will be needed for a general result. In cases where theoretical proofs are not available the link appears borne out by simulations. For stability under eductive learning, IE-stability appears in general to be a necessary but not always sufficient condition. The lack of sufficiency is shown in (Evans and Guesnerie 1993) for the cobweb model with a sufficiently heterogeneous structure, and in (Evans and Guesnerie 1999) for a variation of the dynamic univariate models given in the next Section. In the present paper IE-stability is necessary and sufficient for stability under eductive learning.

Finally, the condition for IE-stability of $\bar{\phi}$, that all eigenvalues of the derivative map $DT(\bar{\phi})$ have modulus less than one, is stronger than the condition for E-stability, that all eigenvalues of $DT(\bar{\phi})$ have real parts less than one. We thus have the following relationships:

$$\begin{aligned} \text{Eductive stability} &\Rightarrow \text{IE-stability} \\ &\Rightarrow \text{E-stability} \Leftrightarrow \text{Stability under adaptive learning.} \end{aligned}$$

This is an informal conjecture, or "working hypothesis," the validity of which has been established for many specific classes of model.

3 A Linear model with Multiplicities

We next consider models of the form

$$y_t = \beta E_t^* y_{t+1} + \delta y_{t-1} + v_t, \tag{2}$$

when expectations are homogeneous, where v_t is an exogenous white noise disturbance. If the model is deterministic then $v_t \equiv 0$. The model has been normalized so that its equilibrium value is zero (this is not innocuous under learning). An initial condition $y_0 = \hat{y}_0$ is given.

Assuming real roots to the quadratic given below, there are two rational expectations solutions of the form

$$y_t = \bar{\lambda} y_{t-1} + \bar{d} v_t,$$

where

$$\beta \bar{\lambda}^2 - \bar{\lambda} + \delta = 0 \text{ and } \bar{d} = (1 - \beta \bar{\lambda})^{-1}.$$

Let the two roots $\bar{\lambda}$ be denoted $|\lambda_1| < |\lambda_2|$. For various reasons we do not consider here explosive solutions, in which $\left|\bar{\lambda}\right| > 1$, nor the case of nonreal roots. There are therefore two cases of interest:

(1) The standard "saddle point" case, in which $|\lambda_1| < 1 < |\lambda_2|$. This case arises if and only if $|\beta + \delta| < 1$.

(2) The "indeterminacy" case, in which $|\lambda_1| < |\lambda_2| < 1$. In (β, δ) space this corresponds to four regions. Region A and B are contiguous, with A bounded by $\delta > 0$, $\beta < -1$ and $\beta + \delta < -1$, and B bounded by $\delta \leq 0$, $\beta < -1/2$, $\beta + \delta < -1$ and $\beta\delta < 1/4$. Similarly regions C and D are contiguous, with region D bounded by $\delta < 0$, $\beta > 1$ and $\beta + \delta > 1$ and C bounded by $\delta \geq 0$, $\beta > 1/2$, $\beta + \delta > 1$ and $\beta\delta < 1/4$.

For adaptive learning we use the LS (least squares) learning approach. Agents make forecasts according to

$$E_t^* y_{t+1} = \lambda_t y_t,$$

where the parameter λ_t is estimated by a least squares regression of y_s on y_{s-1} using data $s = 1, \ldots, t-1$. The question of interest is whether, in some suitable stochastic sense, $\lambda_t \to \lambda_1$ or $\lambda_t \to \lambda_2$ as $t \to \infty$.

Applying the techniques of (Evans and Honkapohja 2001) it can be shown that local stability of λ_1 and λ_2 is determined by the corresponding E-stability condition. To obtain this condition we consider a PLM $y_t = \lambda y_{t-1} + d v_t$. This leads to forecasts $E_t^* y_{t+1} = \lambda y_t$ which, when inserted into (2), yields the ALM $y_t = \delta(1 - \beta\lambda)^{-1} y_{t-1} + (1 - \beta\lambda)^{-1} v_t$. Hence the mapping from PLM to ALM is

$$T(\lambda) = \delta(1 - \beta\lambda)^{-1}$$

and an REE $\bar{\lambda}$ is E-stable if $T'(\bar{\lambda}) = \delta\beta(1 - \beta\bar{\lambda})^{-2} < 1$. We have:

Proposition 3 *Let $\bar{\lambda} = \lambda_1$ or $\bar{\lambda} = \lambda_2$ and assume $|\bar{\lambda}| < 1$. If $\bar{\lambda}$ is E-stable ($T'(\bar{\lambda}) < 1$) then the solution $y_t = \bar{\lambda} y_{t-1} + \bar{d} v_t$ is locally stable under LS (least squares) learning. If $T'(\bar{\lambda}) > 1$ it is unstable under LS learning.*

For the alternative precise interpretations of "locally stable" here, see (Evans and Honkapohja 2001), Chapter 6, and (Evans and Honkapohja 1998).

We consider next the eductive argument, following (Evans and Guesnerie 1999). We adopt the nonstochastic reduced form

$$y_t = \beta \int E_t^* y_{t+1}(\omega_t) d\omega_t + \delta y_{t-1},$$

though it appears straightforward to generalize the argument to allow for a white noise shock, as in the cobweb model. The corresponding REE are of the form $y_t = \bar{\lambda} y_{t-1}$. We make the following CK assumption:

CK Condition: For all $s = 1, 2, \ldots, \infty$, y_s lies between $(\bar{\lambda} - \epsilon) y_{s-1}$ and $(\bar{\lambda} + \epsilon) y_{s-1}$ for some $\epsilon > 0$.

Here we take either $\bar{\lambda} = \lambda_1$ or $\bar{\lambda} = \lambda_2$ and we assume that ϵ is sufficiently small. Thus it is assumed to be CK that for all time the actual growth rate is close to $\bar{\lambda}$.

For each agent the expected growth rate $E_t^* y_{t+1}(\omega_t)/y_t$ lies between $\bar{\lambda} - \epsilon$ and $\bar{\lambda} + \epsilon$. Inserting this condition into the reduced form it follows that the actual growth rate will be between $\bar{\lambda} - \rho\epsilon$ and $\bar{\lambda} + \rho\epsilon$, where $\rho = |T'(\bar{\lambda})|$. If $\rho < 1$ then this argument tightens the initial CK restrictions. Thus the condition for eductive stability in this model is the same as the IE-stability condition $|T'(\bar{\lambda})| < 1$. We obtain:

Proposition 4 *Let $\bar{\lambda} = \lambda_1$ or $\bar{\lambda} = \lambda_2$ and assume $|\bar{\lambda}| < 1$. If $\bar{\lambda}$ is IE-stable ($|T'(\bar{\lambda})| < 1$) then the solution $y_t = \bar{\lambda} y_{t-1}$ is locally eductively stable. If $|T'(\bar{\lambda})| > 1$ then the solution is not locally eductively stable.*

We can now compare the stability of solutions under adaptive (least squares) and eductive learning. We emphasize that all stability results are local.

In the saddle point region $|\beta + \delta| < 1$ there is a unique nonexplosive solution given by λ_1. It can be verified that in this region λ_1 satisfies the IE-stability condition $|T'(\lambda_1)| < 1$. Hence the λ_1 solution is both eductively stable and stable under LS learning.

In the indeterminacy regions A, B, C, D both the λ_1 and λ_2 solutions are nonexplosive. λ_1 continues to be IE-stable and hence both eductively stable and stable under LS learning. The situation for the λ_2 solution is more complicated. The λ_2 solution is never eductively stable in A, B, C, D and it is not stable under LS learning in regions B, C. However in regions A and D the λ_2 solution is stable under LS learning.

Variations to the basic model have been taken up in the literature. If the specification includes a non-zero intercept, i.e. $y_t = \phi + \beta E_t^* y_{t+1} + \delta y_{t-1} + v_t$, and the steady state must also be learned, this will affect the stability conditions under learning. Under LS learning this is because the forecast rule estimated by LS will be augmented to include an intercept, while under eductive learning the CK assumption would be correspondingly weakened. A second variation of the model is to weaken the information assumption by assuming that agents do not have available observations on y_t when time t decisions are made.

LS and eductive learning have been examined for both variations of the model. (See (Evans and Honkapohja 2001) for LS learning and (Evans and Guesnerie 1999) for the analysis of eductive learning). Although the stability conditions are altered, we continue to find that the stability conditions for eductive learning are stricter than for adaptive learning.

4 Persistent Learning Dynamics in the Increasing Social Returns model

In our last example we consider a form of adaptive learning that does not fully converge to RE. The deviation from rationality under adaptive learning now becomes greater than before, since it persists asymptotically. The potential attraction is that the learning dynamics themselves become of greater interest.

We adopt a nonlinear economic model that can have multiple steady states, the "Increasing Social Returns" extension of the OG model. This model is described in Chapter 4 of (Evans and Honkapohja 2001). The ISR (Increasing Social Returns) model is an extension of the basic overlapping generations model of money with production, in which we introduce a positive production externality and random productivity shocks. The reduced form of this model takes the form

$$\begin{aligned} n_t &= H(E_t^* X_{t+1}), \\ X_t &= G(n_t, v_t). \end{aligned}$$

Here n_t is aggregate employment, X_t is a measure (a monotonic transformation) of aggregate output and v_t is an *iid* productivity shock with mean 1. H and G are continuously differentiable functions.

A nonstochastic version of the model is obtained when $v_t \equiv 1$ for all t. Assuming point expectations in this case we can again solve for $n_t = \mathcal{F}(E_t^* n_{t+1})$. Nonstochastic steady states are given by $n = \mathcal{F}(n)$, where $\mathcal{F}(n) = H(G(n, 1)$. For appropriate choices of the parameter values $\mathcal{F}$ is an increasing function with three interior steady states $n_L < n_U < n_H$ satisfying $0 < \mathcal{F}'(n_L), \mathcal{F}'(n_H) < 1$ and $\mathcal{F}'(n_U) > 1$. (Here $\mathcal{F}'(n)$ denotes the derivative of $\mathcal{F}$). The lower two steady states n_L and n_U constitute "coordination failures" since it can be shown that they are Pareto dominated by the high steady state n_H. For some choices of parameter values there can be a single steady state, but we restrict attention to the three steady state case. When the random productivity shock is present, REE "noisy steady states" will exist, at least when the support for v_t is small enough. These take the form $n_t = n$ and $X_t = G(n, v_t)$, where n is near n_L, n_U or n_H.

For eductive learning we focus on the nonstochastic case $v_t \equiv 1$. The model can be linearized around a steady state so that, to a first order approximation $n_t = \bar{n} + \beta(E_t n_{t+1}^* - \bar{n})$ where $\bar{n} = n_L, n_U, n_H$, and $\beta = \mathcal{F}'(\bar{n})$. Suppose we have a CK restriction of the form $\bar{n} - \epsilon \leq n_t \leq \bar{n} + \epsilon$, for $\epsilon > 0$ sufficiently small, for all $t = 1, 2, 3, \ldots$. Then it can be verified that the steady state is eductively stable if $|\beta| < 1$ and not if $|\beta| > 1$. It follows that n_L and n_H are locally stable under eductive learning, but that n_U is not.

Consider next adaptive learning and return to the stochastic case. We adopt the simplest rule, close to the one used in the cobweb model:

$$E_t^* X_{t+1} = \theta_{t-1}, \text{ where } \theta_t = \theta_{t-1} + \gamma_t (X_t - \theta_{t-1}).$$

Under the standard assumption $\gamma_t = 1/t$, an REE noisy steady state near n_L or n_H is locally stable under adaptive learning, while an REE near n_U is not. See (Evans and Honkapohja 2001), Chapter 11.

We now consider adaptive learning with constant gain, i.e. $0 < \gamma_t = \gamma < 1$. Constant gain is often recommended if there is risk of unknown, recurring structural shifts. It can also be a good choice if other agents use it. The results of this section are drawn from (Evans and Honkapohja 2001), Chapter 14. Let $\theta_L = G(n_L, 1)$, $\theta_U = G(n_U, 1)$ and $\theta_H = G(n_H, 1)$ be the values of X corresponding to n_L, n_U and n_H, respectively. We assume that the support of v_t is $[\bar{v}_1, \bar{v}_2]$. We have:

Proposition 5 *There exist $\hat{v}_1 < 1 < \hat{v}_2$ so that for all $\bar{v}_1, \bar{v}_2$, satisfying $\hat{v}_1 < \bar{v}_1 < 1 < \bar{v}_2 < \hat{v}_2$, there are neighborhoods $N(\theta_L) = (a_1, a_2)$ and $N(\theta_H) = (b_1, b_2)$, with $0 < a_1 < \theta_L < a_2 < \theta_U < b_1 < \theta_H < b_2$, such that $\theta_{t-1} \in N(\theta_L)$ implies $\theta_t \in N(\theta_L)$ and $\theta_{t-1} \in N(\theta_H)$ implies $\theta_t \in N(\theta_H)$.*

Thus, for a sufficiently small support for the productivity shock v_t, expectations will remain trapped in a neighborhood of θ_L or θ_H if they start in (or enter) that neighborhood. Note that since $n_t = H(\theta_{t-1})$, this also implies that n_t will be confined to a neighborhoods of n_L or n_H.

Proposition 6 *Suppose $\bar{v}_1 < \hat{v}_1$ and $\bar{v}_2 > \hat{v}_2$. Then for every interval $J = (\bar{\theta}_1, \bar{\theta}_2)$, $0 < \bar{\theta}_1 < \bar{\theta}_2$, and for all neighborhoods $N(\theta_H)$ of θ_H and $N(\theta_L)$ of θ_L there is a positive integer T such that if $\theta_t \in J$ then, for all $s > t + T$, $\theta_s \in N(\theta_H)$ with positive probability and $\theta_s \in N(\theta_L)$ with positive probability.*

Thus, for a given gain γ, if the support of v_t is large enough there are endogenous fluctuations, with the economy occasionally and randomly switching between high and low levels of economic activity near the E-stable steady states. These fluctuations are not fully rational sunspot equilibria (which can also exist in this model), but instead are generated by a combination of the random intrinsic productivity shocks and the particular form of the learning rule. Simulations illustrating the endogenous fluctuations generated are presented in (Evans and Honkapohja 2001), Chapter 14. It is also shown numerically that it is (approximately) optimal for agents to use the constant gain value used in the simulations (in preference to other values of γ or to $\gamma_t = 1/t$), given that all other agents do so. Thus the forecast rules used are (approximately) optimal within a (restricted) class of rules. Whether such reasonable but not fully rational forecast rules can be good descriptions of the world is an open question.

5 Conclusions

Eductive learning is closest to strict rationality. Because eductive reasoning takes place in mental time, the reasoning process could be instantaneous, leading to immediate coordination on an eductively stable REE. A possible alternative view of eductive learning is that the mental stages may correspond to stages in real time, with the possible deviations from REE shrinking (in the eductively stable case) as time progresses. An advantage of eductive learning is that it models agents much like economic theorists.

Adaptive learning models makes agents more boundedly rational. Agents use a statistical forecasting model that is misspecified during the learning transition. However, when an REE is adaptively stable, this misspecification goes away in the limit and their forecasts become fully rational asymptotically. An advantage of adaptive learning is that it models agents much like econometricians. Furthermore, by varying the precise assumptions we make about their econometric specification, we can examine the effects of standard econometric problems such as omitted variables or misspecified functional forms.

Which is the appropriate way to model economic agents will ultimately be a matter for empirical and experimental research. It is likely that the answer depends on the circumstances, for example, in experiments, on the details of the setting and the types of information provided to the subjects. A plausible conjecture is that when a model is simple and transparent, as well as eductively stable, agents will coordinate rapidly on the REE. In such circumstances least squares learning may be too pessimistic in its predictions concerning the speed of convergence.

If a model has no eductively stable REE, but has an REE that is adaptively stable, then a plausible conjecture is that there will still be convergence to the REE, at a rate governed by the accumulation of data. After all, agents still need to make forecasts and standard econometric techniques provide a plausible model of how agents will do so. The eductive results provide a caution, however, that coordination in such cases may not be robust.

Finally, when the model is very complex and agents do not clearly understand the structure, simple constant gain adaptive learning rules may provide useful insights, and indicate the possibility of learning dynamics that persistently deviate from strict rationality.

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