# Learning and the Stock Market: Price Dynamics and Bubbles

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### Introduction

- Macroeconomic models are usually based on optimizing agents in dynamic, stochastic setting and can be summarized by a **dynamic system**, e.g.

$$egin{array}{rcl} y_t &=& Q(y_{t-1},y^e_{t+1},w_t), \ {
m or} \; y_t &=& Q(y_{t-1},\left\{y^e_{t+1}
ight\}_{j=0}^\infty,w_t), \end{array}$$

 $y_t =$  economic variables at time t (unemployment, inflation, investment, etc.),  $y_{t+1}^e =$  expectations of these variables,  $w_t =$  exogenous random factors at t

– The presence of expectations  $y_{t+1}^e$  makes macroeconomics inherently different from natural science. But how are expectations formed?

- Since Lucas (1972, 1976) and Sargent (1973) the standard assumption is **rational expectations** (RE).

- RE assumes too much knowledge & coordination for economic agents. We need a realistic model of **rationality** What form should this take?

- My general answer is given by the **Cognitive Consistency Principle**: economic agents should be about as smart as (good) economists, e.g.

- model agents like economic theorists the eductive approach, or
- model them like **econometricians** the **adaptive learning** approach

- These provide possible mechanisms of coordination. Today I will follow the adaptive approach. Agent/econometricians must select models, estimate parameters and update their models over time.

## Outline

The talk reviews my research (and some others') on expectations and adaptive learning in the context of stock prices.

My interest in asset-price bubbles began with "A Test for Speculative Bubbles in the Sterling-Dollar Exchange Rate: 1981-84" (AER, 1986).

I will discuss:

- Rational bubbles (without learning)
- Instability of rational bubbles under learning

- Asset prices under adaptive learning
  - constant gain learning of fundamentals solution
  - underparameterized dynamics
  - dynamic predictor selection: rules of thumb
  - dynamic predictor selection with least-squares learning
- Learning about risk and return: a simple model of bubbles and crashes
- Near-rational exuberance
- Conclusions

#### **Rational Bubbles**

A good place to start is "rational bubbles", which featured early in the multiple equilibria literature.

The usual consumption Euler equation with risk neutral preferences implies

$$p_t = (1+r)^{-1} E_t (p_{t+1} + d_{t+1})$$
, where  $r = \beta^{-1} - 1 > 0$ ,

where  $d_t$  is exogenous. The standard 'fundamentals' solution is

$$p_t = \bar{p}_t \equiv \sum_{j=1}^{\infty} (1+r)^{-j} E_t d_{t+j}.$$

 $\bar{p}_t$  inherits the time-series properties of  $d_t$ , e.g. if  $d_t$  has a unit root (e.g. is a random walk) then  $p_t$  does, and also  $p_t, d_t$  are cointegrated, i.e.  $p_t - r^{-1}d_t$  is stationary.

There are also explosive 'bubble' solutions

$$p_t = \bar{p}_t + B_t$$
 where  
 $B_t = (1+r)^{-1} E_t B_{t+1}$ 

Bubble solutions, which can be viewed as a kind of sunspot, are explosive since

$$E_t B_{t+j} = (1+r)^j B_t.$$

It seems that one might be able to test empirically for the presence of bubbles by determining whether  $p_t$  has an explosive root and whether  $p_t$  and  $d_t$  are cointegrated.

However in Evans (AER, 1991) I show that "**periodically collapsing**" bubbles exhibit **pseudo-stationarity** in finite samples. Although the excess volatility would be observable, standard unit root and cointegration tests would fail to detect bubbles. (See Figure).

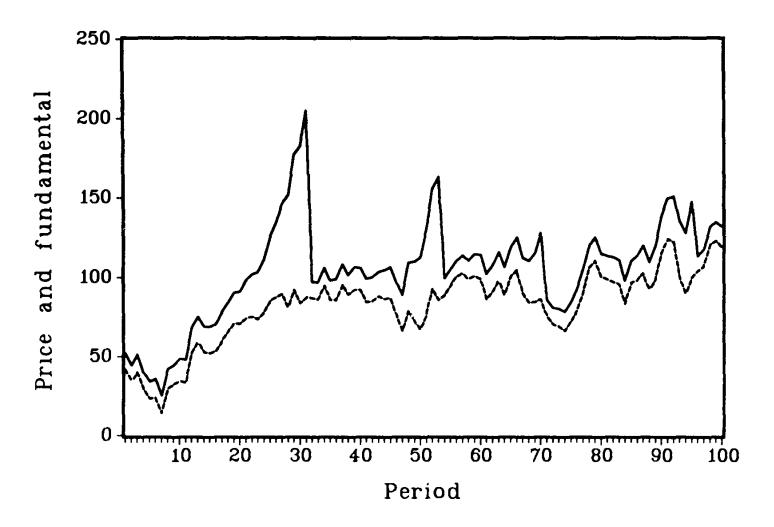


FIGURE 1. PRICE P (Solid Line) and Fundamentals Price F (Dashed Line) for Example Simulation

Despite the apparent appeal of rational bubbles there are several objections to them:

(i) theoretical arguments against explosive solutions (that lean heavily on RE),

(ii) the assumed coordination of agents on a suitable and observable sunspot  $B_t$ , and

(iii) instability under adaptive learning.

We next consider this last point.

Outline of Adaptive Learning Approach

- At t, given exogenous and pre-determined variables, agents form expectations  $E_t^* p_{t+1}$  using a previously estimated forecasting model.
- $E_t^* p_{t+1}$  together with the exogenous variables (e.g. dividends), determines the market ("temporary equilibrium") price  $p_t$ .
- Using time t data, the estimated forecast model parameters are then updated, e.g. by recursive least-squares.
- We then move to t + 1, and the process continues ...
- Is there convergence to RE or another stochastic process?

#### Instability of rational bubbles under adaptive learning

Consider again

$$p_t = \beta E_t^* p_{t+1} + d_t,$$

where now  $E_t^* p_{t+1}$  is the possibly nonrational expectation  $0 < \beta < 1$  and  $d_t$  is the dividend, assumed paid at the beginning of t.

To analyze stability under LS (least-squares) learning consider the special case

 $d_t = k + u_t$ , where  $u_t$  is white noise,

so that

$$\bar{p}_t = (1-\beta)^{-1}k + u_t.$$

We will see whether agents can learn the dynamics of the bubble. For any bubble  $B_t$ , since  $B_t = \beta E_t B_{t+1}$ , we can write

$$B_{t+1} = \beta^{-1}B_t + \varepsilon_{t+1}$$

for some martingale difference sequence (mds)  $\varepsilon_t$ . Using  $B_t = (1 - \beta^{-1}L)\varepsilon_t$ and allowing the mds to depend on  $u_t$  we can write the bubble solutions as

$$p_t = -\beta^{-1}k + \beta^{-1}p_{t-1} - \beta^{-1}u_{t-1} + \zeta u_t + \varepsilon_t,$$

where  $\zeta$  is arbitrary and  $\varepsilon_t$  is an arbitrary mds. Note explosive root in  $p_t$ .

To look at stability under LS learning consider a perceived law of motion (PLM) general enough to include fundamentals and bubble solutions as special cases

$$p_t = a + cp_{t-1} + h_0 u_t + h_1 u_{t-1} + f \epsilon_t.$$

This gives 1-step ahead forecasts  $E_t^* p_{t+1} = a + c E_t^* p_t + h_1 u_t$ , or

$$E_t^* p_{t+1} = a(1+c) + c^2 p_{t-1} + (h_1 + ch_0)u_t + ch_1 u_{t-1} + cf\epsilon_t.$$

Substituting into the model gives the corresponding actual law of motion (ALM)  $p_t = k + \beta a (1+c) + \beta c^2 p_{t-1} + (1 + \beta (h_1 + ch_0)) u_t + \alpha ch_1 u_{t-1} + \alpha cf \epsilon_t.$ The *T* mapping from PLM to ALM is

 $T(a, c, h_0, h_1, f) = (k + \beta a(1 + c), \beta c^2, (1 + \beta (h_1 + ch_0)), \beta ch_1, \beta cf).$ Fixed points of T include both fundamentals and bubble solutions

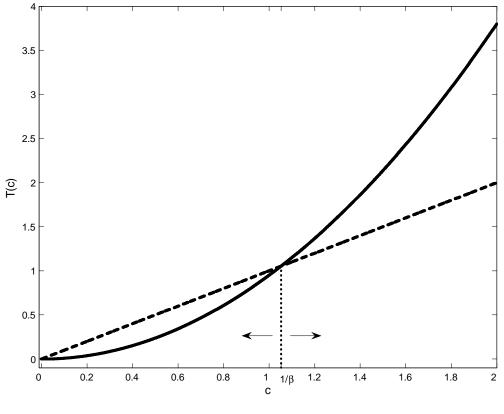
$$\bar{a} = k(1-\beta)^{-1}, \ \bar{c} = \bar{h}_1 = \bar{f} = 0, \ \bar{h}_0 = 1 \text{ and}$$
  
 $\bar{a} = -\beta^{-1}k, \ \bar{c} = \beta^{-1}, \ \bar{h}_1 = -\beta^{-1}, \text{ and } \bar{h}_0, \bar{f} \text{ arbitrary.}$ 

Stability under LS learning is generally governed by E-stability, i.e. by

$$\frac{d\theta}{d\tau} = T(\theta) - \theta$$

where  $\theta = (a, c, h_0, h_1, f)$ . The differential equation governing c is

$$\frac{dc}{d\tau} = \beta c^2 - c.$$



Learning Instability of Rational Bubble

The fundamentals coefficient  $\bar{c} = 0$  is locally stable, whereas the bubble value  $\bar{c} = \beta^{-1}$  is unstable.

Under LS learning initial estimates for the coefficient c will tend to either converge to zero (for initial beliefs less than  $\beta^{-1}$ ) or explode (for initial beliefs more than  $\beta^{-1}$ ).

If  $c \rightarrow 0$  then the other coefficients go to their fundamentals REE values.

This argument can be read as ruling out bubbles, but it can also be viewed as providing a source of bubbles, if they can be "tamed."

### Asset-pricing under adaptive learning

I will next look at several ways to obtain interesting asset price dynamics under adaptive learning that are less radical than rational bubbles. Later I will return to (near-rational) bubbles under adaptive learning. I'll review each of the following

- constant gain learning of fundamentals solution
- underparameterization of dynamics
- dynamic predictor selection: rules of thumb
- misspecification equilibrium (dynamic predictor selection with LS learning).

#### Constant Gain Learning of Fundamentals Solution

This is a minimal deviation of adaptive learning to give persistent dynamics.

The idea goes back to Timmermann's (1993): transitional learning dynamics gives excess volatility. Constant gain learning, or "discounted LS", gives a constant weight to current data and discounts older data. This leads to persistent learning dynamics.

The motivation for constant gain learning is robustness to unknown structural change.

Simple example. Letting  $d_t = \alpha + v_t$ , where  $v_t$  is white noise we have

$$p_t = \alpha + \beta E_t^* p_{t+1} + v_t.$$

The fundamentals solution is

$$p_t = \overline{a} + v_t$$
 where  $\overline{a} = (1 - \beta)^{-1} \alpha$ .

We suppose agents try to learn  $\bar{a}$  statistically by running a regression on an intercept.

Under ordinary LS learning we have

$$a_{t} = a_{t-1} + t^{-1}(p_{t-1} - a_{t-1}),$$
  

$$E_{t}^{*}p_{t+1} = a_{t},$$
  

$$p_{t} = \alpha + \beta a_{t} + v_{t}.$$

It can be shown that  $a_t \to \overline{a}$  as  $t \to \infty$ , i.e. LS learning converges to the fundamentals solution.

Now replace  $t^{-1}$  by  $0 < \gamma < 1$  (e.g.  $\gamma = 0.05$ ). Then  $a_t = a_{t-1} + \gamma(p_{t-1} - a_{t-1}).$ 

Using  $E_t^* p_{t+1} = a_t$  and  $p_t = \alpha + \beta a_t + v_t$  it can be shown that

$$p_t = (1 - \gamma(1 - \beta))p_{t-1} + \alpha\gamma + v_t - (1 - \gamma)v_{t-1}$$

with (asymptotic) variance

$$\operatorname{var}(p_t) = \left(\frac{1 + (1 - \gamma)(1 - 2\beta)}{1 + (1 - \gamma)(1 - 2\beta) - \gamma\beta^2}\right) \operatorname{var}(v_t).$$

See Evans and Honkapohja (2001) for details. Constant gain learning leads to excess volatility. With  $\beta = 0.99$  and  $\gamma = 0.02$ ,  $var(p_t)$  is twice that of RE.

Can be extended to more general  $d_t$  processes, in which agents regress  $p_t$  on  $d_t$  (and possibly lags).

The results carry over to infinite-horizon PV (present value) pricing.

Another application: exchange rates and the forward premium puzzle (Chakraborty and Evans, JME 2008).

#### Underparameterization of Dynamics

Another 'small' deviation from RE: If agents use a misspecified model to forecast this can also lead to novel dynamics. An example is Fuster, Hebert and Laibson (NBER Macro Annual, 2011).

They assume  $\Delta d_t \sim AR(n)$  where *n* is large (e.g. n = 30), but agents fit a parsimonious model with n < 10. In the data  $d_t$  appears mean-reverting over long periods, if we choose *n* large, but if we fit an AR(n) with *n* small, forecasts extrapolate dividend shocks.

Using a Lucas-type model with CARA preferences they show this can fit many asset price anomalies.

From the adaptive learning viewpoint this is an RPE (restricted perceptions equilibrium). It would be natural to also add LS learning of the forecast rule parameters.

#### Dynamic Predictor Selection with Rules of Thumb

Another approach to asset pricing with bounded rationality emphasizes heterogeneous expectations and rules of thumb. Brock and Hommes (JEDC, 1998) is an example. Hommes and coauthors have many recent applications.

The set-up has the usual 1-step ahead structure, except that there is additionally an asset supply shock (due to new issues, repurchases, insider lock-up, etc.), and the equation arises from mean-variance preferences.

Each agent type  $j = 1, \ldots, J$  solves

$$\max_{z_j} RW_t + E_t^j \left( p_{t+1} + y_{t+1} - Rp_t \right) z_{jt} - \frac{a}{2} \sigma^2 E_t^j z_{jt}^2$$

where  $\sigma^2 = Var_t(p_{t+1} + y_{t+1} - Rp_t)$  is the subjective conditional variance of the excess rate of return, assumed constant over time. This leads to the risky asset demand for type j of

$$z_{jt} = \frac{1}{a\sigma^2} E_t^j \left( p_{t+1} + y_{t+1} - Rp_t \right)$$

Financial market equilibrium requires that price adjusts to ensure market clearing. For simplicity assume J = 2.

Let n denote the fraction of agents with expectations  $E_t^1$ . In equilibrium,

$$nz_{1t} + (1-n)z_{2t} = z_{st}$$

which leads to the equilibrium process for stock prices,

$$p_t = \beta \left[ nE_t^1 p_{t+1} + (1-n)E_t^2 p_{t+1} \right] + \beta \rho y_t - \beta a \sigma^2 z_{st}$$

Belief types j = 1, 2 are selected from some simple rules, e.g.

$$p_{t+1}^e = 0.65p_{t-1} + 0.35p_t^e$$
, adaptive expectations  
 $p_{t+1}^e = p_{t-1} + 1.3(p_{t-1} - p_{t-2})$ , strong trend-setting

The proportions of agents using each rule depend on forecast performance

$$U_{j,t} = \pi_{j,t-1} + \eta U_{j-1,t}$$
 where  $0 \le \eta < 1$ ,

where  $\pi_{j,t-1}$  is the rate of return obtained last period using forecast rule j, and  $n_{j,t}$  is given by the logit formula

$$n_{1,t} = \exp(\psi U_{1,t}) / (\exp(\psi U_{1,t}) + \exp(\psi U_{2,t}))$$
 and  $n_{2,t} = 1 - n_{1,t}$ ,

where  $\psi > 0$  measures the 'intensity of choice.'

Brock and Hommes show that complex price dynamics, including bubble-like behavior, can arise with this set-up.

Dynamic predictor selection with least-squares learning

Branch and Evans (Rev. Financial Studies, 2010)

Branch and Evans (2010) use an approach that combines dynamic predictor selection with LS learning.

As with BH we assume that agents select between alternative underparameterized forecasting models.

However we assume that each forecasting model regresses  $p_t$  on an explanatory variable. The exogenous observables, dividends  $\hat{y}_t$  and share supply  $\hat{z}_{st}$ , are AR(1):

$$\hat{y}_t = (1 - \rho)y_0 + \rho \hat{y}_{t-1} + \varepsilon_t$$
  
 $\hat{z}_{st} = (1 - \phi)s_0 + \phi \hat{z}_{s,t-1} + v_t$ 

The two forecasting models use either

$$PLM_1 : p_t = b_0^1 + b_1^1 \hat{y}_t + \eta_t$$
  

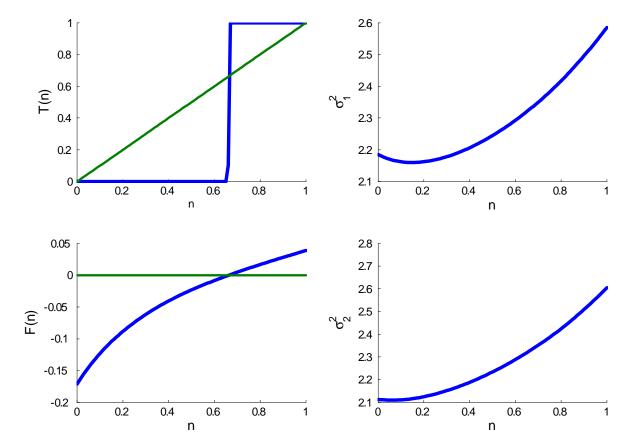
$$PLM_2 : p_t = b_0^2 + b_1^2 \hat{z}_{st} + \eta_t.$$

Agents choose between these two **alternative misspecified forecasting models**, each updated using LS learning.

Motivation: forecasters recommend using parsimonious models.

We show theoretically that in this set-up there can be multiple "misspecification equilibria" in which agents mainly use one of the two models. These have different expected rates of return and different measures of risk.

Let  $F = U_1 - U_2$  where  $U_j$  is risk-adjusted expected excess returns.



Multiple equilibria for large  $\psi$ . n is proportion using model 1 vs. model 2.  $n \rightarrow 0$  and  $n \rightarrow 1$  are both equilibria.

We then add constant-gain adaptive learning of (i) coefficients of each PLM,

(ii) estimates of fitness for each PLM. Under real-time learning this can generate a pattern of **regime-switching and parameter drift**, as agents shift periodically between a high return-high risk equilibrium and a low return-low risk equilibrium. This can match empirical regime-switching models.

These qualitative results hold even if a bivariate regime-switching model is included as a third forecasting model. This would include RE as a special case. But with finite gains the economy under learning tend to switch between all three forecasting models and regime-switching in the data is reinforced.

## Learning about Risk and Return: a simple model of bubbles and crashes

(Branch and Evans, AEJ Macro 2011)

Thus, this vast increase in the market value of asset claims is in part the indirect result of investors accepting lower compensation for risk. Such an increase in market value is too often viewed by market participants as structural and permanent ... Any onset of increased investor caution elevates risk premiums and, as a consequence, lowers asset values and promotes the liquidation of the debt that supported higher asset prices. This is the reason that history has not dealt kindly with the aftermath of protracted periods of low risk premiums.

Alan Greenspan (2005).

We now come back to bubbles and try to tame them under adaptive learning.

We need a mechanism to prevent prices from exploding to infinity (or to zero). Three natural possibilities:

1) Impose a "projection facility" on forecasts that prevents forecasts from becoming too extreme (e.g. Adam, Marcet & Nicolini).

2) Ensure a proportion of "fundamental" forecasters in the economy (e.g. Hommes & colleagues)

3) Include a role for estimates of risk (Branch & Evans, 2011).

We use a simple mean-variance linear asset pricing model, as above.

There is a risky asset with dividend  $y_t$  and price  $p_t$  and a risk-free asset that pays the rate of return  $R = \beta^{-1}$ , where  $0 < \beta < 1$ . Demand for the risky asset is

$$z_{dt} = \frac{E_t^* \left( p_{t+1} + y_{t+1} \right) - \beta^{-1} p_t}{a \sigma_t^2},$$

where  $E_t^*$  are (possibly) non-rational expectations and

$$\sigma_t^2 = Var_t^*(p_{t+1} + y_{t+1} - Rp_t).$$

We allow for the possibility of  $\sigma_t^2$  varying over time.

Writing  $z_{st}$  for risky asset supply and setting  $z_{dt} = z_{st}$  we have

$$p_t = \beta E_t^* \left( p_{t+1} + y_{t+1} \right) - \beta a \sigma_t^2 z_{st}.$$

a > 0 measures risk-aversion.

This is a very simple model that incorporates risk. We keep it simple because we are going to add learning.

We also assume: (i) Dividends  $y_t$  are a constant plus white noise, and (ii) asset supply  $z_{st} = z_0 + v_t$ , white noise, unless price falls below a small proportion of its fundamental value. This implies that the price dynamics are entirely driven by learning.

#### Rational Expectations Equilibria

Under RE, with exogenous supply, there are two solution classes.

- Fundamentals solution:

$$p_t = \frac{\beta(y_0 - a\sigma^2 s_0)}{1 - \beta} - \beta a\sigma^2 v_t$$

Here  $\sigma^2$  is an equilibrium object.

- Rational bubbles solutions

$$p_t = a\sigma^2 s_0 - y_0 + \beta^{-1} p_{t-1} + a\sigma^2 v_{t-1} + \xi_t,$$

where  $\xi_t$  is an arbitrary MDS, i.e.  $E_t \xi_{t+1} = 0$ .

Since  $0 < \beta < 1$  the bubbles solutions are explosive in conditional mean.

#### Stability under Learning

We give agents a PLM (perceived law of motion) that nests the fundamentals solution and also allows for the bubble term in  $p_{t-1}$ ,

$$p_t = k + cp_{t-1} + \varepsilon_t,$$
  

$$\sigma^2 = Var_t(p_{t+1} + y_{t+1})$$

where  $\varepsilon_t$  is perceived white noise with constant variance.

**Under learning** agents estimate k, c and  $\sigma^2$  using an adaptive learning algorithm: (recursive) LS learning for k, c and a recursive estimate of  $\sigma^2$ .

**Proposition:** (1) The fundamentals REE is locally stable under learning. (2) The bubbles REE are unstable under learning.

However, the transitional learning dynamics exhibits paths in which the agents' PLM escapes to a random walk, k = 0, c = 1, with asset prices sensitive to changed estimates of risk, leading to bubbles and crashes.

The random walk PLM behaves like a near-rational bubble.

**Discounted LS**. Furthermore, under **discounted (or "constant gain") learning** (in which agents discount past data at a geometric rate) there can be **recurring bubbles and crashes**.

If the gains (discounting) are small, the dynamics stay near the fundamentals RE. For larger gains (discounting) there are more frequent escapes.

Stochastic simulations.

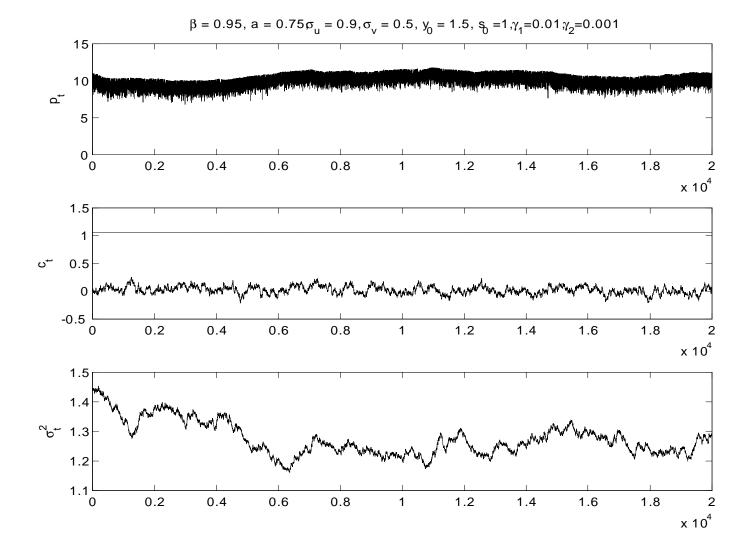
Frequent bubbles and crashes arise when the gain on the estimate of risk ( $\gamma_2$ ) is relatively large. We vary  $\gamma_2 = 0.001$  to 0.04.

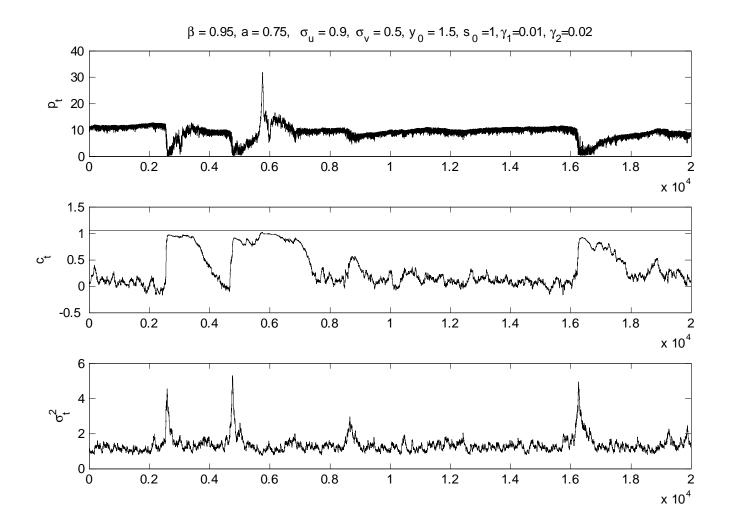
Starting from the fundamentals RE, crashes and bubbles can arise from various sequences of random shocks, e.g.

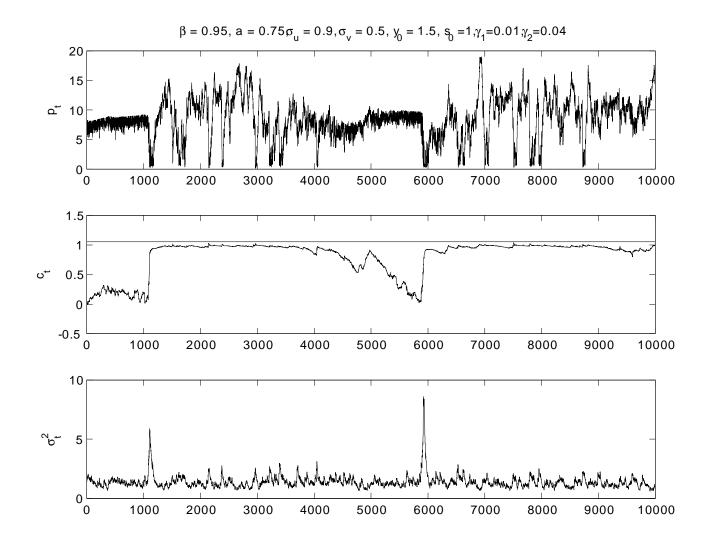
 $u_t \approx 0, v_t \approx 0 \rightarrow \downarrow \sigma_t^2 \rightarrow \uparrow p_t \rightarrow random-walk beliefs.$ 

Random-walk beliefs are almost self-fulfilling and have price high volatility.

Explosive price bubbles  $\rightarrow \uparrow \sigma_t^2 \rightarrow$  crashes.







#### ARCH learning

A natural extension would be to allow agents to use a more sophisticated estimate of risk, e.g. based on an ARCH model

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \eta_t,$$

where estimates of  $\alpha_0$ ,  $\alpha_1$  are updated using a constant-gain recursive algorithm based on observed squared forecast error for stock price returns.

We examine this for random walk beliefs in Branch and Evans (Economics Letters, 2013).

We find that when agents allow for ARCH effects in their estimation of risk this strengthens the effect that risk has in generating bubbles and crashes.

A Model of Near-rational exuberance (Bullard, Evans and Honkapohja, MD 2010)

I consider one last wrinkle on nearly self-fulfilling expectations.

We again use the simple one-step ahead mean-variance asset pricing framework,  $p_{t+1}^e + d - \beta^{-1}p_t = s_t$ , with now a fixed dividend d and a constant estimate of risk.  $s_t$  is a linear function of the risky asset supply, assumed *iid* exogenous and unobserved. This gives

$$y_t = \beta y_{t+1}^e + u_t$$
,  $0 < \beta < 1$ ,

where  $y_t = p_t - \bar{p}$  and  $u_t = -\beta(s_t - \bar{s})$ , with  $\bar{p}$  assumed known.

We consider the possibility that agents add "judgment"  $\xi_t$  to an econometric forecast  $E_t^* y_{t+1}$ 

$$y_{t+1}^e = E_t^* y_{t+1} + \xi_t.$$

This is in line with what Central Banks and expert forecasters actually do, based on the belief that econometric models omit unmeasured factors. We assume  $\xi_t$  has positive first-order autocorrelation  $\rho$ .

What if  $\xi_t$  is extraneous? We look for an "**exuberance equilibrium**" with three properties:

1. Consistent expectations: The econometric forecast  $E_t^*y_{t+1}$  matches the time-series properties of  $y_t$  (here an ARMA(1,1)).

2. Incentives to include judgment: an individual's mean squared forecast error is lower if they include judgment than if they do not.

3. Learnability: The econometric forecasting rule is stable under adaptive learning (RML).

**Theorem:** For  $\beta > 0.5$  there exist suitable  $\xi_t$  such that an exuberance equilibrium exists. These equilibria have excess volatility.

Numerical examples show the SD of price can be over 10 times as high as under RE.

Exuberance equilibria are not fully rational (the forecasts errors are systematic) but they are "near-rational." For  $\beta \rho$  near one, very large sample sizes would be required to detect deviation of forecasts from full rationality.

Conclusion: waves of "judgment" to adjust for perceived omitted fundamentals might become self-fulfilling if the feedback is high.

## Conclusions

- Asset price models are forward-looking with strong positive feedback given by 0 < β < 1, with β near 1.</li>
- Adaptive learning in this setting can yield a range of new price dynamics:

   asset price volatility and other "puzzles" can be explained by small constant gain learning and/or misspecified dynamics.
  - judgmental adjustments can be almost self-fulfilling.
  - dynamic predictor selection can lead to regime-switching dynamics
  - near-rational bubbles and crashes can arise from parameter "escape dynamics." This can happen with constant-gain estimates of risk.

- Key insight:  $\beta < 1$  with  $\beta$  near 1 opens the door to nearly self-fulfilling equilibria under adaptive learning.
- An open question (in my view). In explaining bubbles and asset price dynamics, how important is heterogenous expectations or agents?
  - Heterogeneity plays a role in some models, but (i) bubbles can arise without heterogeneity, and (ii) a similar role can be provided by model averaging. Heterogeneity is clearly present, but does it play a critical role?
- Back to the Cognitive Consistency Principle: modeling agents as economic theorists (eductive) or econometricians (adaptive) is attractive because it defines the limits of genuine rationality.