Policy Change and Learning in the RBC Model[*]

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Abstract

What is the impact of surprise and anticipated policy changes when agents form expectations using adaptive learning rather than rational expectations? We examine this issue using the standard stochastic real business cycle model with lump-sum taxes. Agents combine knowledge about future policy with econometric forecasts of future wages and interest rates. Both permanent and temporary policy changes are analyzed. Dynamics under learning can have large impact effects and a gradual hump-shaped response, and tend to be prominently characterized by oscillations not present under rational expectations. These fluctuations reflect periods of excessive optimism or pessimism, followed by subsequent corrections.

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1 Introduction

Typically economic models are analyzed with an unchanged structure. However, in practice, policy changes do take place, and these often involve long delays. This is well recognized in the case of fiscal policy which involves lags sometimes even exceeding two years. The process of changing taxes involves legislative lags, between when the new tax is proposed and when it is passed, and implementation lags, between when the legislation is signed into law and when it actually takes effect. These changes in policy may well be anticipated by economic actors (often with discussions in the media) and will influence economic decisions even before the actual implementation of the proposed policy change.[1]

The standard assumption in macroeconomics is, of course, rational expectations (RE), and this has been used to analyze the impact of both surprise and preannounced policy changes. Within a nonstochastic perfect foresight setting, see, for example, Sargent and Wallace (1973), Blanchard and Fischer (1989), Romer (2011) and Ljungqvist and Sargent (2004).

The seminal contributions of Baxter and King (1993) and Aiyagari, Christiano, and Eichenbaum (1992) analyze changes to fiscal policy within a RE framework in the stochastic Real Business Cycle (RBC) model. These papers consider changes to government spending and analyze both temporary and permanent changes when the government conducts a balanced budget.[2]

However, the benchmark assumption of RE is very strong and arguably unrealistic when analyzing the effect of policy changes. Economic agents need to have complete knowledge of the underlying structure, both before and after the policy change. They must also rationally and fully incorporate this knowledge in their decision making, and do so under the assumption that other agents are equally knowledgeable and equally rational.

Recently there has been increasing interest in studying situations in which agents have incomplete knowledge of the economy. The assumption that

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[1]See Evans, Honkapohja, and Mitra (2009) for some further discussion. Active fiscal strategies have been adopted recently in various countries around the world (like in the US and UK) in the wake of the recent "Great Recession". These measures include temporary tax cuts and credits and large public works projects; see for instance Auerbach, Gale, and Harris (2010).

[2]Baxter and King (1993) focus on surprise changes and consider two alternative scenarios; one where the government has access to lump-sum taxes only and the second where it has access to distortionary income taxes as well.

economic agents engage in "learning" behavior has been incorporated into macroeconomic theory (see e.g. Sargent (1993) and Evans and Honkapohja (2001)) and used in a wide range of applications in macroeconomics and finance. The standard adaptive learning approach treats economic agents like econometricians who estimate forecast rules, updating the parameter estimates over time as new data become available. It has been shown that in many models, including the RBC model, least-squares learning can converge over time to the RE solution, while at the same time often providing plausible transitional dynamics that are arguably of empirical importance.[3]

However, analyses of learning typically assume an unchanged economic structure.[4] An apparent drawback of least-squares learning rules is that estimated coefficients respond relatively slowly to data, and thus standard learning rules take time to adjust to structural or policy changes. In some cases this is realistic, but in the case of clearly articulated policy changes one would expect even boundedly rational agents to incorporate structural information about future policy.

In this paper we show how to analyze fiscal policy changes in a learning framework for the stochastic RBC model. To do so we assume that agents forecast some key variables using adaptive learning, while simultaneously incorporating structural knowledge about future government spending and taxes. Both permanent and temporary policy changes are examined, and the results contrasted with those from the RE approach. One case we consider in detail is the impact of announced *future* policy changes.

The question of how to analyze known structural changes in a learning framework was taken up in Evans, Honkapohja, and Mitra (2009). They considered announced changes in fiscal policy in a simple endowment economy model and (briefly) in a Ramsey model. However, a major limitation of their framework was its deterministic nature which consequently restricted the type of learning behavior that could be analyzed.[5] In addition, the variable labor supply assumption in the RBC model plays a crucial role in the policy analysis of government spending by Baxter and King (1993).

Our approach uses an adaptive learning model in which agents in effect

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[3]See, for example, Sargent (2008) and Evans and Honkapohja (2010) for extensive references.

[4]See, however, Evans, Honkapohja, and Marimon (2001), Marcet and Nicolini (2003) and Giannitsarou (2006) for partial exceptions.

[5]For a discussion of the differences of learning in deterministic and stochastic models see Evans and Honkapohja (1998).

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also have partial structural knowledge. At each moment in time agents must make consumption and labor supply decisions based on the time path of expected future wages, interest rates and taxes. As is standard with adaptive learning, we assume that agents make forecasts of wages and interest rates based on a statistical model, with coefficients updated over time using least-squares. However, for forecasting future taxes we assume that agents use the path of future taxes announced (credibly) by policymakers.[6]

This approach seems to us very natural. The essence of the adaptive learning approach is that agents are assumed not to understand the general equilibrium considerations that govern the evolution of the central endogenous variables, i.e. capital, labor and factor prices. Agents are therefore assumed to forecast these variables statistically. On the other hand, agents can be expected to immediately incorporate into their decisions the direct effects on their future net incomes of the announced path of future taxes. As noted in Evans, Honkapohja, and Mitra (2009), this general approach to combining statistical learning and limited structural knowledge can be adapted to other economic situations.

Several general features stand out in our analysis of fiscal policy changes in the RBC model. As under RE, announced current or future changes in government spending lead to immediate changes in consumption, employment, and output.[7] However, with adaptive learning the solution exhibits hump-shaped responses and oscillatory convergence to the new steady state, including overshooting not present under RE. These dynamics stem from a combination of inertia in capital accumulation and the adaptation of expectations to data generated by the statistical learning rules used by private agents.

We also show that for changes in policy, announced to take place in the future, the impact effects under learning can be more extreme than under RE, because the wealth effects of future tax changes are immediate, while the partially offsetting price effects are spread out over time and unknown to agents. For both surprise and announced future changes we sometimes find that the dynamics under learning and RE can be qualitatively different for a period of time following the immediate impact.

A final important feature of the model under learning dynamics is that

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[6]For convenience we assume throughout a balanced budget, so that in each period taxes equal government spending.

[7]Surprisingly, it appears that announced future changes of government spending have not previously been systematically studied under RE for the stochastic RBC model.

policy changes can lead to systematic waves of optimism or pessimism. The details depend naturally on the type of policy change considered. For example, a permanent increase in government spending, announced to take place in the future, generates a period of over-optimism concerning wages during much of the pre-implementation period, followed by a correction during the post-implementation period. Such periods of over-optimism or over-pessimism reflect general equilibrium effects, and are a consequence of the agents's incomplete structural knowledge.

Section 2 below describes the basic RBC model in the presence of learning by agents. Section 3 analyzes permanent changes in policy both within a RE framework and under learning and Section 4 does the same for temporary policy changes. The final section concludes.

2   The Model

There is a representative household who has preferences over non-negative streams of a single consumption good c_t and leisure $1 - n_t$ given by

$$\hat{E}_t\{\sum_{s=t}^{\infty} \beta^{s-t} U(c_s, 1 - n_s)\} \tag{1}$$

Here $\hat{E}_t$ denotes potentially subjective expectations at time t for the future, which agents hold in the absence of rational expectations (RE). The analysis of the model under RE is standard. When RE is assumed we indicate this by writing E_t for $\hat{E}_t$. Our presentation of the model is general in the sense that it applies under learning as well as under RE.

We assume the general form

$$U(c_s, 1 - n_s) = \frac{c_s^{1-\sigma}}{1-\sigma} + \zeta \frac{(1 - n_s)^{1-\epsilon}}{1-\epsilon}, \tag{2}$$

for $\zeta > 0$, and often focus on the widely considered special case, $\sigma = \epsilon = 1$, i.e.

$$U(c_s, 1 - n_s) = \ln c_s + \zeta \ln(1 - n_s) \tag{3}$$

as in Ljungqvist and Sargent (2004), p. 324, Long and Plosser (1983) and McCallum (1989).[8]

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[8] As shown in King, Plosser, and Rebello (1988), log utility for consumption is needed for steady state labor supply along a balanced growth path. Campbell (1994), Section 3, uses (2) with $\sigma = 1$.

The household flow budget constraint is

$$a_{t+1} = w_t n_t + r_t a_t - c_t - \tau_{h,t}, \text{ where} \tag{4}$$

$$r_t = 1 - \delta + r_{k,t}. \tag{5}$$

Here a_t is per capita household wealth at the beginning of time t, which equals holdings of capital k_t owned by the household less their debt (to other households), b_{pt}, i.e. $a_t \equiv k_t - b_{pt}$. r_t is the gross interest rate for loans made to other households, w_t is the wage rate, c_t is consumption, n_t is labor supply and $\tau_{h,t}$ is per capita lump sum taxes. Equation (5) is the arbitrage condition arising from loans and capital being perfect substitutes as stores of value; $r_{k,t}$ is the rental rate on capital goods, and δ is the depreciation rate.

Households maximize utility (1) subject to the budget constraint (4) which yields the Euler equation for consumption

$$c_t^{-\sigma} = \beta \hat{E}_t r_{t+1} c_{t+1}^{-\sigma}. \tag{6}$$

We next derive the (linearized) consumption function.

From the flow budget constraint (4) we can get the intertemporal budget constraint (in realized terms)

$$0 = r_t a_t + \sum_{j=1}^{\infty} (D_{t,t+j}(t))^{-1} \chi_{t+j} + \chi_t, \tag{7}$$

$$\text{where } D_{t,t+j} = \prod_{i=1}^{j} r_{t+i}, \ j \geq 1 \text{ and } \chi_t \equiv w_t n_t - c_t - \tau_{h,t},$$

assuming the transversality condition $D_{t,t+j}^{-1} a_{t+j+1} \to 0$ as $j \to \infty$ holds.

Note that (7) involves future choices of labor supply by the household which we next eliminate to derive the linearized consumption function. For this we make use of the static first order condition (between consumption and labor supply) from the household's problem which is

$$\zeta(1 - n_t)^{-\epsilon} = w_t c_t^{-\sigma} \tag{8}$$

This can be written as
$$n_t = 1 - \zeta^{\frac{1}{\epsilon}} c_t^{\frac{\sigma}{\epsilon}} w_t^{-\frac{1}{\epsilon}}, \tag{9}$$

so that $w_t n_t = w_t - \zeta^{\frac{1}{\epsilon}} c_t^{\frac{\sigma}{\epsilon}} w_t^{1-\frac{1}{\epsilon}}$. This gives a relationship between labor supply and consumption choices which can be used to substitute out n_{t+j}

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in (7). Taking expectations we then get the expected value intertemporal budget constraint[9]

$$0 = r_t a_t + \chi_t +$$
$$\sum_{j=1}^{\infty} \hat{E}_t (D_{t,t+j})^{-1} \{ w_{t+j} - \zeta^{\frac{1}{\epsilon}} c_{t+j}^{\frac{\sigma}{\epsilon}} w_{t+j}^{1-\frac{1}{\epsilon}} - c_{t+j} - \tau_{h,t+j} \}$$

To obtain its optimal choice of consumption c_t, we assume that the household uses a consumption function based on a linearization around steady state values. In particular, we assume agents linearize the expected value intertemporal budget constraint and the Euler equations around the initial steady state values $\bar{c}, \bar{a}, \bar{w}, \bar{\tau}_h$ and $\bar{r} = \beta^{-1}$. This is a natural choice since agents can be assumed to have estimated precisely the steady state values before the policy change that takes place.[10]

As shown in the Appendix, substituting the linearized Euler equations into the intertemporal budget constraint, we obtain the consumption function

$$(c_t - \bar{c})C_{AA} = \bar{a}(r_t - \bar{r}) + \beta^{-1}(a_t - \bar{a}) - (\tau_{h,t} - \bar{\tau}_h)$$
$$+ C_{ww}(w_t - \bar{w}) - C_{rr}Sr_t^e - S\tau_{h,t}^e + C_{ww}Sw_t^e \qquad (10)$$

where C_{AA}, C_{ww} and C_{rr} are given in the Appendix and where

$$Sr_t^e \equiv \sum_{j=1}^{\infty} \beta^{j+1} \sum_{i=1}^{j} (r_{t+i}^e - \bar{r}), \qquad (11)$$

$$S\tau_{h,t}^e \equiv \sum_{j=1}^{\infty} \beta^j (\tau_{h,t+j}^e - \bar{\tau}_h), \qquad (12)$$

$$Sw_t^e \equiv \sum_{j=1}^{\infty} \beta^j (w_{t+j}^e - \bar{w}), \qquad (13)$$

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[9]Note we do not assume point expectations as in Evans, Honkapohja, and Mitra (2009); this model cannot be solved exactly so we proceed by linearizing the Euler equation and the intertemporal budget constraint.

[10]Thus we assume that the final steady state values of k, w and r are not initially known to agents. Under least-squares learning agents will eventually come to know the new steady state values as happens in all of the simulations below. We remark that an alternative approach to our procedure would be to assume that agents also update over time the point around which the consumption function is linearized, with the sequence of linearization points chosen to be consistent with the agent's estimates of the new steady-state values. Provided the changes in government spending are not too large, it is satisfactory to use our simpler procedure of using a fixed linearization point.

denote "present value" type expressions. For the case $\sigma = \epsilon = 1$, the linearization coefficients are given by

$$C_{AA} = (1 + \zeta)/(1 - \beta), \ C_{ww} = 1 \text{ and } C_{rr} = \bar{w} - \bar{\tau}_h.$$

Equation (10) specifies a behavioral rule for the household's choice of current consumption based on pre-determined values of initial assets, real interest rates, wage rates, current values of lump-sum taxes and (subjective) expectations of future values of wages, interest rates, and lump-sum taxes. Expectations are assumed to be formed at the beginning of period t and, for simplicity, we assume these to be identical across agents (though agents themselves do not know this to be the case). Equation (10) can then be viewed as the behavioral rule for per capita consumption in the economy.

To implement the behavioral rule, however, the household requires forecasts r_{t+i}^e, w_{t+j}^e, and $\tau_{h,t+j}^e$. For taxes $\tau_{h,t+j}^e$ (and $\bar{\tau}_h$) we assume that agents use "structural" knowledge based on announced government spending rules. For convenience we assume balanced budgets, so that $\tau_{h,t+j} = g_{t+j}$. For r_{t+i}^e and w_{t+j}^e we will assume that household estimate future values using a VAR-type model in $k_t, w_t, r_{k,t}$ and v_t, with coefficients updated over time by RLS (recursive least squares). The detailed procedure is described below in Section 3.1.

Linearizing equation (9) we also obtain the employment equation, which will be useful later:

$$n_t - \bar{n} = -\frac{\sigma}{\epsilon}\zeta^{\frac{1}{\epsilon}}\bar{w}^{-\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}-1}(c_t - \bar{c}) + \frac{1}{\bar{w}\epsilon}\zeta^{\frac{1}{\epsilon}}\bar{w}^{-\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}}(w_t - \bar{w}).$$

To complete the model, we describe the evolution of the other state variables, namely $w_t, r_{k,t}, r_t, y_t$ and k_{t+1}. Households own capital and labor services which they rent to firms. The firm uses these inputs to produce output y_t using the Cobb-Douglas production technology

$$y_t = v_t k_t^\alpha n_t^{1-\alpha}$$

where v_t is the technology shock that follows an AR(1) process

$$\hat{v}_t = \rho \hat{v}_{t-1} + \tilde{u}_t$$

with $\hat{v}_t = (v_t - \bar{v})$. Here $\bar{v}$ is the mean of the process and $\tilde{u}_t$ is an iid zero-mean process with constant variance σ_u^2.[11]

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[11] For simplicity we do not include a trend in technical progress. This would be straightforward to add, but doing so would require choosing between a deterministic and a stochastic trend, and it would substantially complicate the presentation.

Profit maximization by firms implies the standard first-order conditions involving wages and rental rates

$$w_t = (1 - \alpha)v_t(\frac{k_t}{n_t})^\alpha, \tag{14}$$

$$r_{k,t} = \alpha v_t(\frac{n_t}{k_t})^{1-\alpha}. \tag{15}$$

In equilibrium, aggregate private debt b_{pt} is zero, so that $a_t = k_t$, and market clearing determines k_{t+1} from

$$k_{t+1} = v_t k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t - c_t - g_t \tag{16}$$

where g_t is per capita government spending.

For simulations of the model we follow standard procedures and approximate the path using a linearization around the steady state.[12] The linearized wage rate, rental rate, and real interest rate equations are

$$w_t - \bar{w} = \bar{w}[(\frac{v_t}{\bar{v}} - 1) + \alpha(\frac{k_t}{\bar{k}} - 1) - \alpha(\frac{n_t}{\bar{n}} - 1)], \tag{17}$$

$$r_{k,t} - \bar{r}_k = \bar{r}_k[(\frac{v_t}{\bar{v}} - 1) - (1 - \alpha)(\frac{k_t}{\bar{k}} - 1) + (1 - \alpha)(\frac{n_t}{\bar{n}} - 1)], \tag{18}$$

$$r_t - \bar{r} = r_{k,t} - \bar{r}_k. \tag{19}$$

Finally, the linearized output and capital accumulation equations are

$$y_t - \bar{y} = \bar{y}[(\frac{v_t}{\bar{v}} - 1) + \alpha(\frac{k_t}{\bar{k}} - 1) + (1 - \alpha)(\frac{n_t}{\bar{n}} - 1)],$$

$$k_{t+1} - \bar{k} = (y_t - \bar{y}) - (c_t - \bar{c}) - (g_t - \bar{g}) + (1 - \delta)(k_t - \bar{k}).$$

Here the equations giving the steady state are

$$\bar{r} = 1 - \delta + \bar{r}_k = \beta^{-1},$$

$$\bar{c} = \bar{v}\bar{k}^\alpha \bar{n}^{1-\alpha} - \delta\bar{k} - \bar{g},$$

$$\zeta\bar{c}^\sigma = \bar{w}(1 - \bar{n})^\epsilon,$$

$$\bar{w} = (1 - \alpha)\bar{v}(\frac{\bar{k}}{\bar{n}})^\alpha \text{ and } \bar{r}_k = \alpha\bar{v}(\frac{\bar{k}}{\bar{n}})^{\alpha-1}.$$

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[12]It is also straightforward to simulate the model under learning using the exact (non-linear) equations for $y_t, w_t, r_{k,t}, r_t$ and k_{t+1}. For the model at hand we have found the results for the two methods to be very similar. Simulations using linear approximations are much faster, however, so we have used these in the reported results.

These five equations can be solved simultaneously to yield the steady state values of $\bar{c}, \bar{k}, \bar{n}, \bar{w}$, and $\bar{r}_k$ given the value of $\bar{g}$ and the structural parameters $\alpha, \beta, \delta, \zeta, \sigma, \epsilon$.

To examine the impact of policy in the model under learning, we will compare the dynamics to those under RE. At this stage we remark that, as is well known, in the absence of a policy change, under RE the endogenous variables, $k_{t+1}, c_t, n_t, w_t, r_{k,t}, r_t$, can be written as an (approximate) linear function of k_t and v_t, e.g. Campbell (1994). The linearized equations of motion take the form

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\begin{align}
\hat{k}_{t+1} &= \lambda_2 \hat{k}_t + f_{kv}\hat{v}_t, \tag{20}\\
\hat{c}_t &= f_{ck}\hat{k}_t + f_{cv}\hat{v}_t, \tag{21}\\
\hat{n}_t &= f_{nk}\hat{k}_t + f_{nv}\hat{v}_t, \tag{22}\\
\hat{w}_t &= f_{wk}\hat{k}_t + f_{wv}\hat{v}_t, \tag{23}\\
\hat{r}_{k,t} &= f_{rk}\hat{k}_t + f_{rv}\hat{v}_t, \tag{24}\\
\hat{v}_t &= \rho\hat{v}_{t-1} + \tilde{u}_t.
\end{align}
$$

where the hatted values are deviations from the RE deterministic steady state i.e. $\hat{k}_t = k_t - \bar{k}$, $\hat{r}_{k,t} = r_{k,t} - \bar{r}_k$, $\hat{w}_t = w_t - \bar{w}$, etc. The RE solution takes the form of a stationary VAR(1) in the state $\hat{x}_t \equiv \begin{pmatrix} \hat{k}_t \\ \hat{v}_t \end{pmatrix}$

$$
\begin{align}
\begin{pmatrix} \hat{k}_{t+1} \\ \hat{v}_{t+1} \end{pmatrix} &= B \begin{pmatrix} \hat{k}_t \\ \hat{v}_t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{u}_t, \tag{25}\\
B &= \begin{pmatrix} \lambda_2 & f_{kv} \\ 0 & \rho \end{pmatrix}, \tag{26}
\end{align}
$$

with the other variables given by linear combinations of the state. Note also that under RE forecasts of future $\hat{w}_{t+j}$ and $\hat{r}_{k,t+j}$ are given by linear combinations of the forecasted future state $\hat{x}_{t+j}^e = B^j \hat{x}_t$.

We now turn to obtaining the dynamics, under both RE and learning, when there is a policy change.

3   Permanent Policy Changes

At the beginning of period $t = 1$, a policy announcement is made that the level of government spending will change permanently upward from $\bar{g}$ to $\bar{g}'$

at a specified date T_p in the future. The policy announcement is assumed to be credible and known to the agents with certainty. With a balanced budget, this means equivalently that there is an anticipated change in (per capita) taxes, i.e., $\tau_{h,t} = \bar{\tau} = \bar{g}$ when $t < T_p$ and $\tau_{h,t} = \bar{\tau}' = \bar{g}'$ when $t \geq T_p$.

The long run effects on the steady state of an increase in government expenditure are well-known, e.g. Baxter and King (1993). From the steady state equations, it is easy to see that the new steady state involves lower consumption and higher levels of investment, output, labor, and capital, but an unchanged capital-labor ratio. The latter implies that steady state wages and interest rates are unchanged.

The method for obtaining the impact of policy changes under RE is standard, e.g. see Ljungqvist and Sargent (2004), Ch. 11.

3.1   Learning dynamics

We now consider the learning dynamics in the context of the policy change just described. In the standard adaptive learning approach, private agents would formulate an econometric model to forecast future taxes as well as interest rates and wage rates, since these are required in order for agents to solve for their optimal level of consumption. We continue to follow this approach with respect to interest rates and wage rates, but take the radically different approach for forecasting taxes by assuming that agents understand the future course of taxes implied by the announced policy. In effect, we are giving the agents structural knowledge of one part of the economy: the fiscal implications of the announced future change in government spending.

As argued in the Introduction, we think this is a natural way to proceed, since changes in agents' own future taxes have a quantifiable direct effect, while future wages and interest rates are determined through dynamic general equilibrium effects. The adaptive learning perspective is that it is unrealistic to assume that agents understand the economic structure sufficiently well to improve on reduced form econometric forecasts of aggregate variables like wages and interest rates.

To keep things simple, we assume that the government operates and is known to operate under a balanced-budget rule. Given this structural knowledge of the government budget constraint and the announced path of government spending, the agents can thus use $\bar{\tau} = \bar{g}$, for $t < T_p$, and $\bar{\tau}' = \bar{g}'$, for $t \geq T_p$, for their forecasts of future taxes. Of course, for simplicity we are assuming that the announced policy change is fully credible. It would

be possible to relax this assumption within the general framework of our approach.

Since the path of future taxes $\tau_{t+j} = g_{t+j}$ is known to agents, they compute its present value as

$$S\tau_{h,t}^e = \sum_{j=1}^{\infty} \beta^j (g_{t+j} - \bar{g}) = \begin{cases} \frac{\beta^{T_p-t+1}}{1-\beta}(\bar{g}' - \bar{g}), \ 1 \le t \le T_p - 1 \\ \frac{\beta}{1-\beta}(\bar{g}' - \bar{g}), \ t \ge T_p. \end{cases}$$

However, under learning, agents still need to form forecasts of future wages and interest rates since these are needed for their individual consumption choice in (10). Moreover, they need to form forecasts of these variables without full knowledge of the underlying model parameters.

Under RE, in contrast, agents are assumed to know all the underlying parameters involved in the REE solution, i.e. the parameters in (25) and (23) - (24), which they can then use to form future forecasts of wages and rental rates. For anticipated changes in policy the implicit assumptions under RE are even stronger: agents need to know the full structural model and use it to deduce the full equilibrium path that puts the economy on the new saddle path at the exact time at which the policy change takes place. Furthermore this computation by agents must be made under the assumption that other agents are equally "rational" and make the same computation. The learning perspective is that these assumptions are implausibly strong and hence unrealistic.

Under learning wage and interest rate forecasts depend on the perceived laws of motion (PLMs) of the agents, with parameters updated over time in response to the data. We consider PLMs given by (20), (23), and (24) in which future capital, wages, and rental rates depend on the current capital stock and technological shock, k_t and v_t.[13] That is, we consider PLMs that are of the form (including constants)

$$k_{t+1} = b_k + a_{kk}k_t + a_{kv}\hat{v}_t + noise, \tag{27}$$

$$w_t = b_w + a_{wk}k_t + a_{wv}\hat{v}_t + noise, \tag{28}$$

$$r_{k,t} = b_r + a_{rk}k_t + a_{rv}\hat{v}_t + noise, \tag{29}$$

$$\hat{v}_t = \rho\hat{v}_{t-1} + \tilde{u}_t. \tag{30}$$

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[13] We will explore alternative PLMs in future work, for instance PLMs based solely on observed wages and interest rates. Such PLMs may be considered more realistic since (arguably) it is easier to observe market values of wages and interest rates than it is to observe contemporaneous values of capital stock and productivity.

where the PLM parameters b_k, a_{kk} etc. will be estimated on the basis of actual data. The final line is the stochastic process for evolution of the (de-meaned) technological shock, which for simplicity is assumed known to the agents. In real-time learning, the parameters in (27), (28), (29) are time dependent and are updated using recursive least squares (RLS); see for e.g. Evans and Honkapohja (2001) p. 233. We assume agents allow for structural change, which would include policy changes as well as other potential structural breaks, by discounting older data as discussed below.

We remark that in assuming that agents forecast using the PLM (27) - (30), we are implicitly assuming that they do not have useful information available from previous policy changes. We think this is generally plausible, since policy changes are relatively infrequent and since the qualitative and quantitative details of previous policy changes are unlikely to be the same. In particular, any previous fiscal policy changes, of the type considered here, are likely to have varied in terms of the magnitude and duration of the change in government spending, the extent to which it was anticipated, and the state of the economy in which it was announced and implemented. Since older information of this type would probably have limited value, we assume that agents respond to policy change by updating the parameters of the PLM (27) - (29) as new data become available.[14]

Before discussing how the PLM coefficients are updated over time using least-squares learning, we describe how (27) - (29) are used by agents to make forecasts. Given coefficient estimates and the observed state $(k_t, \hat{v}_t)$, equations (27) and (30) can be iterated forward to obtain forecasts k_{t+j}^e and $\hat{v}_{t+j}$ for $j = 1, 2, \ldots$. Then wage and rental rate forecasts $w_{t+j}^e, r_{k,t+j}^e$ are obtained using the relationships (28) - (29) and interest-rate forecasts are then given by $r_{t+j}^e = 1 - \delta + r_{k,t+j}^e$ using (5). Given these forecasts, Sw_t^e and Sr_t^e are computed from (13) and (11), which in turn are used in (10) to help determine consumption in the temporary equilibrium. For further details see the Appendix.

Parameter updating by agents using RLS learning is as follows. We define

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[14]However, if repeated policy changes take place that are qualitatively and quantitatively similar, then agents might plausibly make use of this information using procedures along the lines of Section 4 of Evans, Honkapohja, and Mitra (2009).

the time t parameter estimates as

$$\phi_{k,t} = \begin{pmatrix} b_{k,t} \\ a_{kk,t} \\ a_{kv,t} \end{pmatrix}, \phi_{w,t} = \begin{pmatrix} b_{w,t} \\ a_{wk,t} \\ a_{wv,t} \end{pmatrix}, \phi_{rk,t} = \begin{pmatrix} b_{r,t} \\ a_{rk,t} \\ a_{rv,t} \end{pmatrix}, z_t = \begin{pmatrix} 1 \\ k_t \\ \hat{v}_t \end{pmatrix}.$$

The RLS formulas corresponding to estimates of equation (27) then are

$$\phi_{k,t} = \phi_{k,t-1} + \gamma R_t^{-1} z_{t-1}(k_t - \phi_{k,t-1}' z_{t-1}), \tag{31}$$

$$R_t = R_{t-1} + \gamma(z_{t-1} z_{t-1}' - R_{t-1}). \tag{32}$$

Here we are assuming that agents update parameter estimates using "discounted least squares," i.e. they discount past data geometrically at rate $1 - \gamma$, where $0 < \gamma < 1$ is a (typically) small positive number.[15] In the learning literature the parameter γ is known as the "gain," and discounted least squares is also called "constant-gain" least squares. $\phi_{w,t}$ and $\phi_{rk,t}$ are estimated in the same way, see below.

Constant-gain least squares is widely used in the adaptive learning literature because it weights recent data more heavily. See for example Sargent (1999), Cho, Williams, and Sargent (2002), McGough (2006), Orphanides and Williams (2007), Ellison and Yates (2007), Huang, Liu, and Zha (2009), Carceles-Poveda and Giannitsarou (2008), Eusepi and Preston (2011) and Milani (2011). In the current context constant gain is particularly natural since agents will be aware that the announced policy change will induce changes in forecast-rule parameter values taking a possibly complex and time-varying form. Use of a constant-gain rule allows parameter estimates to more quickly track changes in parameter values than does straight ("decreasing-gain") least squares.

Analogously, the RLS formulas corresponding to estimates of equations (28) and (29) are

$$\phi_{w,t} = \phi_{w,t-1} + \gamma R_t^{-1} z_{t-1}(w_{t-1} - \phi_{w,t-1}' z_{t-1}), \tag{33}$$

$$\phi_{rk,t} = \phi_{rk,t-1} + \gamma R_t^{-1} z_{t-1}(r_{k,t-1} - \phi_{rk,t-1}' z_{t-1}). \tag{34}$$

with R_t being given by (32). Note that we have set the gain to be the same in all of the regressions (this is done only for simplicity and is not essential). The initial values of all parameter estimates ϕ and R are set to the initial steady state values under RE. See the Appendix for details.

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[15] Giving a constant weight of γ to the most recent data point implies discounting older data as the sample size increases.

3.2 Surprise permanent policy change

We first consider the benchmark case of a *surprise* change in government spending that takes place immediately. This is a scenario that is frequently studied in the RE literature (see, e.g., Baxter and King (1993), Aiyagari, Christiano, and Eichenbaum (1992), and Romer (2011)).[16] It would, therefore, be of interest to study a surprise policy change under learning and compare with the corresponding RE dynamics. As we will see this provides interesting insights.[17]

Figure 1 compares the dynamics under RE and learning for an increase in government spending that takes place in period 1 and which was not anticipated by agents. The variables plotted are capital (k_t), gross investment ($i_t = k_{t+1} - (1-\delta)k_t$), consumption ($c_t$), labor ($n_t$), output ($y_t$), capital-labor ratio ($k_t/n_t$), wages ($w_t$) and the interest rate (r_t). In all of the figures below, period $t = 0$ depicts the initial steady state values of the variables. We assume the following parametric form for the figures: $\sigma = \epsilon = 1, \zeta = 4$, $\delta = 0.025, \alpha = 1/3, \beta = 0.985, \rho = 0.9, \bar{v} = 1.359, g_0 = 0.20$, and $\gamma = 0.04$ in the learning rule.

The parameter values used conform to the ones used in the real business cycle literature, see e.g. King and Rebello (1999) or Heijdra (2009). The value of β used implies a quarterly real rate of interest of 1.5% (6% annually); the value of δ implies an annualized rate of depreciation of 10% per annum; $\bar{v} = 1.359$ is chosen to normalize output to (approximately) unity. The government spending/output ratio is 21%, that of investment/output ratios is 20% and that of consumption/output ratio is 59%.[18]

Our choice of the gain parameter $\gamma = 0.04$ is in line with most of the literature, e.g. Branch and Evans (2006), Orphanides and Williams (2007) and Milani (2007). Eusepi and Preston (2011) use a much smaller value for the gain, but they do not consider changes in policy, for which a larger value

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[16]Baxter and King (1993) analyze surprise permanent and temporary changes in government spending in the neoclassical model while Ljungqvist and Sargent (2004), Chapter 11, analyze some anticipated changes in policy in deterministic neoclassical models with elastic and inelastic labor supply.

[17]In the notation of the previous section, for the surprise permanent change, the dynamics under learning has $S\tau_{h,t}^e = \frac{\beta}{1-\beta}(\bar{g}' - \bar{g})$ for all $t \geq 1$ since the anticipatory effects are absent when the policy change takes the agents by surprise.

[18]Note that these values are (approximately) the ones used in Heijdra (2009); p. 510, equations (15.46)-(15.47). In our baseline case, the initial steady state values are $\bar{n} = 0.22, \bar{k} = 8.29, \bar{c} = 0.59, \bar{w} = 3.04$. See also footnote 5, p. 509, in Heijdra (2009).

of γ is more appropriate.[19]

$\tilde{u}_t$ is assumed to be distributed uniformly with a support of $(-0.005, 0.005)$. For the policy exercises, there is an increase in government spending from $g_0 = 0.20$ to $\bar{g} = 0.21$ (a 5% increase) that takes place at $t = 1$. We plot the mean time paths for each endogenous variable over $20,000$ replications.[20] We focus attention on the mean time path across replications since this is the most salient aspect of the differences between the RE and learning dynamics when there is a change in policy.

We first describe the dynamics under RE of the surprise increase in government expenditure financed by lump-sum taxes under a balanced budget regime. These dynamics are standard; see for instance Baxter and King (1993), pp. 321-2 and Heijdra (2009), chapter 15. We can get some (qualitative) intuition from the saddle path dynamics considered in Heijdra (2009), Figures 15.1 and 15.2, in the deterministic continuous-time RBC model for such a surprise, permanent change. This is reproduced as our Figure 5 at the end of the paper. The CSE_0, CE_0 lines represent the initial capital stock and consumption equilibrium lines respectively with E_0 the initial steady state. CSE_1 is the capital stock equilibrium line after the increase in government spending and the new steady state is E_1. Consumption falls immediately on impact from point E_0 to point A on the new saddle path (SP_1) in Figure 5, i.e. consumption under-shoots the new steady state E_1 on impact. Thereafter, the dynamics for consumption and capital are monotonically increasing along SP_1 to the new steady state E_1.

These RE qualitative dynamics are confirmed by the behavior of c_t, k_t in Figure 1 which also illustrates the dynamics of other important endogenous variables $n_t, i_t, y_t, \frac{k_t}{n_t}, w_t$, and r_t.[21] Intuitively, the permanent increase in government spending has a large wealth effect on individuals, reducing their permanent income. Since neither consumption nor leisure are inferior goods, individuals respond by reducing consumption and leisure dramatically, so

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[19]Our results are qualitatively robust to a range of values for the gain parameter, except that very small values of γ slow down convergence to the final steady state, and values that are too large lead to instability. For further discussion of the gain parameter see Evans, Honkapohja, and Mitra (2009).

[20]The learning rule uses a projection facility to keep the dynamics of capital bounded since the autoregressive root in the AR(1) process for capital in the RE equilibrium is close to one for plausible parameter values. The projection facility is used outside the range (0.01, 0.99). In all the reported cases, this is used less than 1% of the times for all replications over all the periods.

[21]All figures and tables are at the end of the paper.

that labor supply increases. Consumption under-shoots (and labor supply over-shoots) the new steady state on impact as shown in Figure 1. Since the capital stock is predetermined, the boost in labor input on impact increases aggregate output, the marginal product of capital and the real interest rate. In the short run, an accelerator mechanism operates to generate a boom in investment (overshooting the new higher steady state); see Baxter and King (1993), p. 321. The investment boom leads to a rising path of capital which increases monotonically towards the new higher steady state (as does output). The increase in the real interest rate on impact leads to a rising path of consumption (and declining path of labor supply) due to intertemporal substitution effects. Rising consumption in turn dampens the investment boom which gradually converges towards the steady state. Rising k_t and falling n_t raise the k_t/n_t ratio gradually towards its (unchanged) steady state value which in turn drives the dynamics of w_t and r_t; r_t declines (and w_t rises) towards the steady state.

Under learning, the most striking difference from RE is in the behavior of capital and investment. Instead of the strong investment boom that characterizes the RE dynamics, in the early periods under learning we have the opposite case of a large drop in investment leading in fact to disinvestment (negative net investment $k_{t+1} - k_t = i_t - \delta k_t$) and hence a falling path of capital in the initial periods after the policy change. Why does this happen under learning? One way to view this is that in the new steady state, all of the perceived parameters in the capital equation (i.e. the constant term $\bar{b}_k$, the auto-regressive root λ_2, and the coefficient of the shock term $\bar{a}_{kv}$) are higher than the initial steady state values. Since agents' parameter estimates are still at the initial steady state at $t = 1$, the "desired" capital stock under learning is lower than under RE, which causes the disinvestment. In effect, agents are yet to realize that the permanent increase in government spending will lead to a higher steady state capital stock; under RLS learning, agents figure this out gradually as they accumulate more data and update their parameter estimates over time.

More specifically, in terms of the equilibrium dynamic system under learning, the mechanism is as follows. At $t = 1$, consumption falls because of the increase in $S\tau_{h,t}^e$. However, because wage and interest rate expectations are predetermined, the fall in consumption and the increases in employment and output are all less than under RE. Under RE the paths of lower future w_t and higher r_t are fully anticipated, magnifying the impact relative to the learning path in which expectations are initially unchanged. Under learning

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w_{t+s}^e, r_{t+s}^e gradually respond to the data, leading initially to a gradual fall in w_{t+s}^e (and rise in r_{t+s}^e) before eventually rising towards the steady state.

As a consequence of the smaller sizes of the impacts on output and consumption at $t = 1$, the increase in g necessarily leads to a lower level of i_t under learning than under RE, and in fact we see a sharp reduction in investment. In the periods immediately following the policy change, expectations of wages and interest rates adjust. Two factors are at work. The lower capital stock in the periods soon after the policy change leads to lower forecasts of future wages and higher forecasts of future interest rates and thus lower Sw_t^e and higher Sr_t^e. This leads to a further reduction in c_t, and increases in n_t and y_t, which results in increases in i_t from its low level at $t = 1$. After several periods this process is sufficient to restore k_t to an upward path, accompanied by a fall in n_t, and an increase in k_t/n_t, drives w_t upwards and r_t downwards to their steady state values. The other factor at work is that over time coefficient estimates under RLS learning gradually adjust in response to the shock and the evolution of the data. Eventually the coefficients converge to the values that correspond to the REE values at the new steady state, so that in the long run their is convergence to the new REE.

This situation is in stark contrast to the RE case where agents, at $t = 1$, are fully aware of the new steady values of all variables including capital. Realizing that the long run capital stock is higher, desired capital stock is higher and that causes the investment boom under RE, with the consequent bigger initial impact effects on consumption and labor supply. Table 1, at the end of the paper, compares these impact effects under RE and learning. Compared to RE the paths of c_t, n_t and k_t/n_t under learning adjust less on impact and respond more sluggishly, leading to a hump-shaped response of c_t, n_t and i_t, with i_t eventually overshooting the new steady state (in effect this compensates for the low levels of investment in the initial periods).[22] This also implies that the paths followed by c_t, n_t and k_t/n_t (and hence w_t and r_t) in the periods following the policy change are *qualitatively* in *opposite* directions under learning compared to that under RE; e.g. c_t, w_t are falling under learning initially whereas they are rising under RE.

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[22]In RBC models with learning, hump-shaped responses to productivity shocks have been observed by Eusepi and Preston (2011), Branch and McGough (2011), and Huang, Liu, and Zha (2009). The latter also emphasize the plausible labor market dynamics that arise from the learning model. However, none of these papers focus on changes in government spending.

3.3 Anticipated permanent policy change

We now examine the effects of an anticipated change in policy that is announced credibly in period 1. We will see that the dynamic effects under both RE and learning depend on how far in advance the policy change is announced. We, therefore, consider two values of T_p in what follows. Figure 2 plots the dynamics for an anticipated, permanent increase in government spending to take place in period $t = 5$ i.e. $T_p = 5$. We interpret a period as a quarter and frequently refer to this as an announcement one year in advance. The parameter values used are the same as those for Figure 1 (and in fact in all of the figures below). Figure 3 illustrates the dynamics when $T_p = 29$ (we refer to this as an announcement seven years in advance).

We first summarize the effects of the policy change under RE. We can again use Figure 5 to help us understand the dynamics. When T_p is small (like $T_p = 5$ in Figure 2), the impact effect on c_t at $t = 1$ is quite large (though smaller than that for the surprise change) and it under-shoots the new steady state E_1. The dynamics, thereafter, is governed by the phase diagram implied by the curves CE_0, CSE_0 since g_t is unchanged until T_p. The phase diagram implies that c_t and k_t rise monotonically during the anticipatory phase until the saddle path SP_1 is hit at the time when g_t increases (and the dynamics are then governed by the CE_0, CSE_1 lines). Thereafter, the paths of c_t and k_t continue to increase monotonically along SP_1 until the steady state E_1 is reached.

When T_p is large (like 29 in Figure 3), the impact effect on c_1 is much smaller and it does not under-shoot the new steady state E_1. The phase diagram then implies that c_t and k_t rise monotonically initially until the dynamics hits the CE_0 line. Thereafter, c_t falls but k_t continues to rise until the new saddle path SP_1 is reached at the time when the actual increase in g_t takes place. The paths of c_t and k_t then monotonically decrease along SP_1 towards the new steady state E_1 (in accordance with the transitional dynamics implied by the CE_0, CSE_1 lines). Thus, k_t increases monotonically until $t = 29$ over-shooting the new steady state before decreasing gradually in the transient phase.

These effects are confirmed by the dynamics displayed under RE in Figures 2 and 3.[23] c_t falls on impact while n_t, gross investment i_t, and y_t all

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[23]Note that for permanent changes in g the RE dynamics for the standard RBC model are significantly different from the Ramsey model discussed in Ljungqvist and Sargent (2004), Chapter 11 or Figures 8-9 of Evans, Honkapohja, and Mitra (2009); steady state

rise on impact (correspondingly, w_t falls on impact). Over-shooting of c_t is observed on impact in Figure 2 but not in Figure 3 which is consistent with the explanation from the phase diagram. The impact effects under RE get smaller as T_p increases. Intuitively, with large T_p, the time period over which the capital stock can be built up is longer, making it possible for agents to smooth out their consumption with a smaller initial fall in consumption.

After these initial impact effects, there are further rises in k_t and n_t until the policy is implemented which further boosts y_t. k_t rises sharply, which raises the k_t/n_t ratio and w_t during this phase. After the policy change, the increase in g crowds out i_t which falls sharply and the other variables converge gradually towards the steady state. An interesting thing to note is that over-shooting of k_t (and w_t, r_t) is observed when $T_p = 29$ whereas c_t and n_t overshoot when $T_p = 5$ (rather than $k_t, w_t,\ r_t$) under RE; this is of course consistent with the explanation given above.

Under learning, only the announced increase in future taxes reduces c_t at $t = 1$, by equation (10), since expectations of wages and interest rates are pre-determined. The impact effects under learning (like that under RE) are reduced as T_p increases. However, compared to RE, the impact effects are smaller under learning when T_p is small (see Figure 2) while they are larger when T_p is large (see Figure 3). Table 1 summarizes the impact effects in percentage terms for the surprise and the announced permanent changes illustrated in Figures 1-3.

We return to the dynamics under learning, and focus on the case $T_p = 29$, which we examine in detail.[24] The initial fall in consumption, due to the higher anticipated future taxes $S\tau_{h,t}^e$, leads to a temporary investment boom and a period of capital accumulation. However, under learning this is soon followed by a considerable period in advance of $T_p = 29$, specifically $t = 4, \ldots, 23$, in which there are higher wages and expected wages, Sw_t^e, and lower interest rates and expected interest rates, Sr_t^e than under RE. These expectations under learning are partially self-fulfilling, in that they are accompanied by higher c_t, lower n_t and higher k_t/n_t, compared to the RE path. As a result, the qualitative dynamics of n_t and y_t under learning are actually opposite to that under RE throughout most of the pre-implementation phase, in the sense that n_t and y_t are falling over time under learning whereas they

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values of capital and labor change in the RBC model.

[24]For $T_p = 5$ the qualitative dynamics are similar under learning, except that the over-shooting of c_t, w_t, and r_t are not observed in the pre-implementation period.

rise over time fairly dramatically under RE.

Continuing with the learning scenario, these optimistic assumptions of high future wages and low future interest rates offset the higher expected taxes $S\tau_{h,t}^e$, and consequently when $T_p = 29$ arrives, employment is back to initial levels and consumption is actually slightly larger than it was initially. During the period $T_p = 29$ itself, when the government spending increase begins, there is virtually no impact on c_t or n_t, or on w_t, r_t, since the tax increases had been almost fully anticipated. Consequently almost the full impact of the increase in $\bar{g}$ at T_p is on i_t and thus on k_{t+1}. This corresponds to a similar decrease in i_t in the RE case. However, in the learning case the fall in the capital stock *after* $T_p = 29$, during periods $t = 30 - 35$, leads to a sharp reduction in wages and a sharp increase in interest rates that were not correctly anticipated by agents. There is then a sustained period for $t > T_p$ of low c_t, low w_t, high r_t and high n_t (with both n_t and i_t overshooting their new higher steady state levels), as agents adjust their expectations to the post-policy implementation reality, with eventual convergence to the new steady state.

To summarize, only the direct wealth effects from the anticipated change in government spending (and taxes) are fully foreseen under learning in the anticipatory phase. Under learning, in contrast to RE, agents do not correctly foresee the path of future wages and interest rates. This leads to overoptimism concerning wages and interest rates in the pre-implementation period, and a substantial correction following implementation, with a period of low wages, low consumption and high interest rates.[25]

3.4 Interpretation of results

For both surprise and anticipated permanent increases in g we see the following main qualitative features:

1. There are large impact effects for both the RE and learning solutions, and these effects get smaller as T_p increases. The impact effects under learning are smaller than under RE for surprise changes but the opposite is true when T_p is large.

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[25]Looking at Figures 1-3, it is evident that the k_t dynamics is qualitatively important in determining the movement of w_t, r_t while the n_t dynamics is influential in determining the behavior of y_t under learning; of course, the interaction between k_t and n_t generally influences the paths of all variables simultaneously.

2. The dynamics of variables under RE and learning can be in qualitatively opposite directions for some periods after the impact effects. For the surprise change, k_t, c_t are falling after the policy change under learning while they are rising under RE (n_t is rising under learning and falling under RE during this time). These features lead to a hump-shaped response in variables under learning that is absent under RE. Similarly, for the announced change, i_t, n_t, and y_t are all falling under learning in the pre-implementation phase whereas they are all rising under RE in this time period.

3. For anticipated future permanent changes in g, under learning there is essentially no impact on c or n on the date when the policy is implemented, and in this respect is like RE. The reasons are the same in each case: the tax change is fully anticipated and agents aim to smooth their consumption path over time.

4. There can be classic "overshooting" results for both learning and RE paths. For example, in the case of an announced increase in g when T_p is large, the path for the capital stock rises above the new higher steady state before eventually converging to it under RE. However, overshooting is a far more prominent feature of learning paths; for an announced change in g, consumption falls instantaneously before gradually rising until T_p; there is then a substantial fall in consumption under-shooting the new steady state before converging to it.

5. Related to this last point, the learning paths exhibit oscillatory convergence that is particularly pronounced for announced policy changes. For example, in the announced case, under learning, the capital stock, after its initial rise, falls for a period before increasing and eventually converging. Other variables like c_t, n_t, y_t, k_t/n_t ratios (hence, w_t and r_t) all exhibit oscillatory convergence as well.

We now discuss the intuition for these results. The key feature is that the effects from the change in expected future government spending and taxes is felt immediately (since agents foresee the path of g_t even under learning), while the implications for expected future wages and interest rates evolve slowly in response to the data.

Consider the effect of an anticipated permanent increase in g illustrated in Figures 2 and 3. At the time of the announcement agents understand that the future higher taxes reduce their overall wealth, leading to lower c_t and higher n_t. Because g_t has not yet increased this leads to an investment boom and a higher capital stock. This in turn leads to higher wages and lower interest rates, offsetting the reduced wealth, so that under learning, for large

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T_p, variables can evolve to steady levels consistent with expectations. Then, under learning, at T_p agents are again surprised because the (anticipated) increase in $g = \tau$ leads to an (unexpected) fall in aggregate capital, leading to lower wages and higher interest rates. This second surprise on implementation under learning leads to a large drop in consumption, overshooting the new lower steady state, and a subsequent sustained period during which the capital stock is built up during the process of convergence to the new REE.

To summarize, on announcement of the future increase in g, agents immediately understand the implications for their wealth of their future higher taxes and they immediately adjust their consumption and labor supply accordingly. During the period $t < T_p$ they also revise their expectations of future wages and interest rates in response to the data. What they do not foresee, however, is that when the policy is implemented this will lead to a crowding out of capital that will in turn eventually reduce wages and increase interest rates. Consequently, for $t > T_p$ there is another period of adjustment as agents learn the properties of the new equilibrium steady state.

How reasonable is our implicit assumption that agents will not foresee the extent to which capital is crowded out by the government spending in the period following implementation? We think this is very plausible. For agents to deduce that there will be the decline in the capital stock following T_p they would need not only to understand the capital accumulation equation (16), but also to accurately forecast *aggregate consumption c_t* and *aggregate labor supply n_t* during the period following T_p. As we have already indicated in our earlier discussion of RE, this in turn requires an implausibly high degree of structural knowledge of the economy, as well as a belief that this structural information is common knowledge, that all agents are fully rational and capable of computing equilibrium paths, and that this is common knowledge.[26] These are precisely the assumptions that the adaptive learning literature aims to avoid.

The approach taken in this paper is to examine the implications of assuming that agents have *some* structural information pertinent to their decision problem, here the path of future taxes, but that they use econometric forecasting procedures for other key variables. An implication of our approach is that agents are likely to make systematic mistakes when confronted with announced future increases in government spending: while the tax implica-

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[26]The strong assumptions required for agents to be able to deduce, and hence coordinate on RE, are discussed in Guesnerie (2002).

tions will be understood, agents may become overoptimistic in advance of the policy implementation, leading to a subsequent correction.

3.5 Oscillatory dynamics under learning

We have noted that oscillatory dynamics is a prominent feature under learning. In general, the oscillatory convergence under learning is due to a combination of surprises and inertia. Given their expectations, households aim to smooth consumption, and this leads to inertia in capital adjustment, which is present under both RE and learning, and to monotone dynamics. Under adaptive learning, there is also inertia in the parameters used in wage and interest rate forecasts, and this can lead to a failure to understand the full dynamics of future wages and prices. This leads to additional learning dynamics that can produce oscillations in the endogenous macroeconomic variables. In the case of anticipated policy changes, the inertia in learning dynamics can also lead to a secondary surprise following implementation, leading to a second round of oscillations.

More formally, the system under learning combines two types of dynamics. First consider the case of the permanent surprise increase in g. This case is simpler since the structure of the system after the shock remains stationary. Under RE the policy change in effect re-initializes the system so that the "initial" capital stock is below its new steady state values. Consumption drops to the new saddle path, and the mean paths of all variables converges over time to their new steady state values. The RE forecast functions immediately jump to the parameter values corresponding to the RE paths associated with the new steady state. Under RE the system dynamics are inherently monotonic since the state is given by (25)-(26), which implies that the mean path of capital simply follows

$$E_t k_{t+1} - \bar{k}' = \lambda_2'(k_t - \bar{k}') \text{ where } 0 < \lambda_2' < 1.$$

Here $\bar{k}', \lambda_2'$ etc. denotes new steady state values.

Under learning the temporary equilibrium and thus the system dynamics are driven by the values of the forecast parameters as well as by the current state (and the random productivity shocks). The forecast functions (27), (28), (29) and (30) are characterized by a vector of estimated coefficients $\theta = (b_k, a_{kk}, a_{kv}, b_w, a_{wk}, a_{wv}, b_r, a_{rk}, a_{rv})$, which are updated over time using RLS. If, at the time of the policy change, the coefficient values for θ changed

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immediately to the new RE values (i.e. if b_k and a_{kk} changed to $a_{kk} = \lambda_2'$ and $b_k = (1 - \lambda_2')\bar{k}'$ and all other coefficients changed analogously) then our temporary equilibrium system would replicate the REE. Under adaptive learning, however, the coefficients gradually evolve towards the new RE values in response to data and the RLS updating scheme. One can show the *actual law of motion* (ALM) dynamics for given parameters θ takes the same form as the PLM but with parameters $T(\theta)$ instead of θ. The mapping $T : \mathbb{R}^9 \to \mathbb{R}^9$ can be computed numerically and REE parameter values are a fixed point $\bar{\theta} = T(\bar{\theta})$. Under learning the parameters $\theta(t)$ evolve under RLS updating. Denoting $\theta^*(t) = T(\theta(t))$ and using $b_k^*(t)$, etc., for the components of θ^*, the (linearized) actual temporary equilibrium dynamics are given by

$$
\begin{aligned}
k_{t+1} &= b_k^*(t) + a_{kk}^*(t)k_t + a_{kv}^*(t)\hat{v}_t \\
w_t &= b_w^*(t) + a_{wk}^*(t)k_t + a_{wv}^*(t)\hat{v}_t \\
r_{k,t} &= b_r^*(t) + a_{rk}^*(t)k_t + a_{rv}^*(t)\hat{v}_t \\
\hat{v}_t &= \rho\hat{v}_{t-1} + \tilde{u}_t,
\end{aligned}
$$

where the temporary equilibrium expression for c_t (and a corresponding expression for n_t) have been used to obtain this system. Thus under learning the system has two types of dynamics: the linear state dynamics corresponding to this system with given parameters $\theta(t)$ (equal to $\bar{\theta}$ at an REE) and the RLS dynamics governing the evolution of $\theta(t)$ over time. The resulting system for the endogenous variables is a nonlinear stochastic dynamic system that can include oscillatory responses to structural change.

In the case of the permanent surprise increase in g illustrated in Figure 1, the hump-shaped response for k_t, i_t, c_t and n_t results from this combined dynamics. Immediately after the policy shock, the PLM parameters are at the old steady state values $\bar{\theta}$, and this leads to smaller impact effects than under RE and a decline in k_t. This then leads to further movements of variables away from the new steady state as discussed above. However, over time $\theta(t)$ evolves towards the new REE values $\bar{\theta}'$, leading to the *eventual monotonic convergence* seen in Figure 1.[27]

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[27]It can be shown that the mean dynamics of the parameter estimates are governed by the "E-stability" differential equation $\dot{\theta} = T(\theta) - \theta$, and that local asymptotic stability of an REE $\bar{\theta}$ is therefore determined by the Jabobian matrix $DT(\bar{\theta})$. Numerically for our baseline parametrization, six of the nine eigenvalues are zero and the remaining are approximately $-4.50, -0.95, -0.64$. Since all the eigenvalues are less than one, the equilibrium is E-stable and therefore stable under least squares learning. For some initial conditions stable

In the case of anticipated permanent increases in g, illustrated in Figures 2 and 3, we have the additional feature under learning, described above, of a second surprise after the policy is implemented, taking the form of unexpected wages and interest rate changes. This leads to a second period of oscillatory dynamics before convergence to the final steady state.

Intuitively, the system under learning exhibits a mixture of state variable dynamics inherited from the rational expectations equilibrium and coefficient dynamics from RLS learning. The rational expectations dynamics deliver a strongly positively serially correlated process for capital and the other variables in the system. In contrast, the learning dynamics can deliver oscillatory behavior around the REE values when the system undergoes either a surprise or an anticipated structural change (here a change in policy).

The importance of cyclical or oscillatory dynamics has been emphasized in RE models by a number of people, e.g. Farmer (1999), Chapter 7, Farmer and Guo (1994) and Azariadis, Bullard, and Ohanian (2004). These papers also argue that such dynamics are a feature of US data. Farmer and Guo (1994) obtain cyclical dynamics in RBC-type models with nonconvexities (see also Baxter and King (1991)). In Azariadis, Bullard, and Ohanian (2004) the oscillatory dynamics arise from the overlapping generations structure. In contrast, we have shown that in the presence of adaptive learning, oscillatory dynamics can be expected to be a prominent feature of changes in fiscal policy in standard RBC models. It would be interesting to examine this feature of adaptive learning in more detail and to compare its implications with the data.[28]

4 Temporary Policy Changes

The other natural fiscal policy experiment to consider is a change in government spending that is known to be temporary. We assume that initially, at $t = 0$, we are in the steady state corresponding to $g = \bar{g}$, and consider the following policy experiment, assumed fully credible and announced at the

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oscillations of parameters could still arise, and large negative eigenvalues can make the system more prone to oscillations under constant gain. These can even generate instability for large gains, as noted in Evans and Honkapohja (2009).

[28] See Eusepi and Preston (2011) and Milani (2011) for empirically oriented studies in models with unchanged policy.

start of period 1:

$$g_t = \tau_t = \left\{ \begin{array}{l} \bar{g}', \, t = 1, ..., T_g - 1 \\ \quad \bar{g}, \, t \geq T_g, \end{array} \right. \tag{35}$$

i.e., government spending and taxes are changed in period $t = 1$ and this change is reversed at a later period T_g. Thus, the experiment is one where the policy change is announced in period 1 to take place in the future for a fixed number of periods.[29] The formal dynamics under RE and learning are summarized in the Appendix.

Before examining the results we remark that our surprise temporary changes include an important anticipated component, since the policy change is assumed to be accompanied by an announced date at which the policy will come to an end. We plot the dynamics for a surprise temporary policy change, which takes place in period 1 and lasts for 8 periods (we interpret this as a two year war).[30] The remaining parameter values are the same as in the earlier Figures.

For the RE case we can again get some intuition from the phase diagram considered in Figure 5. The impact effect on consumption will be less than for the permanent change. In addition, given the transient nature of the shock, c_1 will lie between the CSE_0 and the CSE_1 lines. Thereafter, c_t starts rising and k_t starts falling (since CSE_1 governs the dynamics in this phase) until the dynamics hit the initial saddle path SP_0 passing through E_0 when the policy change is reversed. Thereafter c_t and k_t both rise monotonically towards the initial steady state along this saddle path.

These dynamics are confirmed in Figure 4 and are qualitatively similar to the RE dynamics for a similar policy analyzed in Baxter and King (1993) and Aiyagari, Christiano, and Eichenbaum (1992). k_t falls as long as the policy change is in effect and then increases towards the (unchanged) steady state. c_t falls on impact and then increases monotonically towards the steady state. As emphasized in these papers, the key difference from a surprise permanent change is the behavior of investment. When the change is temporary, consumption smoothing by agents is achieved by a *reduction* in investment. The smaller wealth effect due to the temporary change has a smaller effect on c_t, n_t, and y_t on impact. The k_t/n_t ratio falls on impact which raises r_t and lowers w_t on impact. w_t continues to be low during the war and this reduces

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[29]We have also examined the case of temporary changes in g that are announced in advance, but for reasons of space we omit these results here.

[30]Of course, one could also incorporate uncertainty about the length of the war.

n_t over time. People, however, maintain a rising path of c_t by running down their capital and i_t continues to decline as long as the war lasts which also results in a falling path of y_t over time. There is no longer a need to reduce capital to maintain a rising consumption path once the period of high g is over. There is, therefore, an investment boom at this point and k_t starts increasing towards the (unchanged) steady state. The k_t/n_t ratio starts rising, which raises w_t (lowers r_t). The falling interest rates lead to further declines in n_t which converges towards its steady state.

We now discuss the impacts of the policy under learning. The most marked difference under learning compared to RE is the sharper fall in investment on impact.[31] Under RE, agents foresee the path of low wages (and high interest rates) in the future which reduces initial consumption more on impact compared to learning. With expectations of future wages and interest rates pre-determined, and only a small rise in $S\tau_{h,t}^e$ (due to the temporary change), the reduction in consumption at $t = 1$ is much smaller under learning than under RE (the impact effects on other variables is also muted under learning for the same reason). Consequently, there is a sharp fall in investment with the capital stock run down rapidly.

Under learning, although agents correctly foresee the period of higher taxes, they fail to appreciate the precise form of the wage and price dynamics that result from the policy change. The reduction in k_t over $t = 1, \ldots, T_g - 1 = 8$, leads to lower wages and expected wages, Sw_t^e, and higher interest rates and expected interest rates, Sr_t^e, resulting in a period of excessive pessimism during the period of the war. The resulting reduction in c_t and increase in n_t during this period reverses the fall in investment and stabilizes the capital stock at a level in excess of RE levels. Then, when the war ends at $T_g = 9$, the planned reduction in government spending leads to a sharp spike in investment and build-up of the capital stock. This leads to a period of higher wages and expected wages, and lower interest rates and expected interest rates, and thus to an extended period of correction to the earlier period of overpessimism, before eventual convergence back to the REE steady state. Note that yet again the dynamics of c_t, n_t, y_t, w_t, and r_t display a hump-shaped pattern under learning unlike that under RE.

As in the case of permanent policy changes, one way to view these results is that agents fail to foresee the full impacts of the crowding out or crowding

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[31]See Table 2 for a comparison of impact effects under RE and learning for the endogenous variables.

in of capital from government spending. In the present case, agents tend to extrapolate the low wages during the war, which result from the rundown of capital, and while they understand that their future taxes will fall when the war ends, they fail to recognize the improvement in wages that will occur after the crowding in of capital after the war. This is the source of the excessive pessimism during the war, with a resulting correction after the war ends. These shifts in household sentiment are the origins of the oscillatory response observed under learning to the policy change. As a result one also observes overshooting of all the key variables under learning. The overshooting phenomenon is not observed here under RE. For example, under learning, after the end of the war, c_t, w_t and r_t substantially overshoot the steady state values.

5 Conclusion

Changes in fiscal policy, in an RBC model with adaptive learning, generate mean trajectories that have both common features and significant differences from the mean paths under RE. These dynamics were examined for various types of fiscal changes: surprise vs. announced and permanent vs. temporary. For announced policy changes scheduled to take place in the future, immediate anticipation effects under learning arise from the wealth effects of anticipated future tax changes, followed by additional more gradual impacts arising from changes in expected future wages and interest rates.[32]

The differences in dynamics under RE and adaptive learning therefore arise due to the future path of wages and interest rates being fully foreseen by RE agents, while agents learn only gradually about these variables under incomplete knowledge. In effect, under learning agents understand the direct wealth effects of future changes of government spending and taxes, but fail to fully anticipate the effect on factor prices of the crowding out or crowding in of changes in government spending. Depending on the form of the announced policy change, the size of the impact effects under learning can be either greater or smaller than under RE. In some cases the qualitative dynamics of variables can be in diametrically opposite directions under RE and

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[32]We remark that our focus on anticipated future fiscal changes is reminiscent of the literature on news shocks about future productivity changes, see Jaimovich and Rebelo (2009). The approach used in the current paper could naturally be extended to news shocks within their framework.

learning. Oscillatory dynamics, not present under RE, emerge prominently as agents learn about the full impact of the policy change and its effect on the new steady state. This feature of adaptive learning ought to be explored more in future work since oscillatory dynamics are arguably present in US macroeconomic data as well.

The current work has only considered a scenario with balanced budget and lump-sum taxes, which is the baseline case considered in the RBC literature. In work in progress, we plan to analyze the impact of changes in (distortionary) capital and labor taxes in an RBC type model and compare the dynamics under adaptive learning with those under RE.

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Appendix

A   Details of Solutions under Learning

Under learning, agents need to form forecasts of variables without full knowledge of the underlying model parameters. In the basic formulation, announced policy changes are fully credible and, hence, future forecasts of lump-sum taxes are assumed known to them. However, they still need to form forecasts of future wages and rental rates/interest rates in order to determine their consumption choice in (10). In the learning literature, these forecasts depend on the perceived laws of motion (PLMs) of the agents. We initially start with PLMs that correspond to the REE given in (20), (23) and (24) in which wages, and rental rates are estimated on the basis of data on capital stock and technological shock, k_t and v_t. Thus the PLMs (including constants) of the agents are taken to be of the form of equations (27) - (29), where the PLM parameters b_k, a_{kk}, a_{kv} etc will be estimated on the basis of actual data. The final line is the stochastic process for evolution of the (de-meaned) technological shock which is assumed known to the agents (this is without loss of generality).

We will now write these PLMs in deviation form; with deviations under learning taken from the estimated steady state values of capital, wage rate, and rental rate. Define

$$
\begin{aligned}
\tilde{k}_t &= k_t - \bar{k}_t^e, \\
\tilde{r}_{k,t} &= r_{k,t} - \bar{r}_{k,t}^e, \\
\tilde{w}_t &= w_t - \bar{w}_t^e.
\end{aligned}
\tag{36}
$$

where, for instance, $\tilde{r}_{k,t}$ is the deviation of the rental rate from the steady state rental rate estimated under learning at time t (i.e. $\bar{r}_{k,t}^e$).

Using this notation we have

$$
\begin{aligned}
\tilde{k}_{t+1} &= a_{kk}\tilde{k}_t + a_{kv}\hat{v}_t, \tag{37} \\
\tilde{w}_t &= a_{wk}\tilde{k}_t + a_{wv}\hat{v}_t, \tag{38} \\
\tilde{r}_{k,t} &= a_{rk}\tilde{k}_t + a_{rv}\hat{v}_t. \tag{39}
\end{aligned}
$$

where the estimated steady state values of capital, rental rates, and wages

under learning are (omitting the time subscripts on $\bar{k}_t^e$, etc.)

$$\bar{k}^e = \frac{b_k}{1 - a_{kk}}, \tag{40}$$

$$\bar{r}_k^e = b_r + a_{rk}\frac{b_k}{1 - a_{kk}}, \tag{41}$$

$$\bar{w}^e = b_w + a_{wk}\frac{b_k}{1 - a_{kk}}. \tag{42}$$

Then under learning, the form corresponding to (25) is

$$\begin{pmatrix} \tilde{k}_{t+1} \\ \hat{v}_{t+1} \end{pmatrix} = \tilde{B}\begin{pmatrix} \tilde{k}_t \\ \hat{v}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \tilde{u}_{t+1} \end{pmatrix},$$

$$\tilde{B} = \begin{pmatrix} a_{kk} & a_{kv} \\ 0 & \rho \end{pmatrix}.$$

Defining $\tilde{x}_t \equiv \begin{pmatrix} \tilde{k}_t \\ \hat{v}_t \end{pmatrix}$, we have for $j \geq 1$,

$$\tilde{x}_{t+j}^e = \tilde{B}^j \tilde{x}_t. \tag{43}$$

Using the future forecasts of capital stocks from (43), we can in turn obtain the future forecasts of wages and rental rates from (38) and (39) as

$$\tilde{w}_{t+j}^e = \begin{pmatrix} a_{wk} & a_{wv} \end{pmatrix} \tilde{B}^j \tilde{x}_t,$$

$$\tilde{r}_{k,t+j}^e = \begin{pmatrix} a_{rk} & a_{rv} \end{pmatrix} \tilde{B}^j \tilde{x}_t.$$

We linearize (6) around the deterministic steady state $\bar{c}$ and $\bar{r} = \beta^{-1}$ to get

$$c_t - \bar{c} = \hat{E}_t(c_{t+1} - \bar{c}) - \beta\sigma^{-1}\bar{c}\hat{E}_t(r_{t+1} - \bar{r}) \tag{44}$$

As noted in the main text, we assume agents choose the (known) initial steady state as the point around which to linearize. Iterate equation (44) forward to get

$$c_t - \bar{c} = \hat{E}_t(c_{t+j} - \bar{c}) - \beta\sigma^{-1}\bar{c}\hat{E}_t\sum_{i=1}^{j}(r_{t+i} - \bar{r})$$

which describes current consumption in terms of expected consumption j steps ahead and future short-term interest rates.

Having obtained the future forecasts of wages and interest rates under learning, we reproduce the consumption function below that agents use to determine their current consumption. The linearized consumption function is

$$(c_t - \bar{c})C_{AA} = \bar{a}(r_t - \bar{r}) + \bar{r}(a_t - \bar{a}) - (\tau_{h,t} - \bar{\tau}_h) + C_{ww}(w_t - \bar{w})$$
$$+ S1_t^e + S2_t^e. \tag{45}$$

where $\bar{r} = \beta^{-1}$ in the deterministic steady state and

$$C_{AA} \equiv \frac{1}{1-\beta}(1 + \frac{\sigma}{\epsilon}\zeta^{\frac{1}{\epsilon}}\bar{w}^{\frac{\epsilon-1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}-1}),$$

$$C_{ww} \equiv 1 - \frac{\epsilon-1}{\epsilon}\zeta^{\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}}\bar{w}^{-\frac{1}{\epsilon}},$$

$S1_t^e$ in (45) is defined as

$$S1_t^e \equiv -S_A Sr_t^e, \tag{46}$$

$$S_A = \bar{w} - \zeta^{\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}}\bar{w}^{\frac{\epsilon-1}{\epsilon}} - \bar{c} - \bar{\tau}_h, \tag{47}$$

and $S2_t^e$ is defined as

$$S2_t^e = \sum_{j=1}^{\infty} \bar{r}^{-j}[C_{ww}(w_{t+j}^e - \bar{w}) - (\tau_{h,t+j}^e - \bar{\tau}_h) - \bar{r}^{-1}(\frac{\bar{c}}{\sigma} + \frac{\zeta^{\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}}\bar{w}^{1-\frac{1}{\epsilon}}}{\epsilon})\sum_{i=1}^{j}(r_{t+i}^e - \bar{r})].$$

$S2_t^e$ can be rewritten as

$$S2_t^e = C_{ww}Sw_t^e - S\tau_{h,t}^e - (\frac{\bar{c}}{\sigma} + \frac{\zeta^{\frac{1}{\epsilon}}\bar{c}^{\frac{\sigma}{\epsilon}}\bar{w}^{1-\frac{1}{\epsilon}}}{\epsilon})Sr_t^e. \tag{48}$$

where Sr_t^e, $S\tau_{h,t}^e$, and Sw_t^e are given by equations (11), (12), and (13) in the text.

If we combine the expressions in (46) and (48), we can write the consumption function (45) as

$$(c_t - \bar{c})C_{AA} = \bar{a}(r_t - \bar{r}) + \bar{r}(a_t - \bar{a}) - (\tau_{h,t} - \bar{\tau}_h) + C_{ww}(w_t - \bar{w})$$
$$-(S_A + \frac{\bar{c}}{\sigma} + \frac{\bar{c}^{\frac{\sigma}{\epsilon}}\zeta^{\frac{1}{\epsilon}}}{\epsilon}\bar{w}^{1-\frac{1}{\epsilon}})Sr_t^e + C_{ww}Sw_t^e - S\tau_{h,t}^e$$

which is equation (10) in the text, with

$$C_{rr} = S_A + \bar{c}\sigma^{-1} + \bar{c}^{\frac{\sigma}{\epsilon}}\zeta^{\frac{1}{\epsilon}}\epsilon^{-1}\bar{w}^{1-\frac{1}{\epsilon}}.$$

We note that equation (10) reduces to the following when $\sigma = \epsilon = 1$; the case assumed in the figures,

$$
\begin{aligned}
c_t - \bar{c} = & \frac{1-\beta}{1+\zeta}[\bar{a}(r_t - \bar{r}) + \bar{r}(a_t - \bar{a}) - (\tau_{h,t} - \bar{\tau}_h) + (w_t - \bar{w}) \\
& -(\bar{w} - \bar{\tau}_h)Sr_t^e + Sw_t^e - S\tau_{h,t}^e].
\end{aligned}
$$

For the calibrations assumed in the figures, $\bar{w} > \bar{\tau}_h$, so that increases in Sr_t^e and decreases in Sw_t^e reduce current consumption c_t, as one would intuitively expect.

Since announced policy changes are assumed to be credible, future forecasts of taxes $S\tau_{h,t}^e$ simply coincide with the assumed fiscal rule for the government in the consumption function (10). However, one still needs to obtain analytical expressions for Sw_t^e and Sr_t^e which appear in (10). This is what we do now.

Note that using (19) along with (36) we obtain

$$
r_t - \bar{r} = \tilde{r}_{k,t} + \bar{r}_{k,t}^e - \bar{r}_k,
$$

which after iterating forward gives us

$$
\begin{aligned}
r_{t+i} - \bar{r} & = \left(\tilde{r}_{k,t+i} + \bar{r}_{k,t}^e - \bar{r}_k\right) \\
& = \begin{pmatrix} a_{rk} & a_{rv} \end{pmatrix} \tilde{B}^i \tilde{x}_t + \left(\bar{r}_{k,t}^e - \bar{r}_k\right).
\end{aligned}
$$

since $\bar{r}_{k,t+i}^e = \bar{r}_{k,t}^e$; i.e. the estimated steady state rental rate i steps ahead is still based on time t data and hence equals the time t estimate $\bar{r}_{k,t}^e$ given in (41). We use this to derive Sr_t^e below. Observe that

$$
\sum_{i=1}^{j}(r_{t+i}^e - \bar{r})
$$

$$
= (\bar{r}_{k,t}^e - \bar{r}_k)j \quad + \begin{pmatrix} a_{rk} & a_{rv} \end{pmatrix} [(I - \tilde{B})^{-1}\tilde{B} - (I - \tilde{B})^{-1}\tilde{B}^{j+1}]\tilde{x}_t
$$

since

$$
\sum_{i=1}^{j} \tilde{B}^i = (I - \tilde{B})^{-1}\tilde{B} - (I - \tilde{B})^{-1}\tilde{B}^{j+1}
$$

37

Using this Sr_t^e is finally obtained as

$$Sr_t^e = \frac{\beta^2}{(1-\beta)^2}(\bar{r}_{k,t}^e - \bar{r}_k) + \beta^2 \begin{pmatrix} a_{rk} & a_{rv} \end{pmatrix} (I - \tilde{B})^{-1}\tilde{B}[I(1-\beta)^{-1} - \tilde{B}(I-\beta\tilde{B})^{-1}]\tilde{x}_t.$$

Similarly, since

$$(w_{t+i}^e - \bar{w}) = (\bar{w}_t^e - \bar{w} + w_{t+i}^e - \bar{w}_t^e) = (\bar{w}_t^e - \bar{w}) + \tilde{w}_{t+i},$$

Sw_t^e can be obtained from

$$\begin{aligned}
Sw_t^e &= \sum_{j=1}^{\infty} \beta^j(\bar{w}_t^e - \bar{w}) + \sum_{j=1}^{\infty} \beta^j \tilde{w}_{t+j} \\
&= \frac{\beta}{1-\beta}(\bar{w}_t^e - \bar{w}) + \sum_{j=1}^{\infty} \beta^j \begin{pmatrix} a_{wk} & a_{wv} \end{pmatrix} \tilde{B}^j \tilde{x}_t \\
&= \frac{\beta}{1-\beta}(\bar{w}_t^e - \bar{w}) + \begin{pmatrix} a_{wk} & a_{wv} \end{pmatrix} \beta\tilde{B}(I - \beta\tilde{B})^{-1}\tilde{x}_t
\end{aligned}$$

Finally, we give details concerning the initialization of the parameters under RLS learning discussed in Section 3.1. The initial values of all parameter estimates are set to the initial steady state values under RE, i.e.,

$$\phi_{k,0} = \begin{pmatrix} b_{k,0} \\ a_{kk,0} \\ a_{kv,0} \end{pmatrix} = \begin{pmatrix} (1-\bar{\lambda}_2)\bar{k} \\ \bar{\lambda}_2 \\ \bar{f}_{kv} \end{pmatrix},$$

$$\phi_{w,0} = \begin{pmatrix} b_{w,0} \\ a_{wk,0} \\ a_{wv,0} \end{pmatrix} = \begin{pmatrix} \bar{w} - \bar{f}_{wk}\bar{k} \\ \bar{f}_{wk} \\ \bar{f}_{wv} \end{pmatrix},$$

$$\phi_{rk,0} = \begin{pmatrix} b_{r,0} \\ a_{rk,0} \\ a_{rv,0} \end{pmatrix} = \begin{pmatrix} \bar{r}_k - \bar{f}_{rk}\bar{k} \\ \bar{f}_{rk} \\ \bar{f}_{rv} \end{pmatrix}.$$

We also initialize the R matrix at the initial steady state. Define the variance/covariance matrix of $\begin{pmatrix} \hat{k}_t \\ \hat{v}_t \end{pmatrix}$ as

$$Cov(k,v) = \begin{pmatrix} \bar{\sigma}_k^2 & \bar{\sigma}_{kv} \\ \bar{\sigma}_{kv} & \bar{\sigma}_v^2 \end{pmatrix}$$

where $\bar{\sigma}_k^2, \sigma_v^2 \, (= (1-\rho^2)^{-1}\sigma_u^2)$ are the variances of the steady state capital and technology shock, and $\bar{\sigma}_{kv}$ is the covariance between capital and the shock v_t in the initial steady state. Using standard techniques we can obtain these variances using equations (25) and (26)[33]

$$vec(Cov(k,v)) = (I - B \otimes B)^{-1} vec(\Omega_{kv}),$$
$$\Omega_{kv} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_u^2 \end{pmatrix}.$$

so that $\bar{\sigma}_k^2, \bar{\sigma}_{kv}$, and σ_v^2 are given by the first, second, and fourth elements of $vec(Cov(k,v))$. The second moment matrix of z_t can then be initialized as

$$\bar{R} = \begin{pmatrix} 1 & \bar{k} & 0 \\ \bar{k} & \bar{k}^2 + \bar{\sigma}_k^2 & \bar{\sigma}_{kv} \\ 0 & \bar{\sigma}_{kv} & \sigma_v^2 \end{pmatrix}.$$

which gives the starting point for the algorithm for RLS learning.

B   Details of RE solution with policy change

We obtain the RE solution under a policy change as in Ljungqvist and Sargent (2004) p. 352, to get

$$U_c(c_t, n_t) = \beta E_t[U_c(c_{t+1}, n_{t+1})\{1 + (r_{k,t+1} - \delta)\}], \tag{49}$$

$$\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = -w_t = -(1-\alpha)v_t(\frac{k_t}{n_t})^\alpha, \tag{50}$$

$$r_{k,t+1} = \alpha v_t(\frac{n_{t+1}}{k_{t+1}})^{1-\alpha}, \tag{51}$$

$$c_t = v_t k_t^\alpha n_t^{1-\alpha} + (1-\delta)k_t - g_t - k_{t+1}. \tag{52}$$

We have for the utility function (2)

$$U_c(c_t, n_t) = c_t^{-\sigma}, U_n(c_t, n_t) = -\zeta(1 - n_t)^{-\epsilon}$$

and using these, (50) simplifies to

$$\frac{\zeta c_t^{-\sigma}}{(1 - n_t)^\epsilon} = (1-\alpha)v_t(\frac{k_t}{n_t})^\alpha.$$

---

[33] Here *vec* denotes the operator that stacks the columns of a matrix into a vector.

Using (52) to eliminate consumption we get

$$\zeta(v_t k_t^\alpha n_t^{1-\alpha} + (1-\delta)k_t - g_t - k_{t+1})^\sigma - (1-\alpha)v_t(\frac{k_t}{n_t})^\alpha(1-n_t)^\epsilon = 0. \quad (53)$$

Under policy changes, this (and all subsequent) equations will be linearized around the final steady state.[34] Linearizing (53) we get

$$0 = G_{g0}(g_t - \bar{g}) + G_{k0}(k_t - \bar{k}) + G_{k1}(k_{t+1} - \bar{k}) + G_{n0}(n_t - \bar{n}) + G_{v0}(v_t - \bar{v}). \quad (54)$$

G_{k0} denotes the partial derivatives evaluated for capital at the current time period t (e.g. k_t) and G_{k1} denotes the partial derivatives evaluated for capital at next period $t+1$ (e.g. k_{t+1}) etc. At the steady state these derivatives are

$$
\begin{aligned}
G_{k0} &= \zeta\sigma\bar{c}^{\sigma-1}(\bar{r}_k + 1 - \delta) - (1-\alpha)\bar{r}_k\frac{(1-\bar{n})^\epsilon}{\bar{n}}, \\
G_{k1} &= -\zeta\sigma\bar{c}^{\sigma-1}, \\
G_{n0} &= \zeta\sigma\bar{c}^{\sigma-1}\bar{w} + (1-\alpha)\bar{v}\bar{k}^\alpha\{\alpha\bar{n}^{-\alpha-1}(1-\bar{n})^\epsilon + \epsilon\bar{n}^{-\alpha}(1-\bar{n})^{\epsilon-1}\}, \\
G_{v0} &= \zeta\sigma\bar{c}^{\sigma-1}\bar{k}^\alpha\bar{n}^{1-\alpha} - (1-\alpha)\bar{k}^\alpha\bar{n}^{-\alpha}(1-\bar{n})^\epsilon, \\
G_{g0} &= -\zeta\sigma\bar{c}^{\sigma-1}, \\
G_{\tau n0} &= \bar{w}(1-\bar{n})^\epsilon.
\end{aligned}
$$

(49) on using (52) becomes

$$
\begin{aligned}
&(v_t k_t^\alpha n_t^{1-\alpha} + (1-\delta)k_t - g_t - k_{t+1})^{-\sigma} \\
= \ &\beta E_t[\{v_{t+1}k_{t+1}^\alpha n_{t+1}^{1-\alpha} + (1-\delta)k_{t+1} - g_{t+1} - k_{t+2}\}^{-\sigma} \\
&\{1 + (\alpha v_{t+1}(\frac{n_{t+1}}{k_{t+1}})^{1-\alpha} - \delta)\}].
\end{aligned}
$$

We can linearize this to obtain a solution of the form

$$
\begin{aligned}
0 = \ &H_{k0}(k_t - \bar{k}) + H_{k1}(k_{t+1} - \bar{k}) + H_{k2}E_t(k_{t+2} - \bar{k}) + H_{n0}(n_t - \bar{n}) + \\
&H_{n1}E_t(n_{t+1} - \bar{n}) + H_{g0}(g_t - \bar{g}) + H_{g1}E_t(g_{t+1} - \bar{g}) + H_{v0}(v_t - \bar{v}) + \\
&H_{v1}E_t(v_{t+1} - \bar{v}). \quad (55)
\end{aligned}
$$

---

[34] For convenience we now use $\bar{g}, \bar{k}$, etc., to denote the final steady state. Of course, for temporary policy changes, the initial and final steady states are the same.

Define the H coefficients here.

$$
\begin{aligned}
H_{k0} &= -\sigma \bar{c}^{-\sigma-1}(\bar{r}_k + 1 - \delta), \\
H_{k1} &= \sigma \bar{c}^{-\sigma-1} + \beta \sigma \bar{c}^{-\sigma-1}(\bar{r}_k + 1 - \delta)\bar{r} + \beta(1-\alpha)\bar{c}^{-\sigma}(\alpha \bar{v} \bar{n}^{1-\alpha} \bar{k}^{\alpha-2}), \\
H_{k2} &= H_{g1} = -\beta \sigma \bar{c}^{-\sigma-1}\bar{r} = -\sigma \bar{c}^{-\sigma-1}, \\
H_{n0} &= -\sigma \bar{c}^{-\sigma-1}\bar{w}, \\
H_{n1} &= \beta \sigma \bar{c}^{-\sigma-1}\bar{w}\bar{r} - \beta \bar{c}^{-\sigma}(\alpha(1-\alpha)\bar{v}\bar{n}^{-\alpha}\bar{k}^{\alpha-1}), \\
H_{g0} &= \sigma \bar{c}^{-\sigma-1}, \\
H_{v0} &= -\sigma \bar{c}^{-\sigma-1}\bar{k}^{\alpha}\bar{n}^{1-\alpha}, \\
H_{v1} &= \beta \sigma \bar{c}^{-\sigma-1}\bar{k}^{\alpha}\bar{n}^{1-\alpha}\bar{r} - \beta \bar{r}_k \bar{c}^{-\sigma}, \\
H_{\tau k1} &= \beta \bar{c}^{-\sigma}(\bar{r}_k - \delta).
\end{aligned}
$$

From (54) we get

$$
n_t - \bar{n} = -G_{n0}^{-1}[G_{g0}(g_t - \bar{g}) + G_{k0}(k_t - \bar{k}) + G_{k1}(k_{t+1} - \bar{k}) + G_{v0}(v_t - \bar{v})] \quad (56)
$$

which implies

$$
\begin{aligned}
E_t[n_{t+1} - \bar{n}] &= -G_{n0}^{-1}[G_{g0}E_t(g_{t+1} - \bar{g}) + G_{k0}(k_{t+1} - \bar{k}) + \\
&\quad G_{k1}E_t(k_{t+2} - \bar{k}) + G_{v0}E_t(v_{t+1} - \bar{v})]. \quad (57)
\end{aligned}
$$

k_{t+1} is known in period t, so there is no expectation before this term.

(56) and (57) are substituted in (55) to eliminate n_t and n_{t+1} which gives an equation involving only the endogenous variable capital stock

$$
\begin{aligned}
0 &= J_{k0}(k_t - \bar{k}) + J_{k1}(k_{t+1} - \bar{k}) + J_{k2}E_t(k_{t+2} - \bar{k}) \quad (58) \\
&\quad + J_{g0}(g_t - \bar{g}) + J_{g1}E_t(g_{t+1} - \bar{g}) + J_{v0}(v_t - \bar{v}) + J_{v1}E_t(v_{t+1} - \bar{v}).
\end{aligned}
$$

Define the coefficients J below

$$
\begin{aligned}
J_{k0} &= H_{k0} - H_{n0}G_{n0}^{-1}G_{k0}, \\
J_{k1} &= H_{k1} - H_{n0}G_{n0}^{-1}G_{k1} - H_{n1}G_{n0}^{-1}G_{k0}, \\
J_{k2} &= H_{k2} - H_{n1}G_{n0}^{-1}G_{k1}, \\
J_{g0} &= H_{g0} - H_{n0}G_{n0}^{-1}G_{g0}, \\
J_{g1} &= H_{g1} - H_{n1}G_{n0}^{-1}G_{g0}, \\
J_{v0} &= H_{v0} - H_{n0}G_{n0}^{-1}G_{v0}, \\
J_{v1} &= H_{v1} - H_{n1}G_{n0}^{-1}G_{v0}, \\
H_{\tau n0} &= -H_{n0}G_{n0}^{-1}G_{\tau n0}, \\
H_{\tau n1} &= -H_{n1}G_{n0}^{-1}G_{\tau n0},
\end{aligned}
$$

and $H_{\tau k1}$ is defined after (55).

(58) is a second order difference equation for k_t in terms of the exogenous policy variables g_t and the shock v_t with a condition for initial capital stock k_0. The linear approximation to the solution for the equilibrium k_t sequence is obtained by solving the stable root backward and the unstable root forward (see Ljungqvist and Sargent (2004), Chapter 11 for the details). We finally write (58) as

$$E_t(k_{t+2} - \bar{k}) + A_{k1}(k_{t+1} - \bar{k}) + A_{k0}(k_t - \bar{k})$$
$$= A_{g0}(g_t - \bar{g}) + A_{g1}E_t(g_{t+1} - \bar{g}) + A_{v0}(v_t - \bar{v}) + A_{v1}E_t(v_{t+1} - \bar{v}).(59)$$

where

$$A_{k1} = J_{k1}J_{k2}^{-1}, A_{k0} = J_{k0}J_{k2}^{-1},$$
$$A_{g0} = -J_{g0}J_{k2}^{-1}, A_{g1} = -J_{g1}J_{k2}^{-1},$$
$$A_{v0} = -J_{v0}J_{k2}^{-1}, A_{v1} = -J_{v1}J_{k2}^{-1},$$
$$A_{\tau k1} = -H_{\tau k1}J_{k2}^{-1},$$
$$A_{\tau n0} = -H_{\tau n0}J_{k2}^{-1}, A_{\tau n0} = -H_{\tau n1}J_{k2}^{-1}.$$

For this model, one can show that

$$A_{g0} = \frac{1}{1 + \frac{n_1}{d_1}}, A_{g1} = -1,$$
$$n_1 = \alpha\zeta\bar{c}^\sigma\bar{k}^\alpha(1 - \bar{n})^{1-\epsilon}(\frac{\bar{n}}{\bar{k}})^{1+\alpha}\bar{n}^{-\alpha} > 0,$$
$$d_1 = \beta^{-1}\{(1 - \bar{n})\alpha + \bar{n}\epsilon\} > 0.$$

so that $0 < A_{g0} < 1$ and hence $A_{g1} + A_{g0} < 0$.

The government spending process implies $E_t g_{t+1} = \bar{g}_{t+1}$. Also given the process for v_t

$$E_t(v_{t+j} - \bar{v}) = \rho^j(v_t - \bar{v}).$$

Assuming

$$E_t k_{t+2} = k_{t+2} + \eta_{t+1}; \ E_t\eta_{t+1} = 0,$$

(59) becomes

$$(k_{t+2} - \bar{k}) + A_{k1}(k_{t+1} - \bar{k}) + A_{k0}(k_t - \bar{k})$$
$$= A_{g0}(g_t - \bar{g}) + A_{g1}(\bar{g}_{t+1} - \bar{g}) + A_{v0}(v_t - \bar{v}) + A_{v1}\rho(v_t - \bar{v}) - \eta_{t+1}(60)$$

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The stochastic process (60) can be solved using the techniques in Sargent (1987), p. 393. This yields

$$
\begin{aligned}
k_{t+1} - \bar{k} &= \lambda_2(k_t - \bar{k}) - \lambda_2 A_{k0}^{-1} \sum_{j=0}^{\infty} \lambda_1^{-j} E_t[A_{g0}(g_{t+j} - \bar{g}) + A_{g1}(\bar{g}_{t+j+1} - \bar{g}) + \\
&\quad (A_{v0} + \rho A_{v1})(v_{t+j} - \bar{v}) - \eta_{t+j+1}] \\
&= \lambda_2(k_t - \bar{k}) - \lambda_2 A_{k0}^{-1} \sum_{j=0}^{\infty} \lambda_1^{-j} E_t[A_{g0}(g_{t+j} - \bar{g}) + A_{g1}(\bar{g}_{t+j+1} - \bar{g}) + \\
&\quad (A_{v0} + \rho A_{v1})(v_{t+j} - \bar{v})].
\end{aligned}
$$

This finally gives the stochastic process for capital (using the *hatted* values for deviations from RE steady state)

$$
\hat{k}_{t+1} = \lambda_2 \hat{k}_t - \lambda_2 A_{k0}^{-1} \sum_{j=0}^{\infty} \lambda_1^{-j} E_t[A_{g0}(g_{t+j} - \bar{g}) + A_{g1}(\bar{g}_{t+j+1} - \bar{g}) + (A_{v0} + \rho A_{v1})\hat{v}_{t+j}.
$$

(61)

Here λ_1, λ_2 are given by the roots of the quadratic equation (see Ljungqvist and Sargent (2004) p. 345)

$$
\begin{aligned}
\lambda^2 + A_{k1}\lambda + A_{k0} &= 0, \\
\lambda_1 \lambda_2 &= A_{k0},
\end{aligned}
$$

where it is assumed that $\lambda_1 > 1$ and $0 < \lambda_2 < 1$.

We now specialize the analysis and summarize the details for obtaining a linear approximation to the equilibrium RE capital sequence under a permanent policy change of the type considered in the paper.[35] The capital sequence is given by (61) i.e.

$$
\hat{k}_{t+1} = \lambda_2 \hat{k}_t - \lambda_2 A_{k0}^{-1}(S_g(t) + S_v(t)). \tag{62}
$$

where

$$
S_g(t) \equiv \sum_{j=0}^{\infty} \lambda_1^{-j} E_t\{A_{g0}(\bar{g}_{t+j} - \bar{g}) + A_{g1}(\bar{g}_{t+j+1} - \bar{g})\}, \tag{63}
$$

$$
S_v(t) \equiv \sum_{j=0}^{\infty} \lambda_1^{-j} E_t(A_{v0} + \rho A_{v1})\hat{v}_{t+j} = (A_{v0} + \rho A_{v1}) \sum_{j=0}^{\infty} \lambda_1^{-j} \rho^j \hat{v}_t. \tag{64}
$$

---

[35]The summations below assume $T_p \geq 2$. If $T_p = 1$, then the policy change is immediate and is termed a surprise (from the point of view of the agents) change which is the benchmark case considered in the paper. Equation (62) still gives the dynamics of capital for the surprise permanent policy change by setting $S_g(t) \equiv 0$.

We have

$$\bar{g}_t - \bar{g} = \begin{cases} \bar{g}' - \bar{g}, \ 1 \le t < T_p, \\ 0, \ t \ge T_p, \end{cases}$$

$$\bar{g}_{t+j} - \bar{g} = \begin{cases} \bar{g}' - \bar{g}, \ t + j < T_p, \\ 0, \ t + j \ge T_p. \end{cases}$$

One can show that $A_{g1} = -1$ in (63) which gives us

$$A_{g0}(\bar{g}_{t+j} - \bar{g}) + A_{g1}(\bar{g}_{t+j+1} - \bar{g}) = \begin{cases} (A_{g0} - 1)(\bar{g}' - \bar{g}), \ t + j \le T_p - 2, \\ A_{g0}(\bar{g}' - \bar{g}), \ t + j = T_p - 1, \\ 0, \ t + j \ge T_p. \end{cases}$$

We first compute (64). For all $t \ge 1$, we have

$$S_v(t) \equiv (A_{v0} + \rho A_{v1})\hat{v}_t \sum_{j=0}^{\infty} \lambda_1^{-j} \rho^j = \frac{(A_{v0} + \rho A_{v1})\hat{v}_t}{1 - \frac{\rho}{\lambda_1}}. \tag{65}$$

Then we compute (63). If $1 \le t \le T_p - 2$, we have

$$\begin{aligned} S_g(t) &\equiv \sum_{j=0}^{T_p-2-t} \lambda_1^{-j}(A_{g0} - 1)(\bar{g}' - \bar{g}) + \lambda_1^{-(T_p-1-t)}A_{g0}(\bar{g}' - \bar{g}) \\ &= \left((A_{g0} - 1)\frac{1 - \lambda_1^{-(T_p-1-t)}}{1 - \lambda_1^{-1}} + \lambda_1^{-(T_p-1-t)}A_{g0} \right) (\bar{g}' - \bar{g}) \end{aligned}$$

and if $t = T_p - 1$, then we have

$$S_g(t) \equiv A_{g0}(\bar{g}' - \bar{g})$$

and $S_g(t) = 0$ for $t \ge T_p$.

To summarize,

$$S_g(t) = \begin{cases} \left((A_{g0} - 1)\frac{1 - \lambda_1^{-(T_p-1-t)}}{1 - \lambda_1^{-1}} + \lambda_1^{-(T_p-1-t)}A_{g0} \right) (\bar{g}' - \bar{g}), \ 1 \le t \le T_p - 1, \\ 0, \ t \ge T_p. \end{cases}$$

$$\tag{66}$$

Using the formulas in (65) and (66), we can compute the linearized capital dynamics under RE from (62) for a permanent change in government spending under a balanced budget. This is the dynamics which we compare with the learning dynamics.

TABLES

Impact Effects	Surp RE	Surp RLS	$T_p = 5$ RE	$T_p = 5$ RLS	$T_p = 29$ RE	$T_p = 29$ RLS
c_t	-0.90	-0.34	-0.66	-0.31	-0.10	-0.22
n_t	1.47	0.55	1.08	0.51	0.17	0.36
i_t	2.49	-2.07	5.38	2.55	0.86	1.78
y_t	0.98	0.37	0.72	0.34	0.12	0.24
k_t/n_t	-1.45	-0.55	-1.07	-0.51	-0.17	-0.36
w_t	-0.49	-0.18	-0.36	-0.17	-0.06	-0.12
r_t	0.04	0.015	0.03	0.014	0.005	0.009

Table 1: Impact effects on key variables of a permanent policy change under rational expectations (RE) and under learning (RLS) for the surprise and announced changes.

Impact Effects	RE	RLS
c_t	-0.41	-0.04
n_t	0.68	0.063
i_t	-1.46	-4.51
y_t	0.45	0.04
k_t/n_t	-0.67	-0.063
w_t	-0.22	-0.02
r_t	0.02	0.002

Table 2: Impact effects on key variables of the temporary policy change under rational expectations (RE) and under learning (RLS)

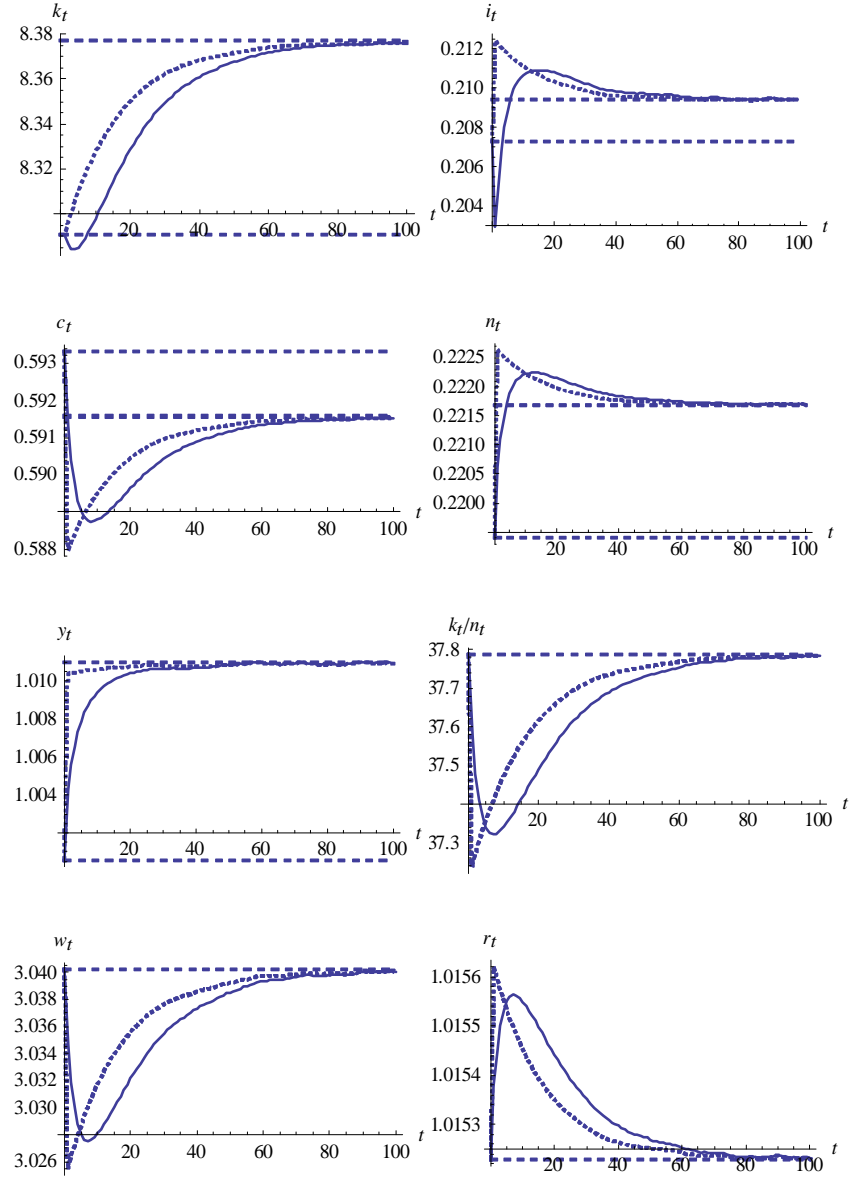Figure 1: Dynamic paths for a surprise permanent increase in government spending. The solid lines are the learning paths while the dashed lines are the RE paths. The horizontal dashed lines depict the old and the new steady states. Mean paths over 20,000 simulations.
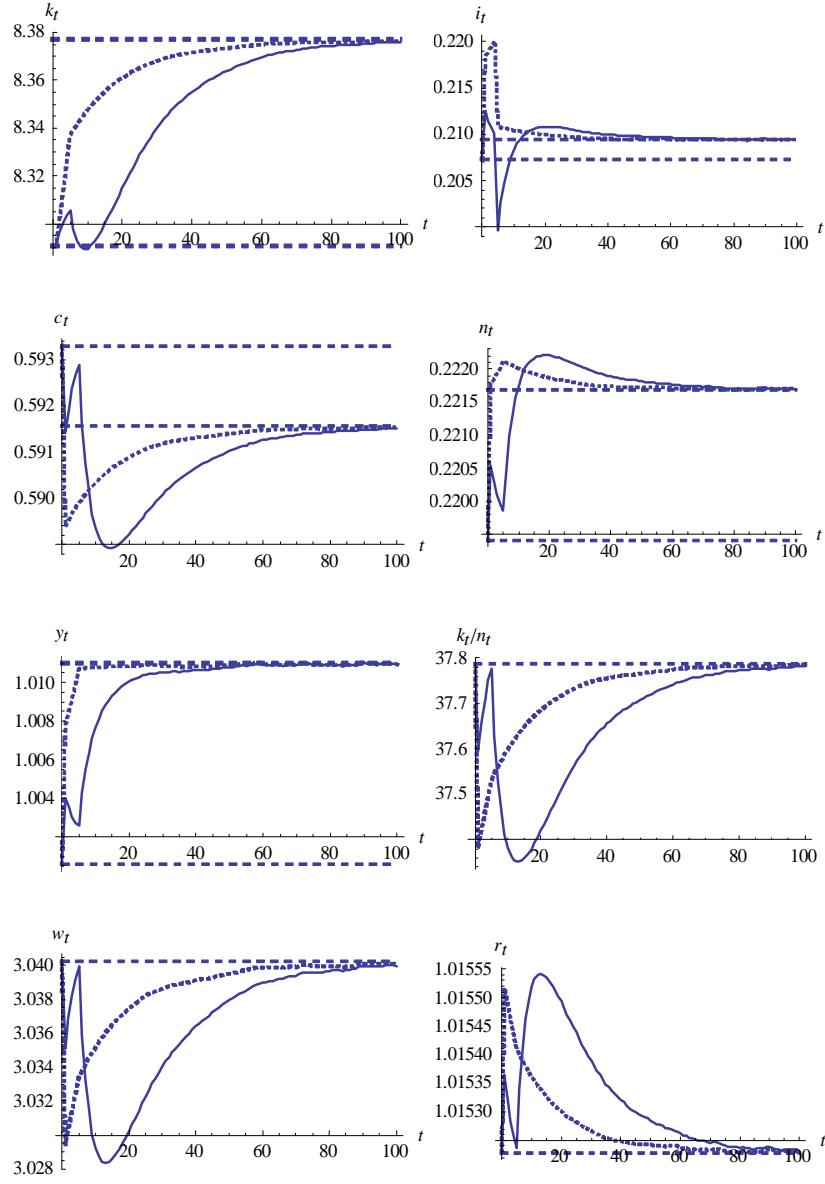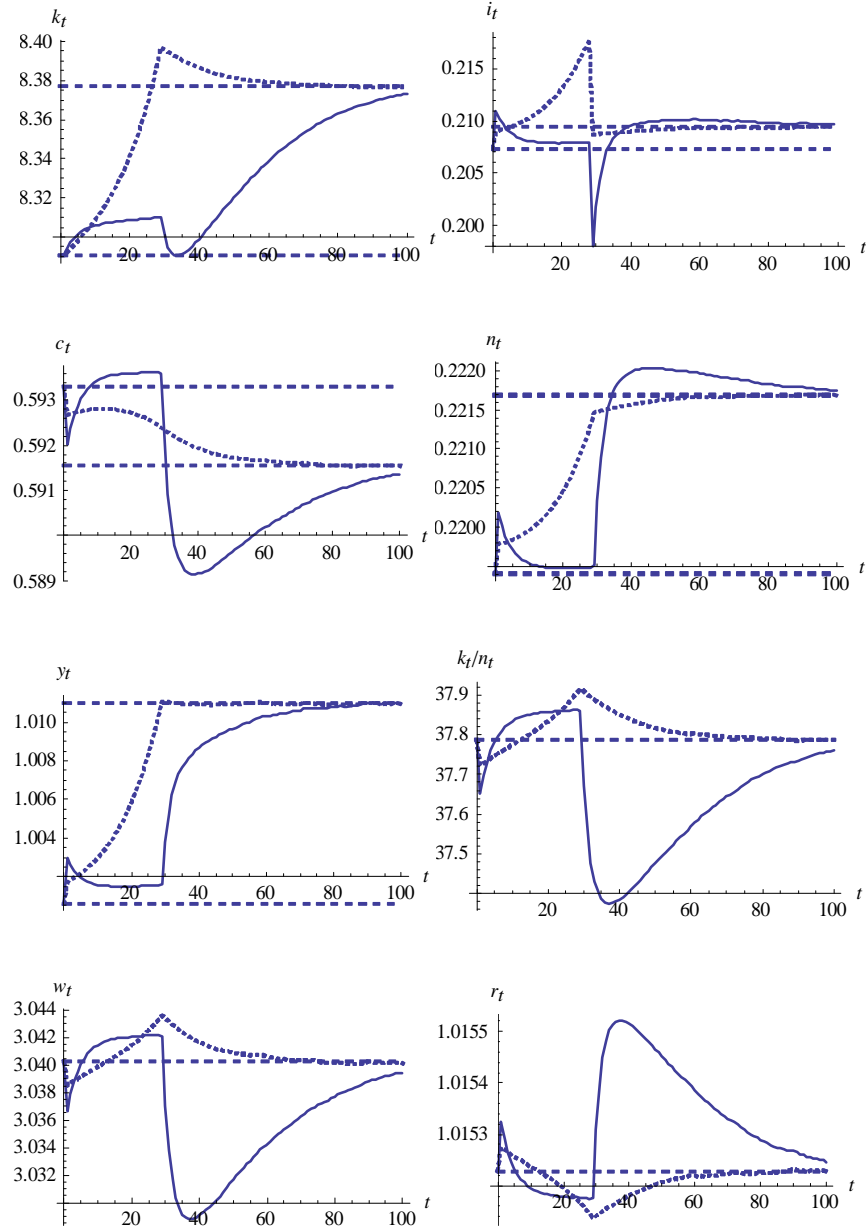
47

Figure 2: Dynamic paths for an anticipated permanent increase in government spending taking place in period 5. The solid lines are the learning paths while the dashed lines are the RE paths. The horizontal dashed lines depict the old and new steady states. Mean paths over 20,000 simulations.

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Figure 3: Dynamic paths for an anticipated permanent increase in government spending taking place in period 29. The solid lines are the learning paths while the dashed lines are the RE paths. The horizontal dashed lines depict the old and new steady states. Mean paths over 20,000 simulations.
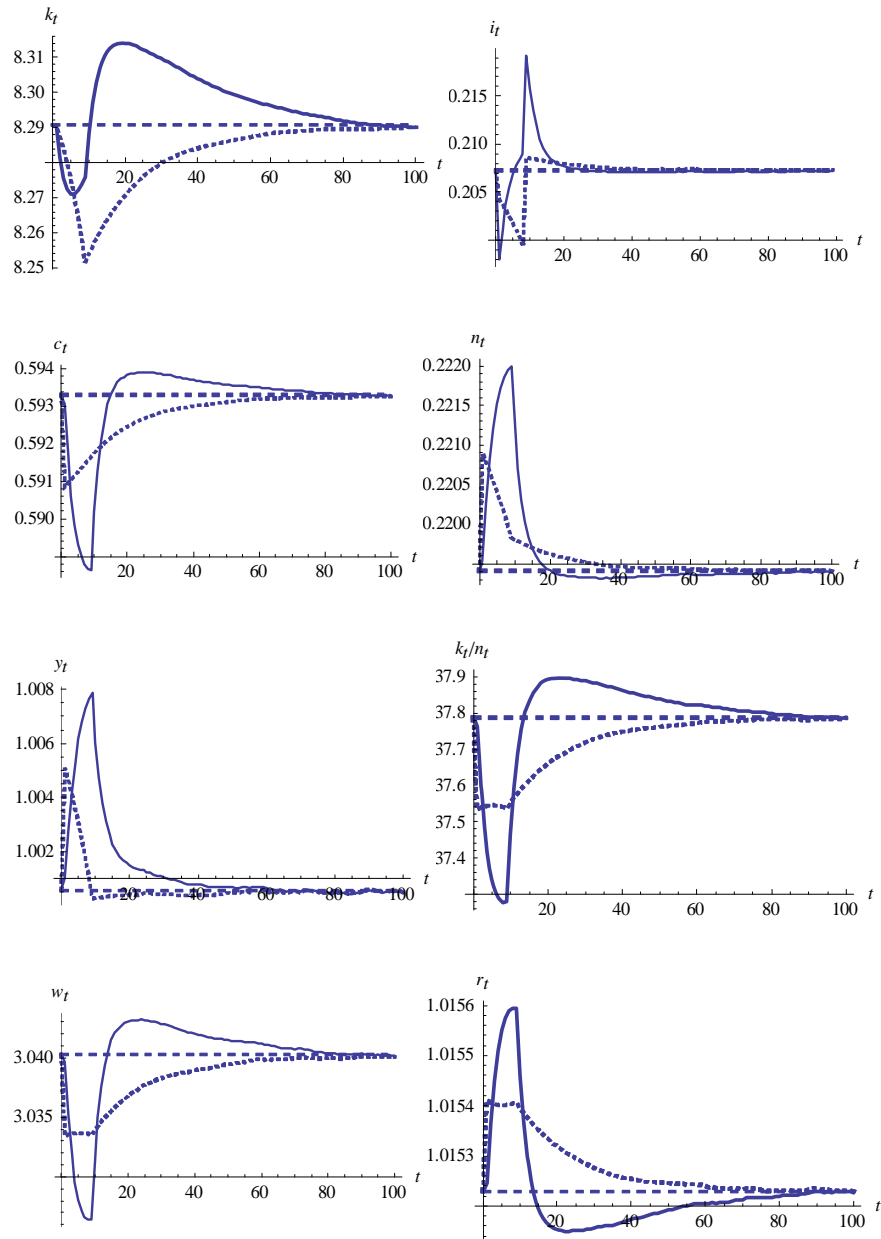
Figure 4: Dynamic paths for a surprise temporary increase in government spending that lasts for two years. The solid lines are the learning paths while the dashed lines are the RE paths. The horizontal dashed lines depict the (unchanged) steady state. Mean paths over 20,000 simulations.
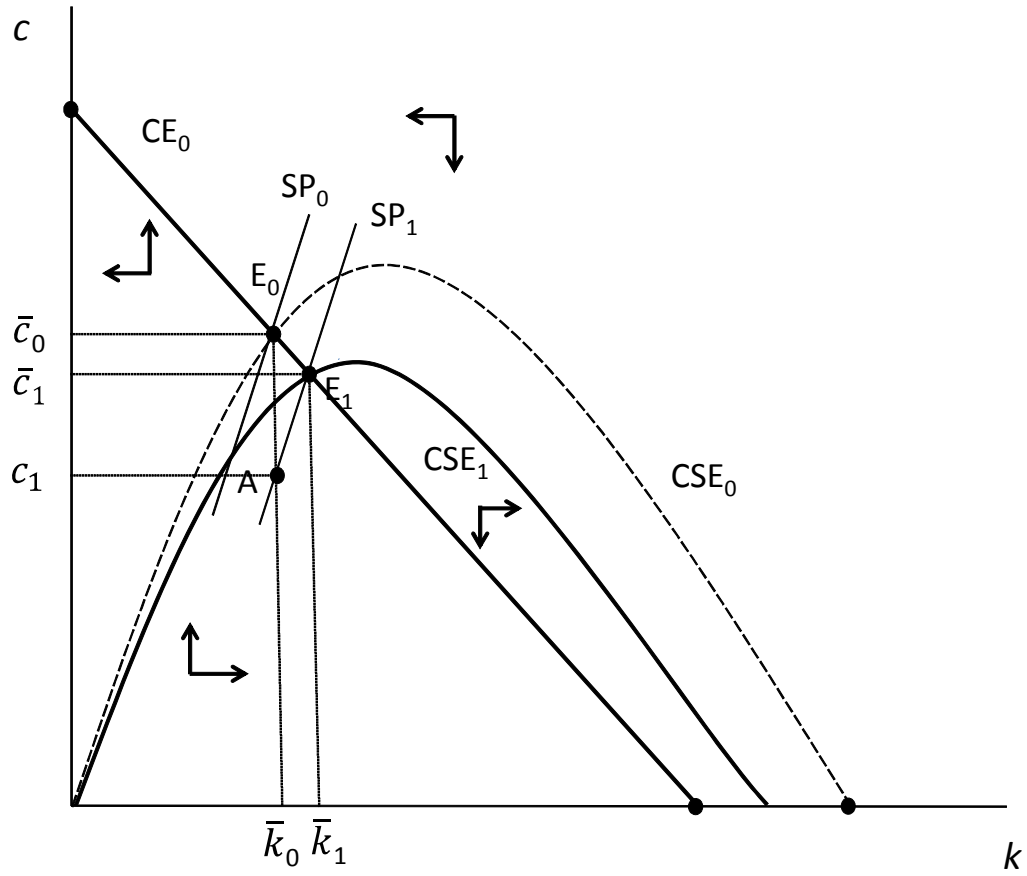
Figure 5: Effects under RE of fiscal policy in deterministic RBC model; based on Heijdra (2009), Figures 15.1-15.2.