# Expectations, Learning and Macroeconomic Policy

George W. Evans (University of Oregon and University of St. Andrews)

26 March 2010

J. C. Trichet: "Understanding expectations formation as a process underscores the strategic interdependence that exists between expectations formation and economics." (Zolotas lecture, 2005)

Ben S. Bernanke: "In sum, many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well." (NBER, July 2007).

# Outline

## Morning Session

Lecture 1: Review of techniques: E- stability & least squares learning
— Muth model. Multivariate Models. Application to New Keynesian model

Lecture 2: Optimal monetary policy and learning
— Optimal policy with learning. Policy with perpetual learning

## Afternoon session

Lecture 3: (i) Recurrent hyperinflations and learning
— (ii) Dynamic Predictor Selection and Endogenous Volatility

Lecture 4: Liquidity traps, learning and stagnation.

# Lecture 1: Review of techniques: E- stability & least squares learning

## Introduction

- Since Lucas (1972, 1976) and Sargent (1973) the standard assumption in the theory of economic policy is rational expectations (RE). This assumes, for both private agents and policymakers,

  - knowledge of the correct form of the model

  - knowledge of all parameters, and

  - knowledge that other agents are rational & know that others know . . . .

- RE assumes too much and is therefore implausible. We need an appropriate model of **bounded rationality** What form should this take?

- My general answer is given by the **Cognitive Consistency Principle**: economic agents should be about as smart as (good) economists. Economists forecast economic variables using. econometric techniques, so a good starting point: **model agents as "econometricians."**

- Neither private agents nor economists at central banks do know the true model. Instead economists formulate and estimate models. These models are re-estimated and possibly reformulated as new data becomes available. Economists engage in *processes of learning* about the economy.

- These processes of learning create **new tasks for macroeconomic policy**

# Starting Points:

- The private sector is forward-looking (e.g. investment, savings decisions).

- Forecasts (including private forecasts) of future inflation and output have a key role in monetary policy:

  1. Empirical evidence e.g. by (Clarida, Gali and Gertler 1998).

  2. Bank of England and ECB discuss private forecasts in addition to internal macro projections.

- Private agents and/or policy-makers are learning.

# Fundamental Problems:

- There may be multiple equilibria, depending on interest rate policy.

- Policy may lead to expectational instability if expectations are not always rational.

These problems necessitate careful design of interest rate rule: Bullard and Mitra (2002), Evans and Honkapohja (2003a, 2006).
=> Survey papers: Evans and Honkapohja (2003b, 2009), Bullard (2006).

- **Central message**: Policy should facilitate learning by private agents.

# A Muth/Lucas-type Model

Before looking at issues of policy, we review the learning approach and the E-stability technique, beginning with a simple univariate reduced form:

$$p_t = \mu + \alpha E^*_{t-1} p_t + \delta' w_{t-1} + \eta_t. \qquad \text{(RF)}$$

$E^*_{t-1} p_t$ denotes expectations of $p_t$ formed at $t-1$, $w_{t-1}$ is a vector of exogenous shocks observed at $t-1$, $\eta_t$ is an exogenous unobserved $iid$ shock, and $w_t$ follows an exogenous stationary VAR process.

**Muth example**. Demand and supply equations:

$$\begin{aligned} d_t &= m_I - m_p p_t + v_{1t} \\ s_t &= r_I + r_p E^*_{t-1} p_t + r_w' w_{t-1} + v_{2t}, \end{aligned}$$

Assuming market clearing, $s_t = d_t$, yields (RF) where $\mu = (m_I - r_I)/m_p$, $\delta = -m_p^{-1} r_w$ and $\alpha = -r_p/m_p$ and $\eta_t = (v_{1t} - v_{2t})/m_p$.
Note that $\alpha < 0$ if $m_p, r_p > 0$.

**Lucas-type Monetary model.** A simple Lucas-type model:

$$q_t = \bar{q} + \pi(p_t - E^*_{t-1}p_t) + \zeta_t,$$

where $\pi > 0$, and aggregate demand function is given by

$$\begin{aligned} m_t + v_t &= p_t + q_t, \\ v_t &= \mu + \gamma' w_{t-1} + \xi_t, \\ m_t &= \bar{m} + u_t + \rho' w_{t-1}. \end{aligned}$$

Here $w_{t-1}$ are exogenous observables. The reduced form is again

$$\begin{aligned} p_t &= \mu + \alpha E^*_{t-1}p_t + \delta' w_{t-1} + \eta_t, \text{ where} \\ \alpha &= \pi(1+\pi)^{-1} \text{ and } \delta = (1+\pi)^{-1}(\rho + \gamma) \end{aligned}$$

In this example $0 < \alpha < 1$.

# RATIONAL EXPECTATIONS

First consider the model under RE:

$$p_t = \mu + \alpha E_{t-1} p_t + \delta' w_{t-1} + \eta_t.$$

The model has a unique RE solution since

$$\begin{aligned} E_{t-1} p_t &= \mu + \alpha E_{t-1} p_t + \delta' w_{t-1} \longrightarrow \\ E_{t-1} p_t &= (1-\alpha)^{-1} \delta + (1-\alpha)^{-1} \delta' w_{t-1} \end{aligned}$$

Hence the unique REE is

$$\begin{aligned} p_t &= \bar{a} + \bar{b}' w_{t-1} + \eta_t, \text{ where} \\ \bar{a} &= (1-\alpha)^{-1} \delta \text{ and } \bar{b} = (1-\alpha)^{-1} \delta. \end{aligned}$$

# LEAST-SQUARES LEARNING

Under learning, agents have the beliefs or perceived law of motion (PLM)

$$p_t = a + bw_{t-1} + \eta_t,$$

but $a, b$ are unknown. At the end of time $t-1$ they estimate $a, b$ by LS (Least Squares) using data through $t-1$, i.e. Then they use the estimated coefficients to make forecasts $E^*_{t-1}p_t$. Here the standard least squares (LS) formula are

$$\begin{aligned} \begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} &= \left(\sum_{i=1}^{t-1} z_{i-1}z'_{i-1}\right)^{-1} \left(\sum_{i=1}^{t-1} z_{i-1}p_i\right), \text{ where} \\ z'_i &= \begin{pmatrix} 1 & w'_i \end{pmatrix}. \end{aligned}$$

The timing is:

– End of $t-1$: $w_{t-1}$ and $p_{t-1}$ observed. Agents update estimates of $a, b$ to $a_{t-1}, b_{t-1}$ using $\{p_s, w_{s-1}\}_{s=1}^{t-1}$. Agents make forecasts

$$E_{t-1}^* p_t = a_{t-1} + b_{t-1}' w_{t-1}.$$

– Period $t$: (i) The shock $\eta_t$ is realized, $p_t$ is determined as

$$p_t = (\mu + \alpha a_{t-1}) + (\delta + \alpha b_{t-1})' w_{t-1} + \eta_t$$

and $w_t$ is realized. (ii) agents update estimates to $a_t, b_t$ using $\{p_s, w_{s-1}\}_{s=1}^{t}$ and make forecasts

$$E_t^* p_{t+1} = a_t + b_t' w_t.$$

The system under learning is a fully specified dynamic system under learning.

Question: Will $(a_t, b_t) \longrightarrow (\bar{a}, \bar{b})$ as $t \longrightarrow \infty$?

LS updating and the system can be set up recursively. Letting

$$\phi_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix} \text{ and } z_t = \begin{pmatrix} 1 \\ w_t \end{pmatrix}$$

the system is:

$$\begin{aligned}
E^*_{t-1}p_t &= a_{t-1} + b'_{t-1}w_{t-1} \\
p_t &= \mu + \alpha E^*_{t-1}p_t + \delta' w_{t-1} + \eta_t, \\
\phi_t &= \phi_{t-1} + t^{-1}R_t^{-1}z_{t-1}(p_t - \phi'_{t-1}z_{t-1}) \\
R_t &= R_{t-1} + t^{-1}(z_{t-1}z'_{t-1} - R_{t-1}),
\end{aligned}$$

This recursive formulation is useful for (a) theoretical analysis, and (b) numerical simulations.

**Theorem:** Consider model (RF) with $E^*_{t-1}p_t = a_{t-1} + b'_{t-1}w_{t-1}$ and with $a_{t-1}, b_{t-1}$ updated over time using least-squares. If $\alpha < 1$ then $\begin{pmatrix} a_t \\ b_t \end{pmatrix} \to \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}$ with probability 1. If $\alpha > 1$ convergence occurs with probability 0.

Thus the REE is stable under LS learning both for Muth model ($\alpha < 0$) and Lucas model ($0 < \alpha < 1$)..

Example of an unstable REE: Muth model with $m_p < 0$ (Giffen good) and $|m_p| < r_p$.

# E-STABILITY

Proving the theorem is not easy. However, there is an easy way of deriving the stability condition $\alpha < 1$ that is quite general. Start with the PLM

$$p_t = a + b' w_{t-1} + \eta_t,$$

and consider what would happen if $(a, b)$ were fixed at some value possibly different from the RE values $(\bar{a}, \bar{b})$. The corresponding expectations are

$$E^*_{t-1} p_t = a + b' w_{t-1},$$

which would lead to the Actual Law of Motion (ALM)

$$p_t = \mu + \alpha(a + b' w_{t-1}) + \delta' w_{t-1} + \eta_t.$$

The implied ALM gives the mapping $T$: PLM $\longrightarrow$ ALM:

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \mu + \alpha a \\ \delta + \alpha b \end{pmatrix}.$$

The REE $\bar{a}, \bar{b}$ is a fixed point of $T$. Expectational-stability ("E-stability) is defined by the differential equation

$$\frac{d}{d\tau}\begin{pmatrix} a \\ b \end{pmatrix} = T\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix}.$$

Here $\tau$ denotes artificial or notional time. $\bar{a}, \bar{b}$ is said to be E-stable if it is stable under this differential equation.

In the current case the $T$-map is linear. Component by component we have

$$\begin{aligned} \frac{da}{d\tau} &= \mu + (\alpha - 1)a \\ \frac{db_i}{d\tau} &= \delta + (\alpha - 1)b_i \text{ for } i = 1, ..., p. \end{aligned}$$

$$\begin{aligned}\frac{da}{d\tau} &= \mu + (\alpha - 1)a \\ \frac{db_i}{d\tau} &= \delta + (\alpha - 1)b_i \text{ for } i = 1, ..., p.\end{aligned}$$

It follows that the REE is E-stable if and only if $\alpha < 1$. This is the stability condition, given in the theorem, for stability under LS learning.

Intuition: under LS learning the parameters $a_t, b_t$ are slowly adjusted, on average, in the direction of the corresponding ALM parameters.

Numerical simulation of learning in Muth model. $\mu = 5$, $\delta = 1$ and $\alpha = -0.5$. $w_t \overset{iid}{\sim} N(0, 1)$ and $\eta_t \overset{iid}{\sim} N(0, 1/4)$. Initial values $a_0 = 1, b_0 = 2$ and $R_0 = \text{eye}(2)$. Convergence to the REE $\bar{a} = 10/3$ and $\bar{b} = 2/3$ is rapid.

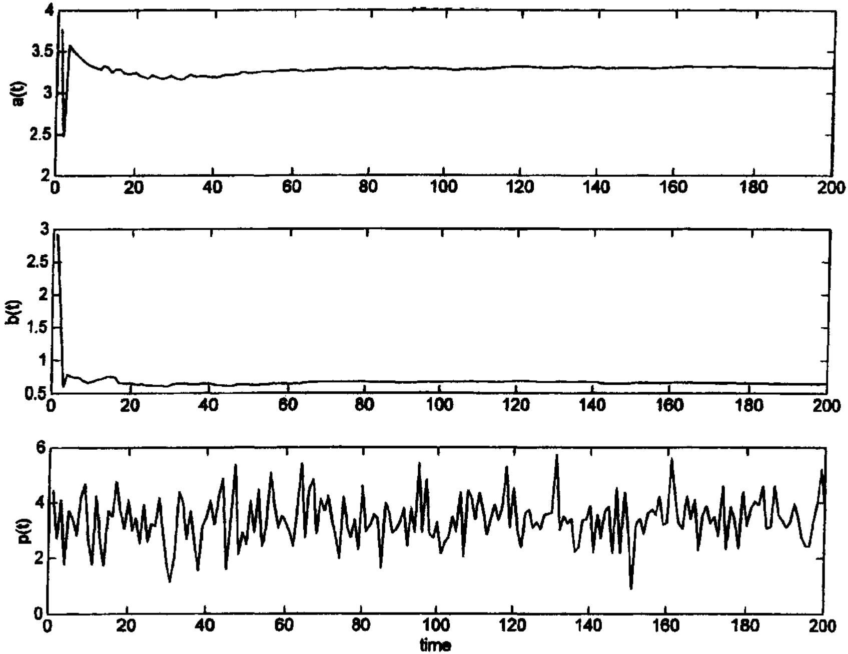



*Figure 2.1.*

A general definition of E-stability is a straightforward extension of the ex-

# The E-Stability Principle

– The E-stability technique works quite generally.

– To study convergence of LS learning to an REE, specify a PLM with parameters $\phi$. The PLM can be thought of as an econometric forecasting model. The REE is the PLM with $\phi = \bar{\phi}$.

– PLMs can take the form of ARMA or VARs or admit cycles or a dependence on sunspots.

– Compute the ALM for this PLM. This gives a map

$$\phi \rightarrow T(\phi),$$

with fixed point $\bar{\phi}$.

– E-stability is determined by local asymptotic stability of $\bar{\phi}$ under

$$\frac{d\phi}{d\tau} = T(\phi) - \phi.$$

The E-stability condition is that all eigenvalues of $DT(\bar{\phi})$ have real parts less than 1.

– The E-stability principle: E-stability governs local stability of an REE under LS and closely related learning rules.

– E-stability can be used as a selection criterion in models with multiple REE.

– These techniques can be applied to multivariate linearized models, and thus to RBC, OLG, New Keynesian and DSGE models.

# The New Keynesian Model

- Log-linearized New Keynesian model (CGG 1999, Woodford 2003 etc.).

1. "IS" equation (IS curve)

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t$$

2. the "New Phillips" equation (PC curve)

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t,$$

where $x_t$ =output gap, $\pi_t$ =inflation, $i_t$ = nominal interest rate. $E_t^* x_{t+1}$, $E_t^* \pi_{t+1}$ are expectations. Parameters $\varphi, \lambda > 0$ and $0 < \beta < 1$.

- Observable shocks follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \; F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

where $0 < |\mu|, |\rho| < 1$, and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$.

- Interest rate setting by a standard **Taylor rule,** e.g.

$$\begin{aligned} i_t &= \chi_\pi \pi_t + \chi_x x_t \text{ where } \chi_\pi, \chi_x > 0 \text{ or} \\ i_t &= \chi_\pi \pi_{t-1} + \chi_x x_{t-1} \text{ or} \\ i_t &= \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1} \end{aligned}$$

- Under learning we treat the IS and PC Euler equations as behavioral equations. Explicit infinite-horizon formations also have been studied: Preston (IJCM, 2005 and JME, 2006), Evans, Honkapohja & Mitra (JME, 2009). For more on Euler equation learning see Evans and McGough (2009).

# Determinacy and Stability under Learning

DETERMINACY

Combining IS, PC and the $i_t$ rule leads to a bivariate reduced form in $x_t$ and $\pi_t$.Letting $y_t' = (x_t, \pi_t)'$ and $v_t' = (g_t, u_t)'$ the model can be written

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = M \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + N \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + P \begin{pmatrix} g_t \\ u_t \end{pmatrix},$$

$$y_t = M E_t^* y_{t+1} + N y_{t-1} + P v_t.$$

If the model is "determinate" there exists a unique stationary REE of the form

$$y_t = \bar{b} y_{t-1} + \bar{c} v_t.$$

Determinacy condition: compare # of stable eigenvalues of matrix of stacked first-order system to # of predetermined variables. If "indeterminate" there are multiple solutions, which include stationary sunspot solutions.

# LEARNING

Under learning, agents have beliefs or a perceived law of motion (PLM)

$$y_t = a + by_{t-1} + cv_t,$$

where we now allow for an intercept, and estimate $(a_t, b_t, c_t)$ in period $t$ based on past data.

- Forecasts are computed from the estimated PLM.
- New data is generated according to the model with the given forecasts.
- Estimates are updated to $(a_{t+1}, b_{t+1}, c_{t+1})$ using least squares.

Question: when is it the case that

$$(a_t, b_t, c_t) \longrightarrow (0, \bar{b}, \bar{c})?$$

We determine this using E-stability

# E-STABILITY

Reduced form

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t.$$

Under the PLM (Perceived Law of Motion)

$$y_t = a + by_{t-1} + cv_t.$$

$$E_t^* y_{t+1} = (I + b)a + b^2 y_{t-1} + (bc + cF)v_t.$$

This $\longrightarrow$ ALM (Actual Law of Motion)

$$y_t = M(I + b)a + (Mb^2 + N)y_{t-1} + (Mbc + NcF + P)v_t.$$

This gives a **mapping from PLM to ALM:**

$$T(a,b,c) = (M(I+b)a, Mb^2 + N, Mbc + NcF + P).$$

The optimal REE is a fixed point of $T(a,b,c)$. If

$$d/d\tau(a,b,c) = T(a,b,c) - (a,b,c)$$

is locally asymptotically stable at the REE it is said to be **E-stable**. See EH, Chapter 10, for details. The **E-stability conditions** can be stated in terms of the derivative matrices

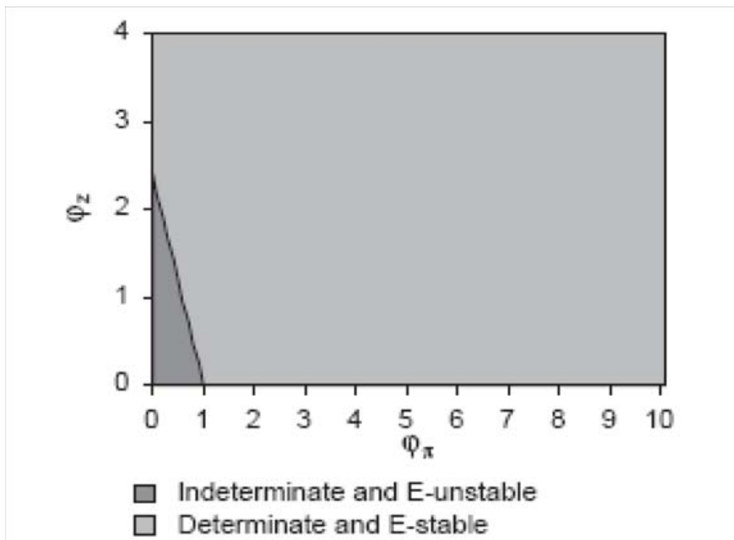
$$\begin{aligned} DT_a &= M(I+\bar{b}) \\ DT_b &= \bar{b}' \otimes M + I \otimes M\bar{b} \\ DT_c &= F' \otimes M + I \otimes M\bar{b}, \end{aligned}$$

where $\otimes$ denotes the Kronecker product and $\bar{b}$ denotes the REE value of $b$.

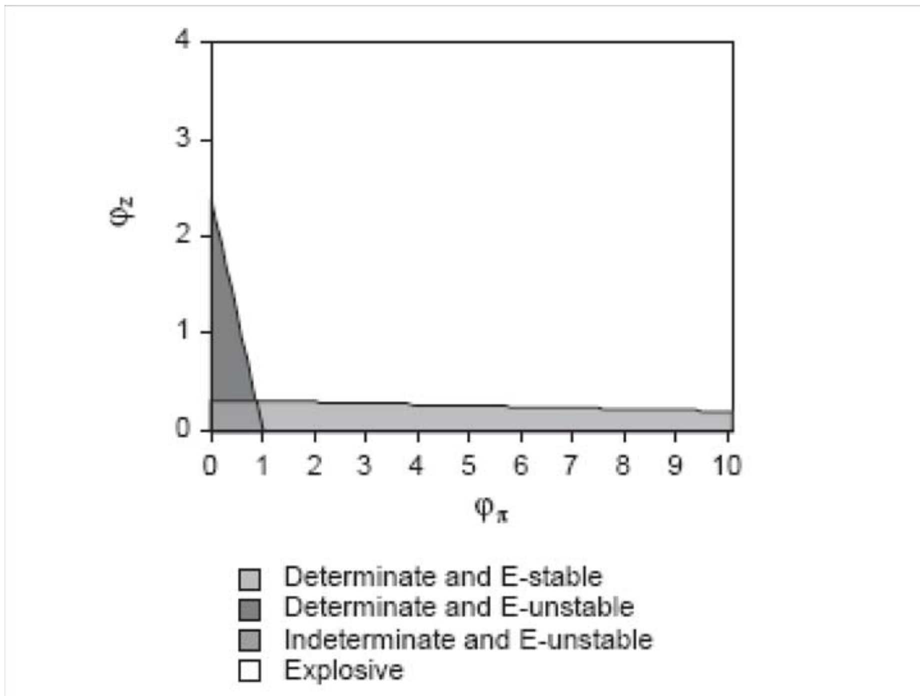
**E-stability governs stability under LS learning**.

Results for Taylor-rules in NK model (Bullard & Mitra, JME 2002)

$i_t = \chi_\pi \pi_t + \chi_x x_t$ yields **determinacy and stability** under LS learning **if** $\lambda(\chi_\pi - 1) + (1-\beta)\chi_x > 0$. Note that $\chi_\pi > 1$ is sufficient.



With $i_t = \chi_\pi \pi_{t-1} + \chi_x x_{t-1}$, **determinacy & E-stability** for $\chi_\pi > 1$ and $\chi_x > 0$ small. Also an **explosive region** ($\chi_\pi > 1$ and $\chi_x$ large) and a **determinate E-unstable** region ($\chi_\pi < 1$ and $\chi_x$ moderate).



For $i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}$, **determinacy & E-stability** for $\chi_\pi > 1$ and $\chi_x > 0$ small. **Indeterminate & E-stable** for $\chi_\pi > 1$ and $\chi_x$ large. Recent work has shown **stable sunspot solutions** in that region.